

Mesoscale Meteorological Modeling

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Preface

The purpose of this monograph is to provide an overview of mesoscale numerical modeling, beginning with the fundamental physical conservation relations. An overview of the individual chapters is given in the introduction. This book is an outgrowth of my article entitled "Mesoscale numerical modeling," which appeared in Volume 23 of *Advances in Geophysics*.

The philosophy of the book is to start from basic principles as much as possible when explaining specific subtopics in mesoscale modeling. Where too much preliminary work is needed, however, references to other published sources are given so that a reader can obtain the complete derivation (including assumptions). Often only an investigator's recent work is listed; however, once that source is found it is straightforward to refer to his or her earlier work, if necessary, by using the published reference list appearing in that paper. An understanding of the assumptions upon which the mathematical relations used in mesoscale modeling are developed is essential for fluency in this subject. To address as wide an audience as possible, basic material is provided for the beginner as well as a more in-depth treatment for the specialist.

The author wishes to acknowledge the contributions of a widely proficient group of people who provided suggestions and comments during the preparation of the book. The reading of all or part of the draft material for this text was required for a course in mesoscale meteorological modeling taught at the University of Virginia and at Colorado State University. Among the students in that course who provided significant suggestions and corrections are Raymond Arritt, David Bader, Charles Cohen, Omar Lucero, Jeffery McQueen, Charles Martin, Jenn-Luen Song, Craig Tremback, James Toth, and George Young. P. Flatau is acknowledged for acquainting me with several Soviet works of relevance to mesoscale me-

teorology. Suggestions and aid were also provided by faculty members in the Atmospheric Science Department at Colorado State University, including Duane E. Stevens, Richard H. Johnson, Wayne H. Schubert, and Richard Pearson, Jr.

Several chapters were also sent to a number of acknowledged experts in certain aspects of mesoscale meteorology. These scientists included André Doneaud (Chapters 1–5, 7, and 8), George Young (Chapter 5), Tzvi Gal-Chen (Chapter 6), Raymond Arritt (Chapter 7), Richard T. McNider (Chapter 7), Steven Ackerman (Chapter 8), Andrew Goorch (Chapter 8), Larry Freeman (Chapter 8), Michael Fritsch (Chapter 9), William Frank (Chapter 9), Jenn-Luen Song (Chapter 9), R. D. Farley (Chapter 9), Harold Orville (Chapter 9), Robert Lee (Chapter 10), Mike McCumber (Chapter 11), Joseph Klemp (Chapter 12), and Mordecai Segal (Chapters 2, 3, 10, 11, and 12), and Robert Kessler (Chapter 12). For their help in reviewing the material I am deeply grateful.

I would also like to thank the individuals who contributed to the summary tabulation of models in Appendix B. Although undoubtedly not a comprehensive list (since not every modeling group responded or could be contacted), it should provide a perspective of current mesoscale modeling capabilities.

I would also like to acknowledge the inspiration of William R. Cotton and Joanne Simpson, who facilitated my entry into the field of mesoscale meteorology. In teaching the material in this text and in supervising graduate research, I have sought to adopt their philosophy of providing students with the maximum opportunity to perform independent, innovative investigations. I would also like to give special thanks to André Doneaud and Mordecai Segal, whose patient, conscientious reading of portions of the manuscript has significantly strengthened the text. In addition, I would like to express my sincere appreciation to Thomas H. Vonder Haar who provided me with an effective research environment in which to complete the preparation of this book.

In writing the monograph, I have speculated in topic areas in which there has been no extensive work in mesoscale meteorology. These speculative discussions, most frequent in the sections on radiative effects, particularly in polluted air masses, also occur in a number of places in the chapters on parameterization, methods of solution, boundary and initial conditions, and model evaluation. Such speculation is risky, of course, because the extensive scientific investigation required to validate a particular approach has not yet been accomplished. Nevertheless, I believe such discussions are required to complete the framework of the text and perhaps may be useful in providing some direction to future work. The introduction of this material is successful if it leads to new insight into the field of mesoscale modeling.

Finally, the writing of a monograph or textbook inevitably results in errors, for which I must assume final responsibility. It is hoped that they will not significantly detract from the usefulness of the book and that the reader will benefit positively from ferreting out mistakes. In any case, I would appreciate comments from users about errors of any sort, including the neglect of relevant current work.

The drafts and final manuscripts were typed by the very capable Ann Gaynor, Susan Grimstedt, and Sara Rumley. Their contribution in proof-reading the material to achieve a manuscript with a minimal number of errors cannot be overstated. The drafting was completed ably by Jinte Kelbe, Teresita Arritt, and Judy Sorbie. Portions of the costs of preparing this monograph were provided by the Atmospheric Science Section of the National Science Foundation under Grants ATM 81-00514 and ATM 82-42931 and ATM-8304042 and that support is gratefully acknowledged.

Finally and most importantly, I would like to acknowledge the support of my family—Gloria, Tara; and Roger Jr.—in completing this time-consuming and difficult task.

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Chapter 1

Introduction

To utilize mesoscale dynamical simulations of the atmosphere effectively, it is necessary to understand the basic physical and mathematical foundations of the models and to have an appreciation of how the particular atmospheric system of interest works. This text provides such an overview of the field and should be of use to the practitioner as well as to the researcher of mesoscale phenomena. Because the book starts from fundamental concepts, it should be possible to use the text to evaluate the scientific basis of any simulation model that has been or will be developed.

Mesoscale can be descriptively defined as having a temporal and a horizontal spatial scale smaller than the conventional rawinsonde network, but significantly larger than individual cumulus clouds. This implies that the horizontal scale is on the order of a few kilometers to several hundred kilometers or so, with a time scale of about 1 to 12 h. The vertical scale extends from tens of meters to the depth of the troposphere. Clearly, this is a somewhat arbitrary limit; however, the shorter scale corresponds to atmospheric features that, for weather forecasting purposes, can only be described statistically, whereas the longer limit corresponds to the smallest features we can generally distinguish on a synoptic weather map. Mesoscale can also be applied to those atmospheric systems that have a horizontal extent large enough for the hydrostatic approximation to the vertical pressure distribution to be valid, yet small enough for the geostrophic and gradient winds to be inappropriate as approximations to the actual wind circulation above the planetary boundary layer. This scale of interest, then, along with computer and cost limitations, defines the domain and grid sizes of mesoscale models. In this text examples of specific circulations will be presented, illustrating scales of mesoscale circulations.

In this text the outline of material is as follows. In Chapters 2 and 3 the fundamental conservation relations are introduced and appropriate simplifications given. In Chapter 4 the equations are averaged to conform to a

mesoscale model grid mesh. In Chapter 5 types of models are discussed and their advantages and disadvantages to properly simulate mesoscale phenomena presented. The transformation of the equations to a generalized coordinate representation is given in Chapter 6, and the parameterizations in a mesoscale model of the planetary boundary layer, electromagnetic radiation, and moist thermodynamics are introduced in Chapters 7–9. In Chapter 10 methods of solution are illustrated, and boundary and initial conditions and grid structure are discussed in Chapter 11. The procedure for evaluating models is given in Chapter 12. Finally, examples of mesoscale simulations of particular mesoscale phenomena are provided in Chapter 13.

Chapter 2

Basic Set of Equations

The foundation for any model is a set of conservation principles. For mesoscale atmospheric models these principles are

- (1) conservation of mass,
- (2) conservation of heat,
- (3) conservation of motion,
- (4) conservation of water, and
- (5) conservation of other gaseous and aerosol materials.

These principles form a coupled set of relations that must be satisfied simultaneously and that include sources and sinks in the individual expressions.

The corresponding mathematical representations of these principles for atmospheric applications are developed as follows.

2.1 Conservation of Mass

In the earth's atmosphere, mass is assumed to have neither sinks nor sources.¹ Stated another way, this concept requires that the mass into and out of an infinitesimal box must be equal to the change of mass in the box. Such a volume is sketched in Fig. 2-1, where $\rho u|_1 \delta y \delta z$ is the mass flux into the left side and $\rho u|_2 \delta y \delta z$ the mass flux out of the right side. The symbols δx , δy , and δz represent the perpendicular sides of the box, ρ the density, and u the velocity component normal to the $\delta z \delta y$ plane.

If the size of the box is sufficiently small, the change in mass flux across the box can be written as

$$\begin{aligned} [\rho u|_1 - \rho u|_2] \delta y \delta z &= \left[\rho u|_1 - \rho u|_1 - \frac{\partial \rho u}{\partial x} \bigg|_1 \delta x - \frac{1}{2} \frac{\partial^2 \rho u}{\partial x^2} \bigg|_1 (\delta x)^2 - \dots \right] \delta y \delta z \\ &= \frac{\delta M}{\delta t}, \end{aligned}$$

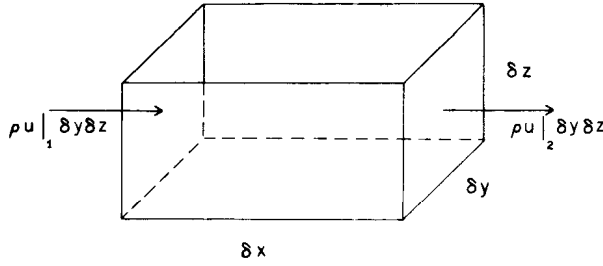


Fig. 2-1. A schematic of the volume used to derive the conservation of mass relation.

where $\rho u|_2$ has been written in terms of a one-dimensional Taylor series expansion and $\delta M/\delta t$ is the rate of increase or decrease of mass in the box. Neglecting terms in the series of order $(\delta x)^2$ and higher, this expression can be rewritten as

$$-\left. \frac{\partial \rho u}{\partial x} \right|_1 \delta x \delta y \delta z \simeq \frac{\delta M}{\delta t},$$

and since the mass M is equal to ρV (where $V = \delta x \delta y \delta z$ is the volume of the box), this expression can be rewritten as

$$-\left. \frac{\partial \rho u}{\partial x} \right|_1 \delta x \delta y \delta z \simeq V \frac{\delta \rho}{\delta t},$$

assuming the volume is constant with time.

If the mass flux through the sides $\delta x \delta y$ and $\delta x \delta z$ is considered in a similar fashion, the complete equation for mass flux in the box can be written as

$$-\left. \frac{\partial}{\partial x} \rho u \right|_1 \delta x \delta y \delta z - \left. \frac{\partial}{\partial y} \rho v \right|_1 \delta x \delta y \delta z - \left. \frac{\partial}{\partial z} \rho w \right|_1 \delta x \delta y \delta z \simeq V \frac{\delta \rho}{\delta t},$$

and dividing by volume, the resulting equation is

$$-\left. \frac{\partial \rho u}{\partial x} \right|_1 - \left. \frac{\partial \rho v}{\partial y} \right|_1 - \left. \frac{\partial \rho w}{\partial z} \right|_1 \simeq \frac{\delta \rho}{\delta t}.$$

If the time and spatial increments are taken to zero in the limit, then

$$\lim_{\substack{\delta x \rightarrow 0, \delta y \rightarrow 0 \\ \delta z \rightarrow 0, \delta t \rightarrow 0}} \left(-\left. \frac{\partial \rho u}{\partial x} \right|_1 - \left. \frac{\partial \rho v}{\partial y} \right|_1 - \left. \frac{\partial \rho w}{\partial z} \right|_1 \right) = \lim_{\substack{\delta x \rightarrow 0, \delta y \rightarrow 0 \\ \delta z \rightarrow 0, \delta t \rightarrow 0}} \frac{\delta \rho}{\delta t},$$

since the remainder of the terms in the Taylor series expansion contain δx , δy , or δz .

Written in an equivalent fashion,

$$-\left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right] = \frac{\partial \rho}{\partial t}, \quad (2-1)$$

where the subscript 1 has been dropped because the volume of the box has gone to zero in the limit. Expression (2-1) is the mathematical statement of the conservation of mass. It is also called the *continuity equation*. In vector notation it is written as

$$-(\nabla \cdot \rho \mathbf{V}) = \partial \rho / \partial t \quad (2-2)$$

2.2 Conservation of Heat

The atmosphere on the mesoscale behaves very much like an ideal gas and is considered to be in local *thermodynamic equilibrium*.² The first law of thermodynamics for the atmosphere states that differential changes in heat content dQ are equal to the sum of differential work performed by an object dW and differential increases in internal energy dI . Expressed more formally, the first law of thermodynamics states that

$$dQ = dW + dI. \quad (2-3)$$

If we represent the region over which (2-3) applies as a box (Fig. 2-2), with volume $\delta x \delta y \delta z$, then an incremental increase in the x direction, caused by a

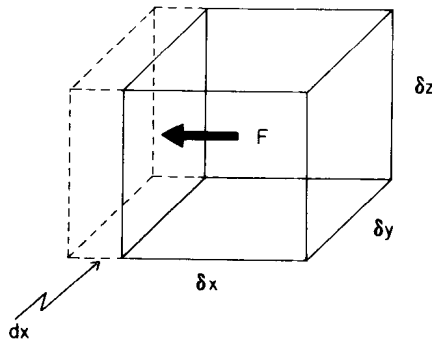


Fig. 2.2. A schematic of the change in size of a volume of gas resulting from a force F exerted over the surface $\delta z \delta y$.

force F , can be expressed as

$$dW = F dx,$$

and since force can be expressed as a pressure P exerted over an area $\delta y \delta z$,

$$dW = p \delta y \delta z dx. \quad (2-4)$$

The term $\delta y \delta z dx$ represents a change in volume dV , so that (2-4) can be rewritten as

$$dW = p dV,$$

For a unit mass of material it is convenient to rewrite the expression as

$$dw = p d\alpha, \quad (2-5)$$

where α is the specific volume (i.e., volume per unit mass). In an ideal gas, which the atmosphere closely approximates as discussed subsequently, the pressure in (2-5) is exerted uniformly on all sides of the gas volume.

The expression for work in (2-3) could also have included external work performed by such processes as chemical reactions, phase changes, or electromagnetism; however, these effects are not included in this derivation of work.

The ideal gas law, referred to previously, was derived from observations of the behavior of gases at different pressures, temperatures, and volumes. Investigators in the seventeenth and eighteenth centuries found that, for a given gas, pressure times volume equals a constant at any fixed temperature (Boyle's law) and that pressure divided by temperature equals a constant at any fixed volume (Charles's law). These two relations can be stated more precisely as

$$p\alpha = F_1(T) \quad (2-6)$$

and

$$p/T = F_2(\alpha), \quad (2-7)$$

where a unit mass of gas is assumed. If (2-6) is divided by T and (2-7) multiplied by α , then

$$p\alpha/T = (1/T)F_1(T) = \alpha F_2(\alpha), \quad (2-8)$$

Since the two right-hand expressions are functions of two different variables, the entire expression must be equal to a constant, conventionally denoted as R . Thus (2-8) is written as

$$p\alpha/T = R, \quad (2-9)$$

where R has been found to be a function of the chemical composition of the

gas. The extent to which actual gases obey (2-9) specifies how closely they approximate an ideal gas.

The value of the gas constant R for different gases is determined using Avogadro's hypothesis that at a given temperature and pressure gases containing the same number of molecules occupy the same volume. From experimental work, for example, it has been shown that at a pressure of 1 atm ($P_0 = 1014$ mb) and a temperature of $T_0 = 273$ K, 22.4 kliter of a gas (V_0) will have a mass in kilograms equal to the molecular weight of the gas μ . This quantity of gas is defined as 1 kmol.

Using this information, the ideal gas law (2-9) and the definition $\alpha_0 = V_0/\mu$, then

$$P_0 V_0 / \mu T_0 = R,$$

or by definition

$$P_0 V_0 / \mu T_0 = R \equiv R^* / \mu, \quad [\text{so that } P_0 V_0 / T_0 = R^*], \quad (2-10)$$

where R^* is called the universal gas constant and μ has units of kilograms per kilomole. From experiments, $R^* = 8.3143 \times 10^3 \text{ J K}^{-1} \text{ kmol}^{-1}$. Since (2-10) is valid for any combination of pressure, temperature, and volume,

$$p\alpha/T = R = R^* / \mu. \quad (2-11)$$

In the atmosphere, the apparent molecular weight of air μ_{atm} is determined by the fractional contribution by mass of each component gas (Table I) from the equation

$$\mu_{\text{atm}} = \sum_{i=1}^N m_i \left/ \sum_{i=1}^N (m_i / \mu_i) \right.,$$

where m_i is the fractional contribution by mass of the N individual gases in the atmosphere ($\sum_{i=1}^N m_i = 1$) and μ_i represents their respective molecular weights.³ For the gaseous components in Table I, excluding water vapor,

TABLE 2-1
Molecular Weight and Fractional Contribution by Mass of Major Gaseous
Components of the Atmosphere^a

Gas	Molecular weight	Fractional contribution by mass
N ₂	28.016	.7551
O ₂	32.00	.2314
Ar	39.94	.0128
H ₂ O	18.02	variable

^a From Wallace and Hobbs (1977).