

---

**Introduction to**

**Statistical  
Quality  
Control**

**Douglas C. Montgomery**

29.403  
M727

---

**Introduction to**

**Statistical  
Quality  
Control**

**Douglas C. Montgomery**

Department of Mechanical Engineering  
University of Washington

**John Wiley & Sons**

New York Chichester Brisbane Toronto Singapore

Copyright © 1985, by John Wiley & Sons, Inc.

All rights reserved. Published simultaneously in Canada.

Reproduction or translation of any part of this work beyond that permitted by Sections 107 and 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful. Requests for permission or further information should be addressed to the Permissions Department, John Wiley & Sons.

***Library of Congress Cataloging in Publication Data:***

Montgomery, Douglas C.

Introduction to statistical quality control.

Bibliography: p. 489

Includes index.

1. Quality control—Statistical methods. 2. Process control—Statistical methods. I. Title.

TS156.M64 1985 620'.0045 84-22019

ISBN 0-471-80870-9

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

# Preface

This book is about the modern practice of statistical quality control. It provides comprehensive coverage from basic principles to state-of-the-art concepts and applications. The objective is to give the reader a sound understanding of the principles and the basis for applying those principles in a wide variety of product and nonproduct environments. While statistical techniques are used throughout, the book has a strong engineering orientation, and extensive knowledge of statistics is not a necessary prerequisite. Readers whose background includes a basic course in statistical methods will find the book easily accessible.

This book is an outgrowth of 15 years of teaching, research, and consulting in the assurance sciences. It is designed as a textbook for students in engineering, management, and technology in colleges and universities who are taking a first course in quality control. Such courses are often taught at the junior or senior level. I have also used the text materials extensively in training programs for professional practitioners, including manufacturing and development engineers, managers, procurement specialists, marketing personnel, technicians and laboratory analysts, inspectors, and operators. Many professionals have also used the material for self-study.

Chapter 1 is an introduction to quality assurance in the modern business and manufacturing environment. It deals with the facts that quality has become a major business strategy and that organizations with successful quality-assurance programs can increase their productivity, enhance their market penetration, and achieve greater profitability and a strong competitive advantage.

Following the introductory chapter, the book is divided into three parts. Part I presents a description of statistical methods useful in quality assurance. Topics covered include sampling and descriptive statistics, the basic notions of

probability and probability distributions, point and interval estimation of parameters, and statistical hypothesis testing. These topics are usually covered in a basic course in statistical methods; however, their presentation in this text is from the quality-assurance viewpoint. My experience has been that even readers with a strong statistical background will find the approach to this material somewhat different than that of the usual statistics textbook.

Part II contains six chapters on statistical process control. The primary process-control technique advocated is the control chart. While the concept of control charts is not new, its use in modern-day industry is of tremendous value. As sensing and measurement technology develops, along with the widespread availability of powerful microcomputers, the implementation of statistical process controls at the work center is becoming a commonplace activity in many businesses. Statistical process controls will play an even greater role in U.S. industry over the next 20 years than it has in the last 50.

Part III contains three chapters dealing with acceptance sampling. The focus is on lot-by-lot acceptance sampling, although there is some discussion of continuous sampling and MIL STD 1235B in Chapter 12. Other sampling topics presented include details of the design of acceptance-sampling plans; a discussion of MIL STD 105D, MIL STD 414, and their civilian counterparts, ANSI/ASQC Z1.4 and ANSI/ASQC Z1.9; and other important techniques useful in a near-zero defects manufacturing environment, such as chain sampling and skip-lot sampling. Throughout Parts II and III guidelines are given for selecting the proper type of process-control and sampling technique to use in a wide variety of product and nonproduct situations.

Many people have generously contributed their time and knowledge of the assurance sciences to the completion of this book. A complete list of colleagues with whom I have interacted in various teaching, research, or consulting projects over the years would be too extensive to include here. However, some of the major contributors and their professional affiliations are as follows: Dr. William W. Hines, Dr. Lynwood A. Johnson, Dr. Russell C. Heikes, Dr. David E. Fyffe, and Dr. H. M. Wadsworth, Jr., Georgia Institute of Technology; Dr. Joseph J. Moder, University of Miami; Dr. Erwin M. Saniga, University of Delaware; Dr. John S. Ramberg, University of Arizona; Dr. Frank B. Alt, University of Maryland; Dr. Kenneth E. Case, Oklahoma State University; Mr. John A. Butora, Mr. Leon V. Mason, Mr. Lloyd K. Collins, Mr. Dana D. Leshner, Mr. Roy E. Dent, Mr. Mark Fazey, Ms. Kathy Schuster, Mr. Dan Fritze, Mr. Jay Gardiner, Mr. Ariel Rosentrater, Mr. Lally Marwah, Mr. Ed Schleicher, Mr. Armin Weiner, and Ms. Elaine Baechtler, IBM; Mr. Larry Newton and Mr. C. T. Howlett, Georgia-Pacific Corporation; Mr. Robert V. Baxley, Monsanto Chemicals; Dr. Craig Fox, Dr. Thomas L. Sadosky, Mr. James F. Walker, and Mr. John Blevins, The Coca-Cola Company; and Mr. Seymour M. Selig, Office of Naval Research. All of these individuals and many others have contributed to my knowledge and understanding of quality assurance. I am in their debt.

I would also like to thank my editor, Mr. William Stenquist, and his assistant, Ms. Jane Kenneally, for exhibiting considerable patience and under-

standing throughout the lengthy completion of this project. They have tolerated the peculiarities of authorship far better than any other editorial team with whom I have worked. I also thank the various professional societies and publishers who have given permission to reproduce their materials. Credit for this permission is acknowledged at appropriate places in the text. In addition I am indebted to the Office of Naval Research, which has sponsored my research activities in the assurance sciences for a number of years. It has provided an open and stimulating environment for conducting research and has generated many opportunities to work with navy fleet and support personnel on a wide range of quality and reliability engineering problems. I also thank Ms. Joene Owen for carefully preparing the final version of this manuscript, and Ms. Lisa Allen for help in proofreading.

Finally, I thank my wife, Edie, for her unbounded love and encouragement. I would never have completed this project without her.

**Douglas C. Montgomery**

# Contents

<b>1</b>	<b>Quality Assurance in the Modern Business Environment</b>	<b>1</b>
1-1	THE MEANING OF QUALITY	1
1-2	QUALITY ASSURANCE, TECHNOLOGY, AND PRODUCTIVITY	3
1-3	QUALITY COSTS	5
1-4	METHODS OF QUALITY ASSURANCE	11
1-5	THE RESPONSIBILITY FOR QUALITY	15
1-6	LEGAL ASPECTS	17
1-7	SUMMARY	19

<b>PART I</b>	<b>STATISTICAL METHODS USEFUL IN QUALITY ASSURANCE</b>	<b>21</b>
---------------	--	-----------

<b>2</b>	<b>Modeling Process Quality</b>	<b>23</b>
2-1	DESCRIBING VARIATION	23
2-1.1	The Frequency Distribution and Histogram	23
2-1.2	Numerical Summary of Data	26
2-1.3	Probability Distributions	28
2-2	IMPORTANT DISCRETE DISTRIBUTIONS	33
2-2.1	The Hypergeometric Distribution	33
2-2.2	The Binomial Distribution	34

2-2.3 The Poisson Distribution	36
2-2.4 The Pascal and Related Distributions	37
2-3 IMPORTANT CONTINUOUS DISTRIBUTIONS	38
2-3.1 The Normal Distribution	38
2-3.2 The Exponential Distribution	44
2-3.3 The Gamma Distribution	46
2-3.4 The Weibull Distribution	47
2-4 SOME USEFUL APPROXIMATIONS	49
2-4.1 The Binomial Approximation to the Hypergeometric	49
2-4.2 The Poisson Approximation to the Binomial	50
2-4.3 The Normal Approximation to the Binomial	50
2-4.4 Comments on Approximations	51
2-5 EXERCISES	52
<b>3 Inferences About Process Quality</b>	<b>57</b>
3-1 STATISTICS AND SAMPLING DISTRIBUTIONS	58
3-1.1 Sampling from a Normal Distribution	59
3-1.2 Sampling from a Bernoulli Distribution	62
3-1.3 Sampling from a Poisson Distribution	63
3-2 ESTIMATING PROCESS PARAMETERS	64
3-2.1 Point Estimation	65
3-2.2 Interval Estimation	66
3-3 HYPOTHESIS TESTING ON PROCESS PARAMETERS	75
3-3.1 Tests on Means, Variance Known	77
3-3.2 Tests on Means of Normal Distributions, Variance Unknown	79
3-3.3 Tests on Variances of Normal Distributions	83
3-3.4 Tests on Binomial Parameters	85
3-3.5 Tests on Poisson Parameters	87
3-3.6 The Probability of Type II Error	89
3-4 EXERCISES	92



<b>PART II    STATISTICAL PROCESS CONTROL</b>	<b>99</b>
<b>4    How the Control Chart Works</b>	<b>101</b>
4-1 CHANCE AND ASSIGNABLE CAUSES OF QUALITY VARIATION	101
4-2 STATISTICAL BASIS OF THE CONTROL CHART	102
4-2.1 Basic Principles	102
4-2.2 Choice of Control Limits	108
4-2.3 Sample Size and Sampling Frequency	110
4-3 RATIONAL SUBGROUPS	111
4-4 ANALYSIS OF PATTERNS ON CONTROL CHARTS	112
4-5 NONMANUFACTURING APPLICATIONS OF CONTROL CHARTS	115
4-6 EXERCISES	116
<b>5    Control Charts for Attributes</b>	<b>119</b>
5-1 INTRODUCTION	119
5-2 THE CONTROL CHART FOR FRACTION NONCONFORMING	119
5-2.1 Development and Operation of the Control Chart	121
5-2.2 Variable Sample Size	133
5-2.3 The Operating-Characteristic Function	139
5-3 CONTROL CHARTS FOR NONCONFORMITIES (DEFECTS)	141
5-3.1 Procedures with Constant Sample Size	142
5-3.2 Procedures with Variable Sample Size	153
5-3.3 Demerit Systems	155
5-3.4 The Operating-Characteristic Function	156
5-4 EXERCISES	158
<b>6    Control Charts for Variables</b>	<b>171</b>
6-1 INTRODUCTION	171
6-2 CONTROL CHARTS FOR $\bar{x}$ AND $R$	173
6-2.1 Statistical Basis of the Charts	173

6-2.2 Development and Use of $\bar{x}$ and $R$ Charts	176
6-2.3 Charts Based on Standard Values	188
6-2.4 Interpretation of $\bar{x}$ and $R$ Charts	189
6-2.5 The Effect of Nonnormality on $\bar{x}$ and $R$ Charts	192
6-2.6 The Operating- Characteristic Function	193
6-3 OTHER CONTROL CHARTS FOR VARIABLES	197
6-3.1 $\bar{x}$ and $S$ Control Charts	197
6-3.2 The $S^2$ Control Chart	199
6-3.3 Control Charts for Individual Units	200
6-3.4 Control Limits Based on a Small Number of Samples	202
6-4 CHOICE BETWEEN ATTRIBUTES AND VARIABLES CONTROL CHARTS	203
6-5 SUMMARY OF PROCEDURES FOR $\bar{x}$ , $R$ , AND $S$ CHARTS	205
6-6 GUIDELINES FOR IMPLEMENTING CONTROL CHART PROGRAMS	206
6-7 EXERCISES	209

<b>7 Other Statistical Process-Control Techniques</b>	<b>221</b>
7-1 MODIFIED CONTROL CHARTS	221
7-1.1 Basic Principles	221
7-1.2 Alternative Designs for the Modified Control Chart	224
7-2 THE CUMULATIVE-SUM CONTROL CHART	225
7-2.1 Basic Principles	225
7-2.2 Designing a Cumulative-Sum Control Chart	229
7-2.3 Related Procedures	230
7-3 CONTROL CHARTS BASED ON WEIGHTED AVERAGES	235
7-3.1 The Moving-Average Control Chart	235
7-3.2 The Geometric Moving-Average (GMA) Control Chart	239
7-4 PRE-CONTROL	243

7-5 METHODS FOR CONTROLLING SEVERAL RELATED QUALITY CHARACTERISTICS	245
7-6 STATISTICAL ALTERNATIVES TO CONTROL CHARTS	252
7-7 EVOLUTIONARY OPERATION	253
7-8 OVERVIEW OF OTHER PROCEDURES	260
7-9 EXERCISES	265
<b>8 Process-Capability Analysis</b>	<b>273</b>
8-1 INTRODUCTION	273
8-2 PROCESS-CAPABILITY ANALYSIS USING A HISTOGRAM OR A PROBABILITY PLOT	275
8-2.1 Using the Histogram	275
8-2.2 Probability Plotting	280
8-3 PROCESS-CAPABILITY ANALYSIS USING A CONTROL CHART	285
8-4 PROCESS-CAPABILITY ANALYSIS USING DESIGNED EXPERIMENTS	288
8-5 SETTING SPECIFICATION LIMITS ON DISCRETE COMPONENTS	293
8-5.1 Linear Combinations	293
8-5.2 Nonlinear Combinations	297
8-6 ESTIMATING THE NATURAL TOLERANCE LIMITS OF A PROCESS	300
8-6.1 Tolerance Limits Based on the Normal Distribution	300
8-6.2 Nonparametric Tolerance Limits	301
8-7 EXERCISES	302
<b>9 Economic Design of Control Charts</b>	<b>307</b>
9-1 INTRODUCTION	307
9-1.1 Process Characteristics	308
9-1.2 Cost Parameters	309
9-1.3 Early Work and Semieconomic Designs	310
9-2 ECONOMIC DESIGN OF THE $\bar{x}$ CONTROL CHART	312
9-2.1 Single Assignable-Cause Models	312
9-2.2 Multiple Assignable-Cause Models	324

9-2.3 Joint Economic Design of $\bar{x}$ and $R$ Control Charts	332
9-3 ECONOMIC DESIGN OF THE CONTROL CHART FOR FRACTION NONCONFORMING	333
9-4 ECONOMIC DESIGN OF OTHER CONTROL CHARTS	336
9-5 SUMMARY	338
APPENDIX 9A A COMPUTER PROGRAM FOR THE ECONOMIC DESIGN OF THE $\bar{x}$ CONTROL CHART	341
9-6 EXERCISES	343

## **PART III ACCEPTANCE SAMPLING 349**

<b>10 Lot-By-Lot Acceptance Sampling for Attributes</b>	<b>351</b>
10-1 THE ACCEPTANCE-SAMPLING PROBLEM	351
10-1.1 Advantages and Disadvantages of Sampling	353
10-1.2 Types of Sampling Plans	354
10-1.3 Lot Formation	354
10-1.4 Random Sampling	355
10-1.5 Guidelines for Using Acceptance Sampling	356
10-2 SINGLE-SAMPLING PLANS FOR ATTRIBUTES	358
10-2.1 Definition of a Single-Sampling Plan	358
10-2.2 The OC Curve	359
10-2.3 Designing a Single-Sampling Plan with a Specified OC Curve	366
10-2.4 Rectifying Inspection	367
10-3 DOUBLE, MULTIPLE, AND SEQUENTIAL SAMPLING	373
10-3.1 Double-Sampling Plans	373
10-3.2 Multiple-Sampling Plans	382
10-3.3 Sequential-Sampling Plans	383
10-4 A LOT-SENSITIVE COMPLIANCE (LTPD) SAMPLING PLAN	387

10-5	MILITARY STANDARD	
	105D (ANSI / ASQC Z1.4)	389
10-5.1	Description of the Standard	389
10-5.2	Procedure	392
10-5.3	Discussion	408
10-6	DODGE-ROMIG SAMPLING PLANS	413
10-6.1	LTPD Plans	414
10-6.2	AOQL Plans	420
10-6.3	Estimation of Process Average	426
10-7	EXERCISES	426
<b>11</b>	<b>Acceptance Sampling by Variables</b>	<b>431</b>
11-1	INTRODUCTION TO VARIABLES SAMPLING	431
11-1.1	Advantages and Disadvantages of Variables Sampling	431
11-1.2	Types of Sampling Plans Available	432
11-1.3	Caution in the Use of Variables Sampling	434
11-2	DESIGNING A VARIABLES SAMPLING PLAN WITH A SPECIFIED OC CURVE	435
11-3	MIL STD 414 (ANSI / ASQC Z1.9)	439
11-3.1	General Description of the Standard	439
11-3.2	Use of the Tables	440
11-3.3	Discussion of MIL STD 414 and ANSI/ASQC Z1.9	453
11-4	OTHER VARIABLES SAMPLING PROCEDURES	455
11-4.1	Sampling by Variables to Give Assurance Regarding the Lot or Process Mean	455
11-4.2	Sequential Sampling by Variables	456
11-4.3	The Lot-Plot Method	456
11-4.4	Narrow-Limit Gaging	457
11-5	EXERCISES	458
<b>12</b>	<b>Other Acceptance-Sampling Procedures</b>	<b>461</b>
12-1	CHAIN SAMPLING	461
12-2	CONTINUOUS SAMPLING	463
12-2.1	CSP-1	464

12-2.2 CSP-2, CSP-3, and Multilevel Plans	468
12-2.3 MIL STD 1235B	470
12-3 SKIP-LOT SAMPLING PLANS	473
12-4 CONSIDERATION OF INSPECTION ERROR	477
12-5 ECONOMIC DESIGN OF ACCEPTANCE-SAMPLING PLANS	481
12-6 EXERCISES	485

<b>BIBLIOGRAPHY</b>	<b>489</b>
---------------------	------------

<b>APPENDIX</b>	<b>497</b>
-----------------	------------

<b>I    Cumulative Poisson Distribution</b>	<b>498</b>
<b>II   Cumulative Standard      Normal Distribution</b>	<b>501</b>
<b>III  Percentage Points      of the <math>\chi^2</math> Distribution</b>	<b>503</b>
<b>IV  Percentage Points      of the <math>t</math> Distribution</b>	<b>504</b>
<b>V   Percentage Points      of the <math>F</math> Distribution</b>	<b>505</b>
<b>VI  Factors for Constructing      Variables Control Charts</b>	<b>510</b>
<b>VII Factors for Two-Sided      Normal Tolerance Limits</b>	<b>511</b>
<b>VIII Factors for One-Sided      Normal Tolerance Limits</b>	<b>512</b>
<b>IX  Random Numbers</b>	<b>513</b>
<b>     Index</b>	<b>515</b>

# Chapter 1

## Quality Assurance in the Modern Business Environment

### 1-1 THE MEANING OF QUALITY

This book is about quality assurance for the products used by our society. Products consist of manufactured goods, such as automobiles, computers, and clothing, as well as services, such as electrical energy, public transportation, and health care. Quality-assurance principles apply to both manufactured goods and services. Our purpose is to present the technical tools necessary to achieve quality assurance in manufacturing and service organizations.

It is essential that products meet the requirements of those who use them. Therefore, our definition of quality is that quality means *fitness for use*. The term *consumer* applies to many different types of users. A purchaser of a product that is used as a raw material in its manufacturing operations is a consumer, and to this manufacturer fitness for use implies the ability to process this raw material with low cost and minimal scrap or rework. A retailer purchases finished goods with the expectation that they are properly packaged, labeled, and arranged for easy storage, handling, and display. You and I may purchase automobiles that we expect to be free of initial manufacturing defects or *nonconformities*, and which should provide reliable and economical transportation over time.

There are two general aspects of quality: quality of design and quality of conformance. All goods and services are produced in various grades or levels of quality. These variations in grades or levels of quality are intentional, and, consequently, the appropriate technical term is *quality of design*. For example, all automobiles have as their basic objective providing safe transportation for

## 2 Quality Assurance in the Modern Business Environment

the consumer. However, automobiles differ with respect to size, appointments, appearance, and performance. These differences are the result of intentional design differences between the types of automobiles. These design differences include the types of materials used in construction, tolerances in manufacturing, reliability obtained through engineering development of engines and drive trains, and other accessories or equipment.

The *quality of conformance* is how well the product conforms to the specifications and tolerances required by the design. Quality of conformance is influenced by a number of factors, including the choice of manufacturing processes, the training and supervision of the work force, the type of quality-assurance system (process controls, tests, inspection activities, etc.) used, the extent to which these quality-assurance procedures are followed, and the motivation of the work force to achieve quality.

There is considerable confusion in our society about quality. The term is often used without making clear whether it is quality of design or quality of conformance about which we are speaking. To achieve quality of design requires conscious decisions during the product or process design stage to ensure that certain functional requirements will be satisfactorily met. For example, the designer of an office copier machine may design a circuit component with a redundant element, because he knows that this will enhance the reliability of the product in the field and will increase the mean time between failures. This in turn will result in fewer service calls to keep the copier running, and the consumer will be far more satisfied with the performance of the product. Designing quality into the product in this fashion often results in a higher product cost. However, such cost increases are actually *prevention costs*, as they are intended to prevent quality problems at later stages in the life cycle of the product. We discuss other aspects of quality costs in Section 1-3.

Improvements in quality of conformance are often made by changing certain aspects of the quality-assurance system, such as the use of statistical process-control procedures, changing the type of inspection procedures used, and so forth. Thus higher quality of conformance is often achieved with a reduction in total costs, because it leads to reduced scrap, rework, and a smaller fraction of nonconforming products and services.

Every product possesses a number of elements that jointly describe its fitness for use. These parameters are often called *quality characteristics*. Quality characteristics may be of several types:

1. **Physical.** Length, weight, voltage, viscosity.
2. **Sensory.** Taste, appearance, color.
3. **Time Orientation.** Reliability, maintainability, serviceability.

Quality control is the engineering and management activity by which we measure the quality characteristics of the product, compare them with speci-



cations or requirements, and take appropriate remedial action whenever there is a difference between actual performance and the standard.

## 1-2 QUALITY ASSURANCE, TECHNOLOGY, AND PRODUCTIVITY

Quality is becoming the basic consumer decision factor in many products and services. This phenomenon is widespread, regardless of whether the consumer is an individual, an industrial corporation, a military defense program, or a retail store. Consequently, quality is a key factor leading to business success, growth, and enhanced competitive position. There is a substantial return on investment from an effective quality-assurance program that provides increased profitability to firms that effectively employ quality as a business strategy. Consumers feel that the products of certain companies are substantially better in quality than those of their competition, and make purchasing decisions accordingly. Effective quality-assurance programs can result in increased market penetration, higher productivity, and lower overall costs of manufacturing and service. Consequently, firms with such programs can enjoy significant competitive advantages.

Achieving quality in the modern business and manufacturing environment is not easy. A significant problem is the rapid evolution of technology. The last 20 years have seen an explosion of technology in such diverse fields as electronics, metallurgy, ceramics, composite materials, and the chemical and pharmaceutical sciences. This has resulted in many new products and services. For example, in the electronics field, the development of the integrated circuit has revolutionized the design and manufacture of computers and many electronic office products. Basic integrated circuit technology has been supplanted by large-scale integration (LSI) and very large-scale integration (VLSI) technology, with corresponding developments in semiconductor design and manufacturing. When technological advances occur rapidly, and when the new technologies are used quickly to exploit competitive advantages, the problems of manufacturing products with adequate levels of quality of design and quality of conformance are greatly complicated.

The basic problem in many industries is that of manufacturing a product in adequate volumes. Often, too little attention is paid to achieving economy, efficiency, productivity, and quality in production. An effective quality-assurance program can be instrumental in increasing productivity and reducing cost. As an example, consider the manufacturer of a mechanical component used in a copier machine. The parts are manufactured in a machining process at a rate of approximately 100 parts per day. For various reasons, the process is operating at a yield of about 75% (that is, about 75% of the process output conforms to specifications, and about 25% of the output is nonconforming). About 60% of the fallout (the 25% nonconforming) can be reworked into acceptable product, and the rest must be scrapped. The direct manufacturing