Engineering Electromagnetic Fields and Waves

Carl T. A. Johnk

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Professor of Electrical Engineering
University of Colorado, Boulder

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Preface

It might indeed be difficult to justify another text on undergraduate electromagnetic fields, except that this work grew out of some specific needs in the electrical engineering curriculum at the University of Colorado that might conceivably extend to similar needs existing elsewhere. Until a few years ago, the junior-level two-semester sequence in introductory field and wave-transmission theory covered static electric and magnetic fields during the first semester (employing a more conventional fields text written from the "historical" point of view), and then (using another text, or texts), the second semester covered principally the theory and applications of transmission lines and waveguides, plus an introduction to antenna theory and radiation. The problems arising from this scheme lay in the fact that in using two or more textbooks, the differences in approach and in symbolic notations contributed to a substantial loss of time required in relearning some of the ideas that should have carried over smoothly from the first semester's work. Moreover, in developing static fields from the historical approach (via the experimental Coulomb and Ampère laws), the understanding of the underlying Maxwell's equations was deferred until nearly the end of the first semester, judged by this author to be a major disadvantage.

The present text represents an effort to allay these difficulties. It is the culmination of several preliminary versions, used in the classroom for more than seven years at the University of Colorado and by colleagues at three other universities. Important features of this book might be summarized as follows:

1. Maxwell's equations are postulated for free space at the outset and then developed for material regions, along with the boundary conditions, within

the first three chapters. Applications to both static and time-varying problems illustrate this development. A full treatment of electrostatic and magnetostatic fields as special cases is then offered in Chapters 4 and 5, permitting a smooth transition to quasi-static time-varying fields in those chapters. This much material ordinarily comprises one semester's work.

2. Prerequisites assumed of the student are a freshman- or sophomore-level course covering differential and integral calculus. Vector analysis concepts as needed throughout this text are presented in the first two chapters. While the principal applications of vector ideas in this text involve the rectangular, circular cylindrical, and spherical coordinate systems, the generalized coordinate system is chosen for developing the concepts of dot and cross products as well as the gradient, divergence, and curl operators. Some years of experience with various approaches to vector analysis presentations as well as observations of students' responses have convinced the author that introducing these subjects in generalized form proves a time-saver by embracing the various coordinate systems within a single treatment.

3. In Chapters 4 and 5, on static and quasi-static electric and magnetic fields, the topics of capacitance and inductance are given detailed treatments. Besides the usual approaches via energy and voltage, capacitance is attacked by use of the flux-plotting method and extended to the capacitance-conductance analog. Self- and mutual inductances are given more than average consideration, with their energy definitions developed for both nonlinear and linear devices, together with seven detailed examples worked out in Chapter 5.

4. Chapters 6–11 are suitable for a second semester emphasizing waveguides and transmission lines plus an introduction to antennas. Chapter 6 represents a departure from the conventional manner in which transmission-line methods are usually broached. The student is introduced simultaneously to simple boundary-value problems of electromagnetics and to the analysis of wave-transmission systems via the problem of reflected and transmitted plane waves at normal incidence in a multilayered system. This generic plane-wave approach emphasizes the universality of impedance, reflection coefficient and Smith chart concepts. It has been found to provide additional insight into the wave structures of transmission lines with reflections, a topic considered in detail later in Chapters 9 and 10, while developing basic abilities in handling reflection and transmission problems of radio-wave and optical systems.

5. Chapter 7 gives an in-depth treatment of the real-time and complex forms of the Poynting theorem relative to electromagnetic energy and power, with applications to plane waves in lossless and lossy regions. A thoroughgoing discussion of rectangular hollow waveguide modes is found in Chapter 8, including the concept of group velocity and wall-loss attenuation. The TEM

waves of two-conductor transmission lines are described in Chapter 9, using static-field theory developed in Chapters 4 and 5 to derive line parameters for the lossless and lossy cases. Chapter 10 continues with reflections on transmission lines, drawing from the background of Chapter 6 and including additional applications of the Smith chart, in both impedance and admittance forms, to standing-wave and impedance-matching problems. A consideration of time-domain nonsinusoidal wave reflections on lossless lines rounds out the chapter.

6. In Chapter 11, several aspects of antenna radiation are covered in greater depth than in most texts at this level. These include the Green's theorem to develop the radiation integral, Pocklington's theorem for current distributions on thin wires, the radiation from a center-fed dipole, and the use of the equivalence theorem to find the radiation fields of aperture sources such as horn antennas and lasers.

7. An effort has been made to achieve a balance between the depth of presentation of the theoretical background and the applications via solved problems. Numerous worked-out examples throughout the text provide the student with a useful self-study aid, while giving the instructor greater flexibility in his classroom presentations.

A new book must invariably draw from the works of many authors; here a clear indebtedness to the authors listed in the references should be mentioned. Special gratitude to two of my former teachers, J. L. Glathart and E. C. Jordan, from whom much of my early encouragement was derived, is acknowledged.

While preparing the several earlier versions of this book, many discussions with colleagues and students were of inestimable benefit. Comments by Robert Bond, Ivar Pearson, James Lindsay, Ray King, Paul Klock, David Chang, and Ezekiel Bahar were most helpful, as well as those of anonymous reviewers. The development of this text has been quite rewarding, due in great part to the unflagging spirit of my students, whose remarks have been greatly appreciated. Also acknowledged are the encouragement and support of F. S. Barnes, whose leadership, vision, and indefatigability as Chairman of the Electrical Engineering Department have added materially to this text.

Special thanks go to Mrs. Charlotte Beeson and Mrs. Marie Krenz for their excellent typing efforts. Lastly, the author would like to thank any readers who forward corrections or suggestions for improvements.

Boulder, Colorado *Carl T. A. Johnk*

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Vector Analysis and Electromagnetic Fields in Free Space

The important beginnings of vector analysis as a branch of mathematics date back to the middle of the nineteenth century, and since that time it has developed into an essential ingredient of the background of the physical scientist. The object of the treatment of vector analysis contained in the first two chapters is to serve the needs of the remainder of the book. Scalar and vector products, and certain integral processes involving vectors are developed, providing a groundwork for the Lorentz force effects which define the electric and magnetic fields, and for the postulated Maxwell integral relations among these fields in free space. Attention is focused on the generalized orthogonal coordinate system, with examples framed in the more common cartesian, circular cylindrical, and spherical systems.

1-1 Scalar and Vector Fields

A field is taken to mean a mathematical function of space and time. Fields can be classified as *scalar* or *vector* fields. A scalar field is a function having, at each instant in time, an assignable magnitude at every point of a region in space. Thus, the temperature field $T(x, y, z, t)$ inside the block of material of Figure 1-1(*a*) is a scalar field. To each point $P(x, y, z)$ there exists a corresponding temperature $T(x, y, z, t)$ at any instant t in time. The velocity of a fluid moving inside the pipe shown in Figure 1-1(*b*) illustrates a vector field.

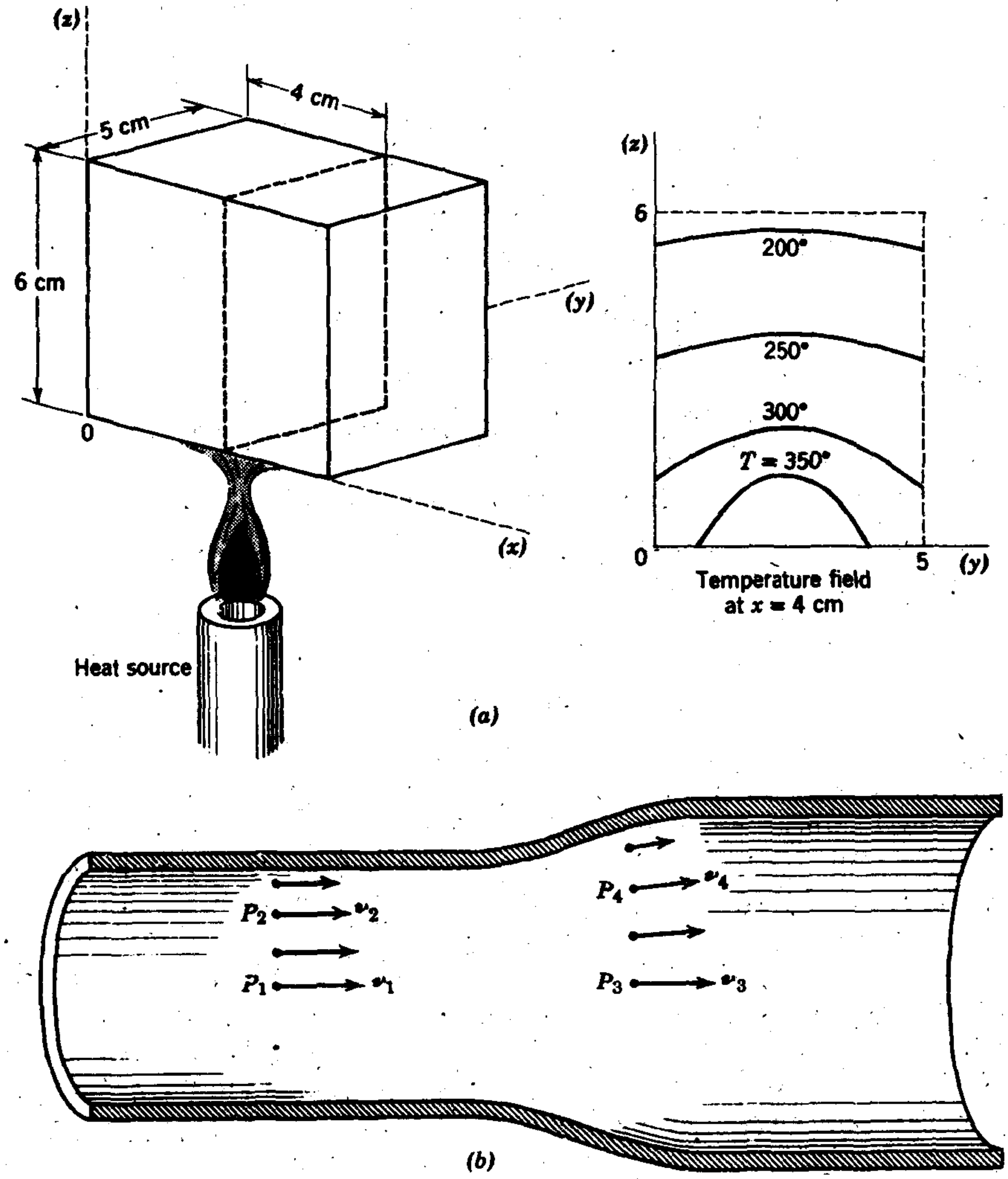

Figure 1-1. Examples of scalar and vector fields. (*a*) Temperature field inside a block of material. (*b*) Fluid velocity field inside a pipe of changing cross-section.

A variable direction, as well as magnitude, of the fluid velocity occurs in the pipe where the cross-sectional area is changing. Other examples of scalar fields are mass, density, pressure, and gravitational potential. A force field, a velocity field, and an acceleration field are examples of vector fields.

The mathematical symbol for a scalar quantity is taken to be any letter: for example, A, T, a, f. The symbol for a vector quantity is any letter set in

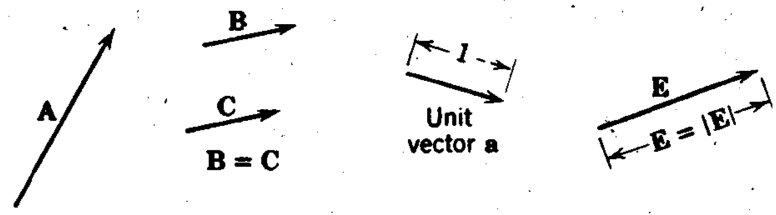

Figure 1-2. Graphic representations of a vector, equal vectors, a unit vector, and the representation of magnitude or length of a vector.

boldface roman type, for example, **A**, **H**, **a**, **g**. Vector quantities are represented graphically by means of arrows, or directed line segments, as shown in Figure 1-2. The magnitude or length of a vector **A** is written |**A**| or simply A, a positive real scalar. The *negative* of a vector is that vector taken in an opposing direction, with its arrowhead on the opposite end. A *unit vector* is any vector having a magnitude of unity. The symbol **a** is used to denote a unit vector, with a subscript employed to specify a special direction. For example, $\mathbf{a}_x$ means a unit vector having the positive-x direction. Two vectors are said to be *equal* if they have the same direction and the same magnitude. (They need not be collinear, but only parallel to each other.)

1-2 Vector Sums

The vector sum of **A** and **B** is defined in relation to the graphic sketch of the vectors, as in Figure 1-3. A physical illustration of the vector sum occurs in combining displacements in space. Thus, if a particle were displaced consecutively by the vector distance **A** and then by **B**, its final position would be denoted by the vector sum **A** + **B** = **C** shown in Figure 1-3(*a*). Reversing the order of these displacements provides the same vector sum **C**, so that

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \tag{1-1}$$

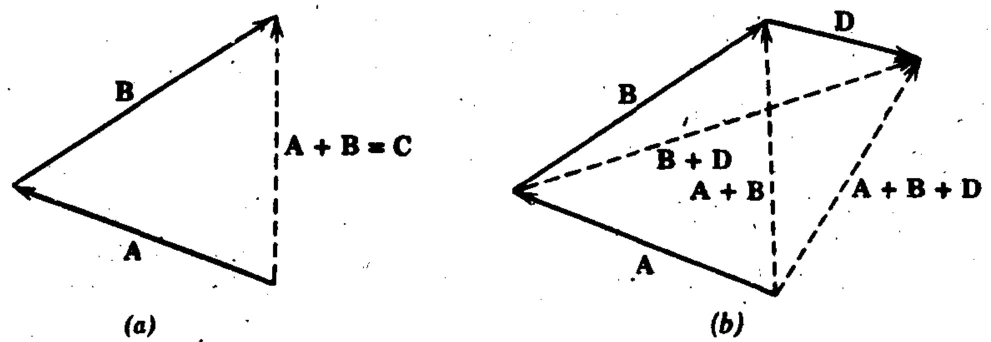

Figure 1-3. (*a*) The graphic definition of the sum of two vectors. (*b*) The associative law of addition.

the commutative law of the addition of vectors. If several vectors are to be added, an associative law

$$(\mathbf{A} + \mathbf{B}) + \mathbf{D} = \mathbf{A} + (\mathbf{B} + \mathbf{D}) \tag{1-2}$$

follows from the definition of vector sum and from Figure 1-3(*b*).

1-3 Product of a Vector and a Scalar

If a scalar quantity is denoted by u and if $\mathbf{B}$ denotes a vector quantity, their product $u\mathbf{B}$ means a vector having a magnitude u times the magnitude of $\mathbf{B}$, and having the same direction as $\mathbf{B}$ if u is a positive scalar, or the opposite direction if u is negative. The following laws hold for the products of vectors and scalars:

$$u\mathbf{B} = \mathbf{B}u \qquad \text{Commutative law} \tag{1-3}$$

$$u(v\mathbf{A}) = (uv)\mathbf{A} \qquad \text{Associative law} \tag{1-4}$$

$$(u + v)\mathbf{A} = u\mathbf{A} + v\mathbf{A} \qquad \text{Distributive law} \tag{1-5}$$

$$u(\mathbf{A} + \mathbf{B}) = u\mathbf{A} + u\mathbf{B} \qquad \text{Distributive law} \tag{1-6}$$

1-4 Coordinate Systems

The solution of physical problems often requires that the framework of a coordinate system be introduced, particularly if explicit solutions are being sought. The system most familiar to engineers and scientists is the cartesian, or *rectangular* coordinate system, although two other frames of reference often used are the *circular cylindrical* and the *spherical* coordinate systems. The symbols employed for the independent coordinate variables of these orthogonal systems are listed as follows:

1. Rectangular coordinates: (x, y, z)
2. Circular cylindrical coordinates: (ρ, ϕ, z)
3. Spherical coordinates: (r, θ, ϕ)

A useful application of the product of a vector and a scalar occurs in the specification of a vector quantity in terms of its components. The present discussion of this idea is limited to orthogonal coordinate systems. A typical point P is identified in space with each of the common coordinate systems in Figure 1-4(*a*). At that point, the unit vectors of each system are defined to lie in the positive increasing direction of the appropriate coordinate variable, as shown in Figure 1-4(*b*). The symbol $\mathbf{a}$ subscripted with the desired coordinate variable is used to denote the *unit vectors* of a particular coordinate

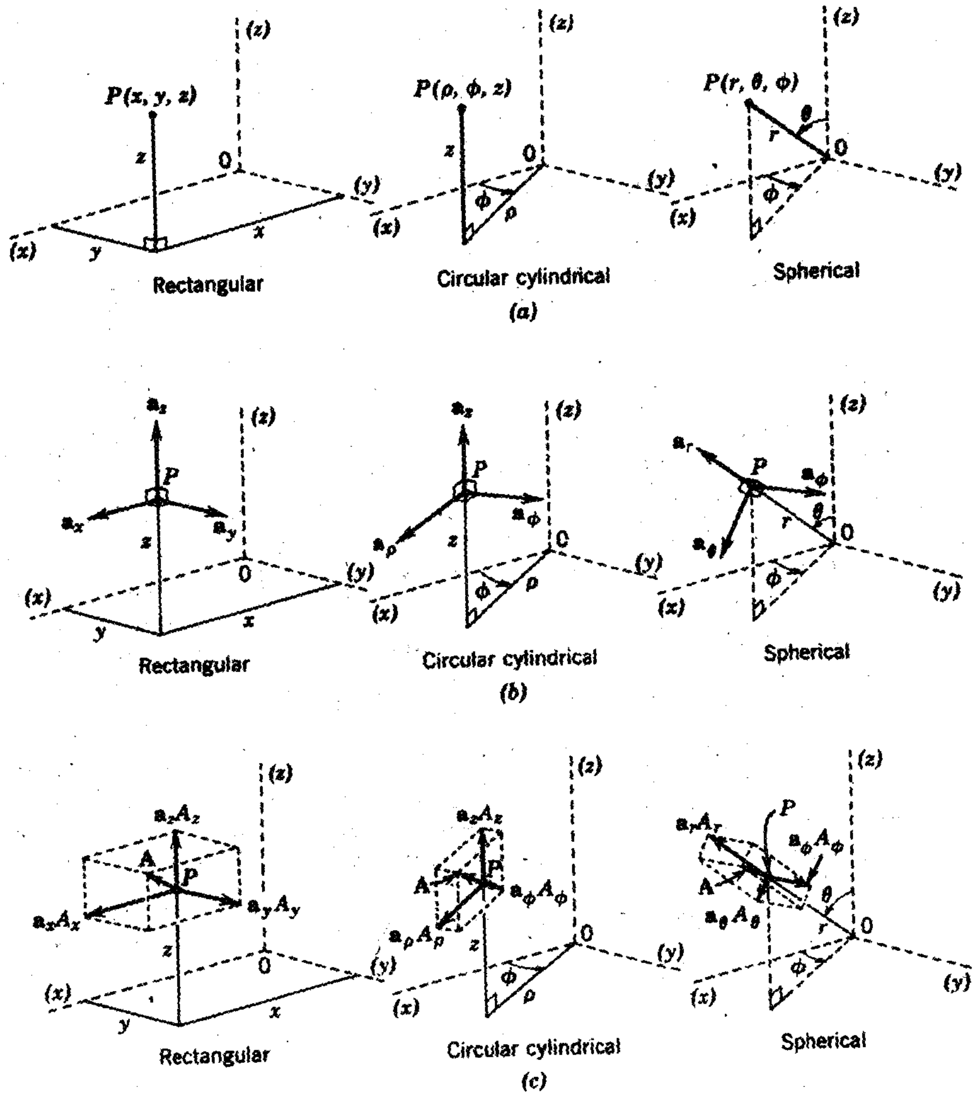

Figure 1-4. Notational conventions adopted in the three common coordinate systems. (*a*) Location of a point P in space. (*b*) The unit vectors at the typical point P. (*c*) The resolution of a vector **A** into its orthogonal components.

system. Thus $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ are the unit vectors in the rectangular system. In (*c*) of that figure is shown a typical vector A resolved into its components† in each of those coordinate systems, denoted symbolically as follows:

$$
\begin{aligned}
\mathbf{A} &= \mathbf{a}_x A_x + \mathbf{a}_y A_y + \mathbf{a}_z A_z && \text{Rectangular} \\
\mathbf{A} &= \mathbf{a}_\rho A_\rho + \mathbf{a}_\phi A_\phi + \mathbf{a}_z A_z && \text{Circular cylindrical} \qquad (1\text{-}7) \\
\mathbf{A} &= \mathbf{a}_r A_r + \mathbf{a}_\theta A_\theta + \mathbf{a}_\phi A_\phi && \text{Spherical}
\end{aligned}
$$

while the magnitude (length) is given by:

$$
\begin{aligned}
A &= [A_x^2 + A_y^2 + A_z^2]^{1/2} && \text{Rectangular} \\
A &= [A_\rho^2 + A_\phi^2 + A_z^2]^{1/2} && \text{Circular cylindrical} \qquad (1\text{-}8) \\
A &= [A_r^2 + A_\theta^2 + A_\phi^2]^{1/2} && \text{Spherical}
\end{aligned}
$$

For the sake of unifying and compacting the subsequent developments concerning scalar and vector fields, a system of *generalized* orthogonal coordinates is introduced, in which u_1, u_2, u_3 denote the coordinate variables. The typical point $P(u_1, u_2, u_3)$ in space is the intersection of the three constant surfaces $u_1 = C_1$, $u_2 = C_2$, and $u_3 = C_3$. The intersections of pairs of these surfaces define the *coordinate lines*; e.g., the coordinate line labelled u_1 is defined by the surfaces $u_2 = C_2$ and $u_3 = C_3$ shown. These ideas are exemplified in Figure 1-5 by the three common coordinate systems; thus, in spherical coordinates, the intersection of the coordinate surfaces r = constant and θ = constant is a circle.

The unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are mutually perpendicular and lie tangent to the coordinate lines through the typical point P of Figure 1-5. If a vector A is associated with the point $P(u_1, u_2, u_3)$ in that figure, it may be expressed symbolically in terms of its generalized orthogonal components by

$$\mathbf{A} = \mathbf{a}_1 A_1 + \mathbf{a}_2 A_2 + \mathbf{a}_3 A_3 \quad \text{Generalized orthogonal} \qquad (1\text{-}9)$$

its magnitude being

$$A = [A_1^2 + A_2^2 + A_3^2]^{1/2} \qquad (1\text{-}10)$$

The scalars A_1, A_2, and A_3 are called the *components* of the vector A. Examples of these expressions specialized to the three common coordinate systems have already been given in (1-7) and (1-8).

† Thus, the *components* of A in the rectangular coordinate system are the vectors $\mathbf{a}_x A_x$, $\mathbf{a}_y A_y$, and $\mathbf{a}_z A_z$. Another common usage is to refer to only the scalar multipliers A_x, A_y, and A_z as the components of A, although it may be considered more proper to call these scalars the *projections* of A onto the respective coordinate axes.