BRIAN D. O. ANDERSON
JOHN B. MOORE

Optimal Filtering

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OPTIMAL FILTERING

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PREFACE

This book is a graduate level text which goes beyond and augments the undergraduate exposure engineering students might have to signal processing; particularly, communication systems and digital filtering theory. The material covered in this book is vital for students in the fields of control and communications and relevant to students in such diverse areas as statistics, economics, bioengineering and operations research. The subject matter requires the student to work with linear system theory results and elementary concepts in stochastic processes which are generally assumed at graduate level. However, this book is appropriate at the senior year undergraduate level for students with background in these areas.

Certainly the book contains more material than is usually taught in one semester, so that for a one semester or quarter length course, the first three chapters (dealing with the rudiments of Kalman filtering) can be covered first, followed by a selection from later chapters. The chapters following Chapter 3 build in the main on the ideas in Chapters 1, 2 and 3, rather than on all preceding chapters. They cover a miscellany of topics; for example, time-invariant filters, smoothing, and nonlinear filters. Although there is a significant benefit in proceeding through the chapters in sequence, this is not essential, as has been shown by the authors' experience in teaching this course.

The pedagogical feature of the book most likely to startle the reader is the concentration on discrete-time filtering. Recent technological developments as well as the easier path offered students and instructors are the two reasons for this course of action. Much of the material of the book has been with us in one form or another for ten to fifteen years, although again, much is relatively recent. This recent work has given new perspectives on the earlier material; for example, the notion of the innovations process provides helpful insights in deriving the Kalman filter.

We acknowledge the research support funding of the Australian Research Grants Committee and the Australian Radio Research Board. We are indebted also for specific suggestions from colleagues, Dr. G. Goodwin and Dr. A. Cantoni; joint research activities with former Ph.D. students Peter Tam and Surapong Chirarattananon; and to the typing expertise of Dianne Piefke. We have appreciated discussions in the area of optimal filtering with many scholars including Professors K. Åstrom, T. Kailath, D. Mayne, J. Meditch and J. Melsa.

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CHAPTER 1

INTRODUCTION

1.1 FILTERING

Filtering in one form or another has been with us for a very long time. For many centuries, man has attempted to remove the more visible of the impurities in his water by filtering, and one dictionary gives a first meaning for the noun filter as "a contrivance for freeing liquids from suspended impurities, especially by passing them through strata of sand, charcoal, etc."

Modern usage of the word filter often involves more abstract entities than fluids with suspended impurities. There is usually however the notion of something passing a barrier: one speaks of news filtering out of the war zone, or sunlight filtering through the trees. Sometimes the barrier is interposed by man for the purpose of sorting out something that is desired from something else with which it is contaminated. One example is of course provided by water purification; the use of an ultraviolet filter on a camera provides another example. When the entities involved are signals, such as electrical voltages, the barrier—in the form perhaps of an electric network—becomes a filter in the sense of signal processing.

It is easy to think of engineering situations in which filtering of signals might be desired. Communication systems always have unwanted signals, or

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noise, entering into them. This is a fundamental fact of thermodynamics. The user of the system naturally tries to minimize the inaccuracies caused by the presence of this noise—by filtering. Again, in many control systems the control is derived by feedback, which involves processing measurements derived from the system. Frequently, these measurements will contain random inaccuracies or be contaminated by unwanted signals, and filtering is necessary in order to make the control close to that desired.

1.2 HISTORY OF SIGNAL FILTERING

Filters were originally seen as circuits or systems with frequency selective behaviour. The series or parallel tuned circuit is one of the most fundamental such circuits in electrical engineering, and as a "wave trap" was a crucial ingredient in early crystal sets. More sophisticated versions of this same idea are seen in the IF strip of most radio receivers; here, tuned circuits, coupled by transformers and amplifiers, are used to shape a passband of frequencies which are amplified, and a stopband where attenuation occurs.

Something more sophisticated than collections of tuned circuits is necessary for many applications, and as a result, there has grown up an extensive body of filter design theory. Some of the landmarks are constant k and m-derived filters [1], and, later, Butterworth filters, Chebyshev filters, and elliptical filters [2]. In more recent years, there has been extensive development of numerical algorithms for filter design. Specifications on amplitude and phase response characteristics are given, and, often with the aid of sophisticated computer-aided design packages which allow interactive operation, a filter is designed to meet these specifications. Normally, there are also constraints imposed on the filter structure which have to be met; these constraints may involve impedance levels, types of components, number of components, etc.

Nonlinear filters have also been used for many years. The simplest is the AM envelope detector [3], which is a combination of a diode and a low-pass filter. In a similar vein, an automatic gain control (AGC) circuit uses a low-pass filter and a nonlinear element [3]. The phase-locked-loop used for FM reception is another example of a nonlinear filter [4], and recently the use of Dolby® systems in tape recorders for signal-to-noise ratio enhancement has provided another living-room application of nonlinear filtering ideas.

The notion of a filter as a device processing continuous-time signals and possessing frequency selective behaviour has been stretched by two major developments.

The first such development is digital filtering [5–7], made possible by recent innovations in integrated circuit technology. Totally different circuit

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modules from those used in classical filters appear in digital filters, e.g., analog-to-digital and digital-to-analog converters, shift registers, read-only memories, even microprocessors. Therefore, though the ultimate goals of digital and classical filtering are the same, the practical aspects of digital filter construction bear little or no resemblance to the practical aspects of, say, m-derived filter construction. In digital filtering one no longer seeks to minimize the active element count, the size of inductors, the dissipation of the reactive elements, or the termination impedance mismatch. Instead, one may seek to minimize the word length, the round-off error, the number of wiring operations in construction, and the processing delay.

Aside from the possible cost benefits, there are other advantages of this new approach to filtering. Perhaps the most important is that the filter parameters can be set and maintained to a high order of precision, thereby achieving filter characteristics that could not normally be obtained reliably with classical filtering. Another advantage is that parameters can be easily reset or made adaptive with little extra cost. Again, some digital filters incorporating microprocessors can be time-shared to perform many simultaneous tasks effectively.

The second major development came with the application of statistical ideas to filtering problems [8–14] and was largely spurred by developments in theory. The classical approaches to filtering postulate, at least implicitly, that the useful signals lie in one frequency band and unwanted signals, normally termed noise, lie in another, though on occasions there can be overlap. The statistical approaches to filtering, on the other hand, postulate that certain statistical properties are possessed by the useful signal and unwanted noise. Measurements are available of the sum of the signal and noise, and the task is still to eliminate by some means as much of the noise as possible through processing of the measurements by a filter. The earliest statistical ideas of Wiener and Kolmogorov [8, 9] relate to processes with statistical properties which do not change with time, i.e., to stationary processes. For these processes it proved possible to relate the statistical properties of the useful signal and unwanted noise with their frequency domain properties. There is, thus, a conceptual link with classical filtering.

A significant aspect of the statistical approach is the definition of a measure of suitability or performance of a filter. Roughly the best filter is that which, on the average, has its output closest to the correct or useful signal. By constraining the filter to be linear and formulating the performance measure in terms of the filter impulse response and the given statistical properties of the signal and noise, it generally transpires that a unique impulse response corresponds to the best value of the measure of performance or suitability.

As noted above, the assumption that the underlying signal and noise processes are stationary is crucial to the Wiener and Kolmogorov theory. It was not until the late 1950s and early 1960s that a theory was developed that did not require this stationarity assumption [11–14]. The theory arose because of the inadequacy of the Wiener-Kolmogorov theory for coping with certain applications in which nonstationarity of the signal and/or noise was intrinsic to the problem. The new theory soon acquired the name *Kalman filter theory*.

Because the stationary theory was normally developed and thought of in frequency domain terms, while the nonstationary theory was naturally developed and thought of in time domain terms, the contact between the two theories initially seemed slight. Nevertheless, there is substantial contact, if for no other reason than that a stationary process is a particular type of nonstationary process; rapprochement of Wiener and Kalman filtering theory is now easily achieved.

As noted above, Kalman filtering theory was developed at a time when applications called for it, and the same comment is really true of the Wiener filtering theory. It is also pertinent to note that the problems of implementing Kalman filters and the problems of implementing Wiener filters were both consistent with the technology of their time. Wiener filters were implementable with amplifiers and time-invariant network elements such as resistors and capacitors, while Kalman filters could be implemented with digital integrated circuit modules.

The point of contact between the two recent streams of development, digital filtering and statistical filtering, comes when one is faced with the problem of implementing a discrete-time Kalman filter using digital hardware. Looking to the future, it would be clearly desirable to incorporate the practical constraints associated with digital filter realization into the mathematical statement of the statistical filtering problem. At the present time, however, this has not been done, and as a consequence, there is little contact between the two streams.

1.3 SUBJECT MATTER OF THIS BOOK

This book seeks to make a contribution to the evolutionary trend in statistical filtering described above, by presenting a hindsight view of the trend, and focusing on recent results which show promise for the future. The basic subject of the book is the Kalman filter. More specifically, the book starts with a presentation of discrete-time Kalman filtering theory and then explores a number of extensions of the basic ideas.

There are four important characteristics of the basic filter:

1. Operation in discrete time
2. Optimality

3. Linearity
4. Finite dimensionality

Let us discuss each of these characteristics in turn, keeping in mind that derivatives of the Kalman filter inherit most but not all of these characteristics.

Discrete-time operation. More and more signal processing is becoming digital. For this reason, it is just as important, if not more so, to understand discrete-time signal processing as it is to understand continuous-time signal processing. Another practical reason for preferring to concentrate on discrete-time processing is that discrete-time statistical filtering theory is much easier to learn first than continuous-time statistical filtering theory; this is because the theory of random sequences is much simpler than the theory of continuous-time random processes.

Optimality. An optimal filter is one that is best in a certain sense, and one would be a fool to take second best if the best is available. Therefore, provided one is happy with the criterion defining what is best, the argument for optimality is almost self-evident. There are, however, many secondary aspects to optimality, some of which we now list. Certain classes of optimal filters tend to be robust in their maintenance of performance standards when the quantities assumed for design purposes are not the same as the quantities encountered in operation. Optimal filters normally are free from stability problems. There are simple operational checks on an optimal filter when it is being used that indicate whether it is operating correctly. Optimal filters are probably easier to make adaptive to parameter changes than suboptimal filters.

There is, however, at least one potential disadvantage of an optimal filter, and that is complexity; frequently, it is possible to use a much less complex filter with but little sacrifice of performance. The question arises as to how such a filter might be found. One approach, which has proved itself in many situations, involves approximating the signal model by one that is simpler or less complex, obtaining the optimal filter for this less complex model, and using it for the original signal model, for which of course it is suboptimal. This approach may fail on several grounds: the resulting filter may still be too complex, or the amount of suboptimality may be unacceptably great. In this case, it can be very difficult to obtain a satisfactory filter of much less complexity than the optimal filter, even if one is known to exist, because theories for suboptimal design are in some ways much less developed than theories for optimal design.

Linearity. The arguments for concentrating on linear filtering are those of applicability and sound pedagogy. A great many applications involve linear systems with associated gaussian random processes; it transpires that the optimal filter in a minimum mean-square-error sense is then linear. Of course, many applications involve nonlinear systems and/or nongaussian random processes, and for these situations, the optimal filter is nonlinear. However, the plain fact of the matter is that optimal nonlinear filter design and implementation are very hard, if not impossible, in many instances. For this reason, a suboptimal linear filter may often be used as a substitute for an optimal nonlinear filter, or some form of nonlinear filter may be derived which is in some way a modification of a linear filter or, sometimes, a collection of linear filters. These approaches are developed in this book and follow our discussion of linear filtering, since one can hardly begin to study nonlinear filtering with any effectiveness without a knowledge of linear filtering.

Finite dimensionality. It turns out that finite-dimensional filters should be used when the processes being filtered are associated with finite-dimensional systems. Now most physical systems are not finite dimensional; however, almost all infinite-dimensional systems can be approximated by finite-dimensional systems, and this is generally what happens in the modeling process. The finite-dimensional modeling of the physical system then leads to an associated finite-dimensional filter. This filter will be suboptimal to the extent that the model of the physical system is in some measure an inaccurate reflection of physical reality. Why should one use a suboptimal filter? Though one can without too much difficulty discuss infinite-dimensional filtering problems in discrete time, and this we do in places in this book, finite-dimensional filters are very much to be preferred on two grounds: they are easier to design, and far easier to implement, than infinite-dimensional filters.

1.4 OUTLINE OF THE BOOK

The book falls naturally into three parts.

The first part of the book is devoted to the formulation and solution of the basic Kalman filtering problem. By the end of the first section of Chapter 3, the reader should know the fundamental Kalman filtering result, and by the end of Chapter 3, have seen it in use.

The second part of the book is concerned with a deeper examination of the operational and computational properties of the filter. For example, there is discussion of time-invariant filters, including special techniques for computing these filters, and filter stability; the Kalman filter is shown to have a signal-to-noise ratio enhancement property.

In the third part of the book, there are a number of developments taking off from the basic theory. For example, the topics of smoothers, nonlinear and adaptive filters, and spectral factorization are all covered.

There is also a collection of appendices to which the reader will probably refer on a number of occasions. These deal with probability theory and random processes, matrix theory, linear systems, and Lyapunov stability theory. By and large, we expect a reader to know some, but not all, of the material in these appendices. They are too concentrated in presentation to allow learning of the ideas from scratch. However, if they are consulted when a new idea is encountered, they will permit the reader to learn much, simply by using the ideas.

Last, we make the point that there are many ideas developed in the problems. Many are not routine.

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