The Finite Element Method Displayed

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FOREWORD

This book by my two colleagues of Compiègne is a welcome addition to the finite element literature. It is the result of many years of teaching the finite element methods in France and also in America. As a collaborator in the pedagogical experiments of Compiègne, the undersigned has seen early manuscripts of this book; it agrees well with his own views on what should be taught in such a course. It contains very concisely all that is needed to understand finite element methods in general.

The mathematical treatment is rigorously correct without being too sophisticated; this aspect should appeal to a vast readership. The choices of numerical algorithms for the solution of the discretized problems follow standard practice and are up to date. Implementation of the methods into computer codes is very well done and the reader should acquire valuable skills in writing engineering analysis software after studying the numerous examples in the text.

Finally it should be said that this book is written for those potential users of finite element methods who are not steeped in the theories of structural mechanics. It contains some structural mechanics applications but even the casual reader should have no problems with the concepts used in these examples.

Gilles Cantin
Monterey, June 1982

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INTRODUCTION

0.1 The Finite Element Method

0.1.1 *Generalities*

Modern engineering projects have become extremely complex, costly and subject to severe reliability and safety constraints. Space vehicles, aircraft and nuclear reactors are examples of projects where reliability and safety are of crucial importance. Other preoccupations of modern technology with the protection of the environment, the control of thermal, acoustical and chemical pollution, the management of natural resources like aquifers and weather predictions, have forced engineers to reexamine all methods of analysis. For a proper understanding, analysts need mathematical models that can be used to simulate the behaviour of complex physical systems. These models are then used during the design of the projects.

Engineering sciences (mechanics of solids, fluids, thermodynamics, etc.) allow physical systems to be described in the form of partial differential equations. Today, the finite element method has become the most popular method for solving such equations. The method coupled with developments in computer technology has successfully been applied to the solution of steady and transient problems in linear and non-linear regions for one-, two-, and three-dimensional domains. It can easily handle discontinuous geometrical shapes as well as material discontinuities.

The finite element discretization process, like the finite difference process, transforms partial differential equations into algebraic equations. It draws heavily on the following three disciplines:

— continuum mechanics, for the correct formulation of mathematical models;

— numerical analysis, for the elaboration of algorithmic solutions of the discretized equations;

— computer programming, to produce parametrized codes applicable to large classes of problems.

0.1.2 *Historical Evolution of the Method*

For the past hundred years structural mechanics has been used to analyse frames and trusses [1]. Stiffness matrices of bars and beams could be constructed using elementary strength of materials. The direct stiffness method then allowed the assembly of elementary matrices into global matrices of coefficients for the system of algebraic equations relating forces and displacements. Simple modifications of the global system of equations allowed boundary conditions to be satisfied. The solution of the system of linear equations gave the displacements at the nodes and the reactions at the supports. In the early nineteen-fifties, computers made it possible to solve structural problems very effectively, but the method was slowly accepted by industry. Turner, Clough, Martin, and Topp [2] introduced the finite element concepts in 1956. Almost simultaneously, Argyris and Kelsey [3] developed similar concepts in a series of publications on energy theorems. Courant [4], Hrennikoff [5] and McHenry [6] are also early precursors of the finite element methods.

During the nineteen-sixties, the finite element method became widely accepted, research was pursued simultaneously in various parts of the world in several directions:

— the method was reformulated as a special case of the weighted residual method [7–10];

— a wide variety of elements were developed including bending elements, curved elements and the isoparametric concept was introduced [11–15];

— the method was reorganized as a general method of solution for partial differential equations; its applicability to the solution of non-linear and dynamic problems of structures was amply demonstrated; extension in other domains, soil mechanics, fluid mechanics, thermodynamics, produced solutions to engineering problems hitherto intractable [14–26];

— a mathematical basis was established using concepts of functional analysis [27–28].

Starting in 1967, many books have been written on the finite element method [29–56]. The three editions of the book authored by Professor Zienkiewicz [30] received worldwide diffusion. Until now the books available in French were the translations of the books authored by Zienkiewicz [53] (second edition), Gallagher [54], Rockey *et al.* [55], as well as the books written by Absi [56] and Imbert [56a]. During the same period a number of journals devoted most of their pages to the finite element method [57–62].

0.1.3 *State of the Art*

The finite element method (FEM) has now been widely accepted for all kinds of structural engineering application in aerospace, aeronautics, naval architecture and nuclear-powered electrical generating stations. Fluid mechanics applications are currently being developed for studying tidal motions, thermal and chemical transport and diffusion problems as well as fluid structure interactions. A number of general purpose finite element computer codes have been successfully developed for industrial users in the field of solid and fluid mechanics. Without any pretence at being comprehensive we may mention: NASTRAN, ASKA, SAP, MARC, ANSYS, TITUS, ADINA [2, 65, 66, 67]. These codes were conceived to run on large computers. Today, many codes have been modified so they can run on mini- and microcomputers (Rammant in [25]).

A major defect of most codes has been the preparation of input information and the interpretation of voluminous outputs. The absence of control during executions of the various phases of a finite element analysis has always been felt to be a major problem as well as the ability to visualize what is being accomplished in the computer. Today the trend is to design software with facilities for model generation, program interaction and graphical display capabilities.

0.2 Object and Organization of the Book

0.2.1 *Teaching of the Finite Element Method*

Although the finite element method has gained wide acceptance and usage in industry, its teaching is not yet widespread. The method draws from many disciplines thereby complicating coherent teaching. Amongst the various skills to be mastered we may cite:

— deep understanding of the physical principles involved in the problem at hand;

— approximations by sub-domains and interpolation theory;

— mathematical modelling of principles involved leading to variational formulations; these are obtained using either energy principles or the method of weighted residuals;

— matrix algebra for the discretized formulation;

— numerical methods for integration, solution of systems of linear and non-linear algebraic equations;

— computer programming involving massive data files.

It is hard to conceive a balanced formation in all these diverse disciplines. Moreover, the necessary software adapted to finite element teaching, keeping at the same time essential features of a general purpose program, is still under active development. The teaching of the method requires so many prerequisites that it is almost invariably relegated to senior and graduate level courses.

0.2.2 *Objectives of the Book*

This text is an attempt to simplify the teaching of the finite element method to engineers. The mathematical preparation required is that commonly dispensed in an engineering undergraduate curriculum. Tables of formulas and constants are presented in sufficient detail to allow the reader to construct his own code.

After reading this text, the reader should be better equipped to use most of the large commercial finite element codes.

0.2.3 *Organization of the Book*

The book is divided into six chapters that are nearly independent of one another. The necessary numerical and programming techniques are contained in the body of the text.

Chapter 1

Approximation of continuous functions over sub-domains in terms of nodal values, introduction of interpolation concepts, reference elements and error estimations.

Chapter 2

Interpolation functions for classical elements in one, two, and three dimensions.

Chapter 3

Application of the weighted residual method for the construction of integral or variational forms from partial differential equations.

Chapter 4

Discretization of integral forms using matrix algebra. Fundamental element vectors and matrices and assemby techniques.

Chapter 5

Numerical methods needed to construct and solve linear and non-linear system of algebraic equations. Numerical methods of integration for propagation problems in the time domain. Matrix eigenvalue and eigenvector problem.

Chapter 6

FORTRAN programming techniques illustrated with the two programs (BBMEF) and (MEF).

Figure 0.1 shows the relationship between the various chapters. Note that Chapters 1, 3, and 4 are devoted to the fundamental concepts underlying the finite element method and that Chapters 2 and 5 are mostly devoted to prerequisite material. Examples of short FORTRAN programs are used throughout the book to illustrate the practical implementation of the method.