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SUPERCONDUCTING MAGNETS

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PREFACE

Most books about superconductivity have been written by physicists for physicists. This is hardly surprising, for superconductivity was discovered in the physics laboratory and has ever since remained one of the livelier areas of experimental and theoretical physics. In recent years however, one particular application of superconductivity—the construction of high-field electromagnets—has moved from the laboratory to the factory. Superconducting magnets are now being used in a variety of applications, some of them on a very large scale and calling for the highest quality of engineering. In writing this book, I have attempted to meet the needs of the growing number of engineers, technologists, and applied physicists who are now engaged in the practical exploitation of superconductivity. With these needs in mind, I have generally adopted an engineering approach, paying little attention to microscopic processes within the superconductor itself, but rather regarding it as a given material whose properties must be utilized in the most effective way. This is not intended to imply any lack of thoroughness in the engineering approach. A complete appreciation of the macroscopic behaviour of superconductors, particularly their response to electromagnetic fields, is needed before one can undertake the most basic engineering calculations on, for example, the stability of current flow or the losses produced by alternating fields. As the size and cost of magnet systems continues to increase, designers must pay particular attention to questions of reliability, safety, and cost optimization. I have tried to present a satisfactory theoretical basis for such design calculations and for the other important aspects of magnet design. The theory is frequently illustrated by numerical examples and, wherever possible, by comparison with experiment.

Much of the book is based on work carried out at the Rutherford Appleton Laboratory (RAL) and I am grateful to Dr G. Manning, Director of RAL, and to Dr D. B. Thomas, Head of Technology Division, for their encouragement and support in the task of writing. Many other colleagues at RAL have contributed to the book—by furthering my education, by making experimental results available, or by commenting constructively on the manuscript—notably Dr P. F. Smith, Mr C. R. Walters, Dr C. A. Scott, Mr C. W. Trowbridge, Dr D. E. Baynham, and Mr D. Evans. I am also grateful to colleagues in other laboratories for their helpful comments on the manuscript, particularly Dr B. Turck, Dr W. Schauer, Dr A. M. Campbell, Dr K. P. Jungst, Dr L. Dresner, and Professor D. C. Larbalestier. Finally, my thanks to Mrs P. M. Morgan and Mrs B. Y. Triplow for their work in producing a well-ordered typescript from a much-edited manuscript and to Mrs E. Marsh and the RAL library staff for checking the references.

Chilton 1982

M. N. W.

NOMENCLATURE

The principal symbols used are listed below, together with the chapter and section in which they are first introduced. Consistency with general usage has been maintained as far as possible, but this has inevitably caused certain symbols to acquire more than one meaning, e.g. H for magnetic field and enthalpy. Double meanings rarely occur within the same chapter however and it is hoped that readers will not be seriously misled. SI units are used throughout. Vector quantities are denoted by bold type.

Symbol	Meaning	Section first used
a	radius	3.1
a	half-width	6.8
$\mathbf{A}$	magnetic vector potential	3.5
A	area of cross-section	5.3
b	outer radius	3.1
B, $\mathbf{B}$	magnetic field (induction)	1.1
B_{w}	maximum field on winding	3.1
B_0	central field of coil	3.1
B_a B_b	field at inner and outer radius of solenoid	4.1
$B_{\mathrm{i}} B_0$	inner and outer dipole field	4.2
B_{m}	peak-to-peak amplitude of oscillating field	8.1
B_{p}	penetration field	8.2
c	penetration radius	7.3
C	specific heat	5.2
C_{p}	specific heat of gas at constant pressure	11.1
C_{L}	latent heat of evaporation	6.9
d	characteristic distance for heat conduction within superconductor	6.8
D	diffusivity	7.4
D	hydraulic diameter	6.10
e	normalized energy	5.5
e	ratio of ellipse axes	8.2
E	solenoid error coefficients	3.1
E	elliptic integral	3.5
$\mathbf{E}$	electric field	5.4
E	energy	4.1
f	transient heat transfer function	6.9
f	flux screening factor	8.4

f	heat transfer fraction	11.1
F	solenoid magnetic field factor	3.1
F	complex field working variable	3.2
F, $\mathbf{F}$	force	4.1
F_{pc}	flux pinning strength	12.1
g	linear current density in A m^{-1}	3.2
G	ohmic heat generation per unit volume of conductor	5.4
G_c	generation at critical temperature θ_c	5.4
G	Gibbs free energy	12.1
h	heat transfer coefficient	6.1
h	normalized external field	8.2
h	height	10.3
H	enthalpy	5.2
H	heat transfer flux	6.3
H, $\mathbf{H}$	magnetic field	8.1
i	ratio I_t/I_c	7.3
I	current	3.2
I_c	critical current	4.4
I_m	magnet current	5.4
I_t	transport current	8.2
I_0 I_1	modified Bessel functions	6.8
J, $\mathbf{J}$	current density	1.1
J_c	critical current density	1.1
J_{c0}	critical current density at θ_0	5.5
J_m	density of magnet current	5.5
J_0 J_1	Bessel functions	7.6
k	thermal conductivity	5.3
k	coupling coefficient	9.7
K	elliptic integral	3.5
K	force function	4.1
l	solenoid half-length	3.1
l	length	5.3
l	alternating twist pitch	8.5
L_0	Lorentz number	7.4
L	Inductance coefficient	8.1
L	twist pitch	8.3
$\mathbf{m}$	magnetic moment	8.1
m	normalized magnetization	8.2
$\dot{m}$	mass flow rate	11.1

M, $\mathbf{M}$	magnetization	3.5
M	force function	4.11
M	bending moment	4.2
M	mutual inductance	9.7
n	number of filaments	8.4
N_{F}	density of states	12.1
p	penetration depth	7.1
P	cooled perimeter	6.1
q	loss	8.1
q	heat per unit area	6.9
Q	strain energy per unit volume	5.6
Q	heat energy per unit volume	6.9
Q	a.c. loss per unit volume per cycle	8.1
r, $\mathbf{r}$	radius	3.3
R	radius	3.5
R_{g}	radius of generation zone	5.5
R	resistance	9.2
s	thickness of dipole winding	3.2
S	heat flow function	6.2
t	time variable	6.9
t	normalized time	9.4
T	hoop tension	4.3
T	time	6.9
T_{m}	time of field ramp B_{m}	8.3
T_{Q}	quench time for unbounded normal zone	9.4
u	radial displacement	4.1
u	ratio $C_{\mathrm{p}}/C_{\mathrm{L}}$	11.1
U	protection function	9.1
v	quench propagation velocity	9.3
V	volume	3.1
V	potential difference	9.1
w	reduced radial displacement	4.1
w	width of composite annulus	8.3
w	low-temperature heat leak	11.1
W	power	10.1
W_{r}	refrigeration power	11.1
x	distance—usually transverse to a conductor	5.5
X	superfluid heat flow function	6.7
y	normalized temperature	5.5
Y	Young's modulus	4.1

Y_0 Y_1	Bessels functions of the second kind	7.9
z	distance—usually along a conductor	5.5
z	substituted length parameter	11.1
Z_2	substituted length of lead	11.1
α	solenoid radius ratio	3.1
α	aspect ratio of MPZ	5.3
α	ratio of transverse: longitudinal propagation velocities	9.3
β	solenoid length ratio	3.1
β	stability parameter	7.2
β	field ratio B_m/B_p	8.1
γ	density	5.2
γ	Sommerfeld constant	12.1
Γ	loss function	8.1
Δ	energy gap	12.1
ε	normalized radius r/a	4.1
ε	strain	5.6
η	ratio of thermal and magnetic diffusivities	7.7
η	transient heat transfer function	6.9
θ	angle	3.1
θ	absolute temperature	1.1
θ_c	critical temperature	1.1
θ_g	temperature at which power generation starts in composite conductors	5.2
θ_o	base temperature—usually that of the helium bath	5.2
Θ	absolute temperature of conductor	6.10
κ	Ginsburg–Landau constant	12.1
λ	volumetric proportion of superconductor in a composite	5.4
Λ	characteristic length	10.1
μ_o	permeability of free space	3.1
ν	Poisson's ratio	4.1
ν	heat transfer stability factor	7.5
ρ	radius of curvature	4.3
ρ	resistivity	5.3
ρ_{et}	effective tranverse resistivity	8.3
σ	tensile stress	4.1
τ	time constant	8.3
ϕ	angle	3.3

ϕ	magnetic flux	7.1
χ	angle	3.5
χ	helix angle of twisted composite	8.3
ω	angular frequency	8.3

CONTENTS

1

INTRODUCTION

Superconductivity is a strange and remarkable phenomenon whereby certain metals, when cooled to very low temperatures, become perfect conductors of electricity. Unlike the gradual changes in electrical resistance shown by all metals at more familiar temperatures, the superconducting state appears quite abruptly at a critical temperature θ_c which is characteristic of the metal in question. Below this temperature the resistance is not just very small; as far as can be seen from the results of some very sensitive experiments, it is absolutely zero. Critical temperatures are very low, a few degrees kelvin, demanding the use of liquid helium as a refrigerant. Indeed, the discovery of superconductivity by Kammerlingh Onnes in 1911 was a direct consequence of his development of helium liquefaction techniques some three years previously—a good example of the way in which technological developments can pave the way to fundamental discoveries.

Onnes quickly realized that his new discovery could have technological as well as scientific implications and was soon contemplating the construction of large electromagnets which would be capable of generating very high fields and yet would consume no power. His hopes in this direction were quickly dashed when he discovered that, in addition to their critical temperature, superconductors also have a critical field, above which they revert from the superconducting state to a normal resistive state. For the metals which Onnes investigated, i.e. mercury, lead, and tin, the critical field B_c is small $\sim$ 1/20 tesla (T). With commendable objectivity, Onnes noted '... an unforeseen difficulty is now found in our way, but this is well counterbalanced by the discovery of the curious property which is the cause of it...'. Many other curious properties of superconductors were discovered in subsequent years and there have been profound advances in our understanding of this fascinating phenomenon, but it was to be more than fifty years before Onnes' vision of large powerful superconducting magnets could become a practical possibility.

The breakthrough came in the late-1950s and early-1960s when workers in the USA, notably Matthias and Kunzler, discovered a new class of high-field superconducting alloys. Unlike the pure metals which Onnes had worked with, these materials were able to remain superconducting up to very high fields and were also able to carry extremely high-current densities. Fig. 1.1 illustrates the properties of niobium titanium, currently the most popular superconducting alloy. To describe these properties fully, it is necessary to add the concept of a critical current density J_c to those of critical temperature θ_c

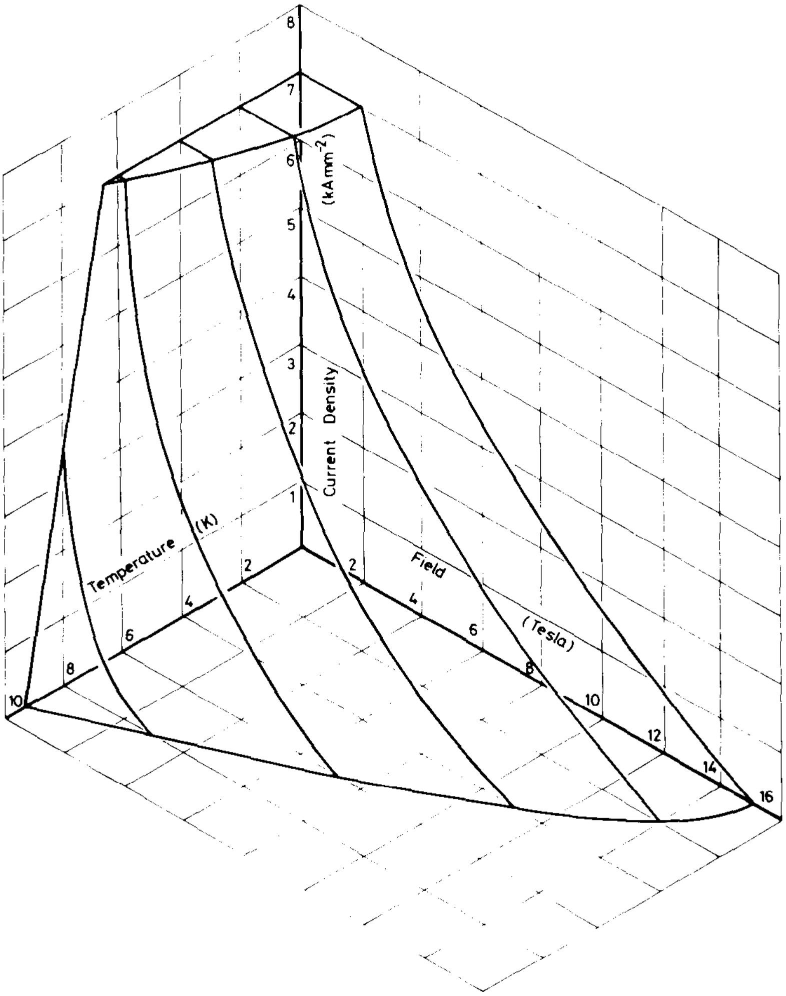

Fig. 1.1 Critical-current surface for a commercial superconducting alloy of niobium–titanium. (Based on recent measurements at 4.2 K, together with earlier measurements at variable temperature by Hampshire, R., Sutton, J., and Taylor, M. T. (1969).)

and field B_c. All these properties are related to each other by the critical surface in $BJ\theta$ space which is characteristic of the material in question; superconductivity prevails everywhere below this surface with normal resistivity everywhere above it. It may be seen that an increase in any one of the properties invariably produces a decrease in the other two. For example, the critical temperature θ_c of niobium titanium as shown in Fig. 1.1 is 9.3 K, but it would not be interesting to operate a superconducting device at 9.3 K because at this temperature the critical field and current density have both fallen to zero. In practice, the usual operating temperature for superconducting magnets is 4.2 K, the boiling point of liquid helium under atmospheric pressure. Fig. 1.2 shows a 'slice' through the critical surface at 4.2 K for niobium titanium and also for niobium tin, a material which has much better performance but is more difficult to deal with because it is weak and brittle.

The current densities shown in Fig. 1.2 are quite extraordinary by everyday standards. Normal house wiring for example is rated at about 10^7 A m^{-2} and fuses blow at about 10^8 A m^{-2}. Conventional electromagnets have water-cooled copper windings running at about 10^7 A m^{-2}. They make extensive use of soft iron yokes to minimize the number of ampere turns required and hence the power consumption. Iron saturates at a field of about 2 T, thereby imposing an upper limit on field and restricting the operating range of conventional electromagnets to the shaded area shown in Fig. 1.2. Much higher fields and current densities are possible in conventional magnets, the well-known Bitter solenoids for example, but only at the cost of very high power consumption and the related need for enormous quantities of cooling

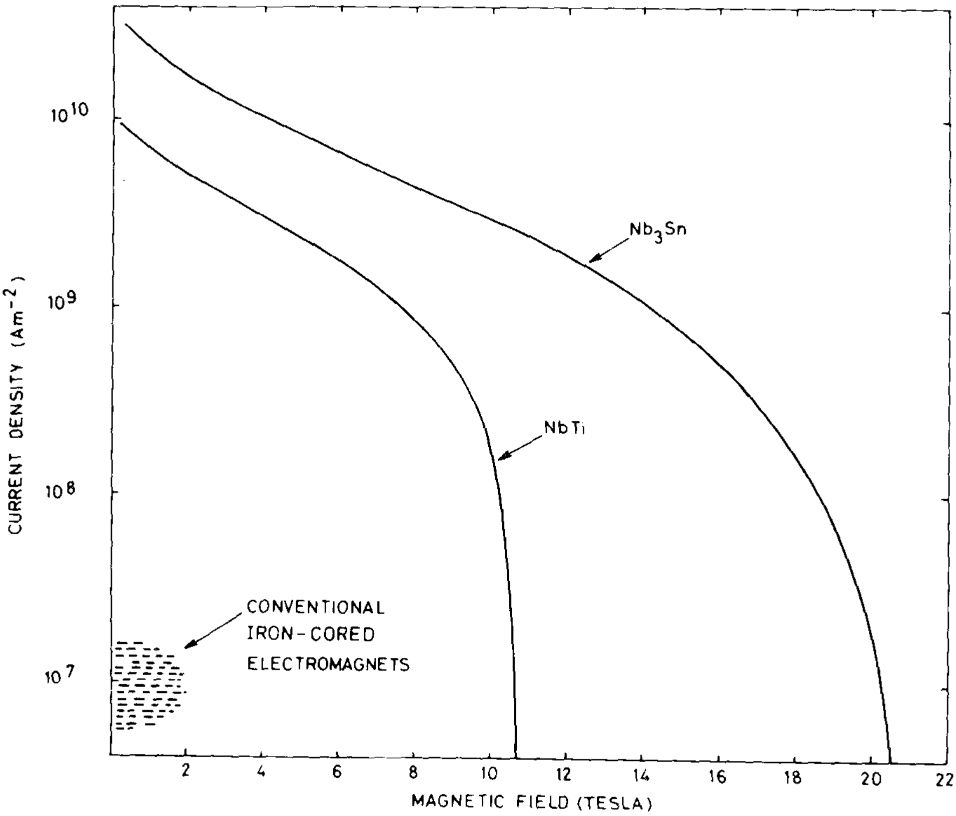

Fig. 1.2 Critical curves for the two common technological superconductors, niobium titanium and niobium tin, at a constant temperature of 4.2 K. Superconductivity prevails below the curves and normal resistivity above. Shaded area at bottom left illustrates the usual operating range for conventional electromagnets.

water. The only power required by a superconducting magnet is the refrigeration power needed to cool it to low temperatures and a small current supply needed to initiate the flow of current round the superconducting circuit. Refrigeration power can be significant however because the performance coefficient of low-temperature refrigerators is small, for both thermodynamic and practical reasons. Typically a cooling power of 1 W at 4.2 K will demand the expenditure of some 500–1000 W at room temperature. Fortunately the cooling requirements of most superconducting magnets can be kept down to a very modest level.

It therefore became very clear, following the discovery of high-field superconductors, that superconducting magnets have the potential for much better performance than their conventional counterparts. They should be able to produce large volumes of high field for a small consumption of electric power. They should be able to operate at high overall current densities, allowing a given number of ampere-turns, i.e. a given field, to be provided by a rather small volume of winding. Apart from reducing the capital cost by making the magnet very compact and light in weight, this should also make it possible to produce higher gradients of field than are conventionally available.

To a considerable extent, this potential has now been realized, as witnessed by the growing number of large superconducting magnet systems and the thousands of small magnets now in routine operation all over the world. Success only became possible however when a number of quite difficult technical problems had been solved. The principal purpose of this book is to discuss the cause of these problems, to describe the most successful techniques which have been developed to overcome them, and to formulate design procedures which should enable them to be avoided in future. To fill in the background for these detailed discussions, the principal applications of superconducting magnets will first be reviewed and the requirements of each application, in terms of field shape and magnet configuration, will be mentioned. Practical aspects of magnet construction and operation and the fabrication of superconducting materials will also be described in later chapters.

For most of the book, the superconductors themselves will be regarded quite simply as engineering materials, with a given set of properties which must be used as effectively and economically as possible. Only in Chapter 12 will the basic mechanisms of superconduction be briefly touched upon, but no attempt will be made to explain the origins of superconductivity or to give a comprehensive description of its many features. For a better coverage of these broader aspects, the reader is referred to one of the many excellent books on superconductivity, for example Grassie (1975) or Rose-Innes and Rhoderick (1969).

To maintain the low-temperature environment so essential for their operation, magnets must be placed in special liquid helium vessels or 'cryostats'. These are vacuum-insulated containers, which usually have an intermediate liquid nitrogen-cooled shield interposed between room temperature and the low-temperature region. Designs vary widely, from the simple 'tub' shown in Fig. 2.1 to more complicated vessels of the re-entrant type as shown in Fig. 2.6. The latter have the advantage of allowing room-temperature access to the high-field region, which is filled with liquid helium in a tub cryostat, but they are more expensive and difficult to assemble. Small-scale laboratory installations are usually cooled by helium dispensed from a storage vessel. The liquid may either be produced by a local laboratory liquefier or bought-in from a commercial supplier. Larger magnet systems must have their own built-in helium refrigerator. In either case, considerable care and skill are needed to design a cryogenic system which will function reliably over long periods of time while maintaining an acceptably low 'heat leak' from its room-temperature surroundings. For more information on refrigeration and cryogenic engineering, the reader is referred to one of the many standard works, such as Barron (1966), Croft (1970), Scott (1959), or White (1979).

Although magnets have so far been the major application of superconductivity, there are many other possibilities: the transmission of electrical power for example or the continuous operation of high-powered microwave cavities. The Josephson effect in all its varied manifestations has given rise to a whole range of sensitive detectors and the possibility of an ultra-fast superconducting computer. Such applications are quite beyond the scope of this book but are well covered in the literature, for example in the biennial Proceedings of the Applied Superconductivity Conference.

Reference

BARRON, R. B. (1966). *Cryogenic systems.* McGraw Hill.

CROFT, A. J. (1970). *Cryogenic laboratory equipment.* Plenum Press.

GRASSIE, A. D. C. (1975). *The superconducting state.* Sussex University Press.

HAMPSHIRE, R. SUTTON, J., and TAYLOR, M. T. (1969). *Proc. conf. low temperatures and electric power.* International Institute of Refrigeration, London.

PROCEEDINGS OF THE APPLIED SUPERCONDUCTIVITY CONFERENCE are published in the following issues of *IEEE Transactions on Magnetics*: (1975) **MAG-11** (2); (1977) **MAG-13** (1); (1979) **MAG-15** (1); (1981) **MAG-17** (1).

ROSE-INNES, A. C. and RHODERICK, E. H. (1969). *Introduction to superconductivity.* Pergamon Press.

SCOTT, R. B. (1959). *Cryogenic engineering.* Van Nostrand.

WHITE, G. K. (1979). *Experimental techniques in low temperature physics* (3rd edn). Oxford University Press.