

Introduction to Lasers and Their Applications

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Preface

The laser is now found not only in the research laboratory but in the automobile factory, on the construction site, and even in the supermarket. A great need exists for those outside the ranks of research scientists and engineers to have a broader familiarity with this recent addition to today's technology. Most important, we believe, are the students of the technical professions preparing for tomorrow's world and its challenges. The course on which this textbook is based is but one of a great number of modern device-oriented courses that are now being taught at today's colleges, universities, and institutes.

There presently exists a gap between the brief reviews of lasers provided in modern physical optics texts and the thorough, graduate-level texts on lasers and quantum mechanics. This book is designed to fill this gap and to serve as the basis for a short, self-contained course that gives the undergraduate a feel for modern laser technology. For those students who may not want to invest a substantial amount of their elective time in extensive course work in this area, it represents a reasonable alternative to a more lengthy treatment. The text is based upon a series of notes used in a one-quarter, three-credit-hour course at the Georgia Institute of Technology, open to any student who has passed the calculus-based physics sequence (text: Halliday and Resnick, *Fundamentals of Physics*, Wiley, 1974). The mathematical level of the text is kept relatively simple (a few integrals and some simple differential equations) to ensure that it will be useful to a large number of students, not just the mathematically more sophisticated physics and engineering students. Stated concisely, the objectives of the text are the following: (1) to give the student the concepts and vocabulary needed to understand laser advances and applications discussed in the technical magazines of the laser field (see references at the end of Chapter 1), and (2) to enable the student to understand a laser specification sheet. If these objectives are met, the student should be able to understand most laser applications in his or her field of specialization.

We expect the text to be of use to some who are not students. Laser technicians using the preliminary version of the text have found it useful. We have not avoided mathematical descriptions where they were needed, but would note that most of the math leads to a figure or an expression that gives an understandable result and serves as a point of continuation for the balance of the discussion. Not wishing to overwhelm the student, we have in places backed off from an exhaustive explanation. A teacher may feel it worthwhile to add additional material in these areas.

The text is organized into two general parts. The early portion (Chapters 1–4) emphasizes the physical theory needed to understand lasers; the latter portion (Chapters 5–9) emphasizes the devices themselves, their construction, and their applications. After a brief introductory chapter, the properties of laser light are discussed in some detail. We have chosen to begin with a discussion of these properties rather than with the conventional exposition on atomic energy levels, Einstein relations, and the concept of stimulated emission for two reasons. (1) Our approach allows a number of laser demonstrations at the beginning of the course, and (2) it provides a review and expansion of necessary physical optics principles that are referred to throughout the text. Particularly important are the discussions of interferometers and coherence. We realize that Chapter 2 is rather lengthy, but for those students who have taken a physical optics course as a prerequisite, it can be quickly reviewed.

The next three chapters constitute a spiral approach to understanding the laser. Chapter 3 provides a first look at the basic requirements of an idealized light amplifier and a laser. The idealization is dropped in Chapter 4, where the finite laser lineshape and gain saturation are discussed. This leads to a description of the output of a laser of unspecialized design. In Chapter 5, we examine modifications of the laser output obtained by introducing various devices into the laser cavity. This chapter is a transition chapter in that it is a mixture of physical theory and device description.

Chapter 6, a detailed discussion of various types of lasers, is almost completely device-oriented. We have chosen to describe those lasers that are most widely used or that illustrate an interesting point of laser theory. To be all-inclusive, we feel, would require too much introductory material for specialized laser systems. For example, the tunable spin-flip Raman laser is not discussed.

Chapters 7, 8, and 9 cover three areas of laser applications: holography, communications, and power. Just as with the types of lasers, we could not present the mass of material necessary to give adequate descriptions of all the laser applications known today. These three areas were chosen because they are individually important and because they are representative of the broad range of possible applications. Other areas were omitted for a number of reasons. Applications of lasers to alignment and precision measurement are for the most part easily grasped and need little exposition. The area of medical applications re-

sembles that of power applications (focusing of a laser beam to cut or heat a small area), but would require considerable introduction of medical terminology and techniques. In the area of optical identification and scanning, the laser is used as a glorified light bulb of high brightness and directional output. The applications of lasers to spectroscopy, while of interest to many scientists are so varied and require so much introductory preparation that only a very lengthy chapter would do justice to this application. Still, an instructor who is working in one of these laser-related fields could supplement or substitute for the three we have chosen those of his or her own interest. In addition, a strong student interest in a particular field should be taken into account. If this course is taught on a semester basis, the instructor should have no trouble filling the time available; additional applications can be added very easily.

In all demonstrations and experiments, the safety of the student and the instructor should be a major consideration. Rather than provide a separate chapter on laser safety, we have included cautions and discussions on laser safety within the body of the text. This makes for a series of continuing reminders instead of a single section that could easily be ignored. At this writing, the regulations for lasers in some states and at the federal level are still being debated. The instructor should keep informed of the restrictions on the use of lasers and pass this information on to his or her students.

We wish to acknowledge the help, patience, and criticisms of our students who have taken the laser course over the past five years. We also thank Earl McDaniel and Charles Braden of the faculty of the School of Physics, Georgia Institute of Technology, for critical readings of an early version of the notes. We are particularly indebted to Hugo Wichel, Charles Hathaway, David Burch, Clifford Fairchild, Murray Sargent, Stephen Jacobs, and Shaoul Ezekial for their excellent suggestions and criticisms of the manuscript. We appreciate the cooperation and support of James Stevenson, Director of the School of Physics, and Demetrius Paris, Director of the School of Electrical Engineering, Georgia Institute of Technology. Finally, having produced this book, we now understand why other authors always thank their spouses with such fervor. It is they who must put up with the greatest amount of hassle. Thank you, dear wives . . . you were great!

Atlanta, Georgia
August 1976

D.C.O.
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Lasers:

A Short Introduction

The laser is now approaching the end of its second decade. During this period, it has developed from what was once described as "an invention in search of an application"* into one of the most important technological developments of this half-century. It has brought about a rebirth of the science and technology of optics and has led to the development of whole new industries. Viewed from almost any perspective, the laser is a remarkable device. Consider:

- A laser time standard accurate to a small fraction of a second per year.
- A laser beam so directional it can easily be seen from the moon—or reflected back to earth and detected here.
- A glass bead supported in air only by a shaft of green light from a laser.
- Laser-based measuring systems so accurate that they determine the altitude of earth-orbiting satellites to within several meters; the surface deformations of vibrating objects to 0.05 nanometers (billionths of a meter).
- Focused laser beams so intense that they initiate nuclear reactions.
- Huge industrial lasers with many thousands of watts of output in beams the diameter of one's finger.

The variety of lasers and the wealth of laser applications developed since 1960 is enormous.

Short History of the Laser

Until 1917, no one conceived that there was a basic process that would allow light to be amplified as it is in a laser. In that year, Albert Einstein showed that the process of stimulated emission must exist, and from that time the invention

* A description applied by Dr. A. L. Schawlow, coinventor of the laser principle.

of the laser was possible. During the 1920's, 30's, and 40's, physicists were pre-occupied with the new discoveries of quantum mechanics, particle physics, and nuclear physics. For the most part, the possibility of laser action lay dormant, although the needs of science and technology for such a device grew. Besides the science-fiction writer, there were others who conceived of uses for a high-power, highly directional light beam. Telecommunications engineers envisioned highly directional, line-of-sight communication systems where the information was carried on a light beam. Ophthalmologists needed intense beams of light that could be focused onto small areas at the back of the eyeball to weld detached and torn retinas.

Experience gained in the development of radar during World War II and the continuation of such work at higher microwave frequencies prompted scientists to explore the conditions that were necessary for laser action to be achieved. In the early 1950's, a group at Columbia University headed by Charles H. Townes operated a microwave device that amplified radiation by the stimulated emission process. The device was termed a MASER, an acronym for Microwave Amplification by Stimulated Emission of Radiation. During the remainder of the fifties, the maser principle was employed in many materials. In 1958, Townes and Arthur L. Schawlow published an important paper in which they discussed the extension of maser principles to the optical region of the electromagnetic spectrum. By 1960, a number of groups were investigating systems that might serve as the basis for an optical maser, as it was called by some, or *laser*. Credit for first achieving laser action at optical frequencies is given to T. H. Maiman of the Hughes Research Laboratories. His laser consisted of a pink ruby rod with silvered ends for mirrors inserted in the helical coil of a photographic flashlamp.

Types of Lasers—Edible and Inedible

Within six months of the invention of the ruby laser, laser action was obtained in a mixture of helium and neon gases. Research quickly mushroomed. Many materials were investigated as the active laser medium, including impure crystals, semiconductors, ionized gases, molecular gases, and dye solutions.

Because it is possible to dissolve dyes in many mediums, some of the more exotic types of lasers are dye lasers. A case in point is the edible laser. By dissolving a laser dye in ordinary gelatin (following the directions on the package, according to researchers), a quivering laser medium is produced. A pulse of ultraviolet light is used to excite the medium, and the jellylike material lases. Researchers have even found that the medium will withstand higher pulse repetition rates if it is allowed to quiver than if it is clamped in a rigid enclosure.

The sizes and shapes of lasers are varied. They can be as small as the experimental miniature lasers that are the heart of optical integrated circuits, the future companions of today's semiconductor marvels, or they can fill a room, as do the

high-power gaseous lasers that are contorted to achieve a maximum amount of active laser medium in a minimum amount of space. They can be as modest looking as the one-milliwatt educational lasers, or as fearsome as the pulsed, high-power, experimental lasers.

The Present and Future of Lasers

The laser is presently being used in a variety of materials-processing applications. It has become a part of many specialized communication systems, and it is the keystone of modern optical data processing and holographic technology. We will cover these applications in considerable detail in later chapters. The laser is also finding its way into an ever-growing number of other fields: long-distance measurement, construction and airframe alignment, optical spectroscopy, wind velocity measurements, and so on, all of which we cannot cover here. However, these applications are continually being discussed in the trade journals, particularly in those listed among the references for this chapter. Other more specialized books referenced throughout this text discuss some of these fields in detail.

In the future, we can expect lasers of even higher power to be developed. More lasers will have outputs in the ultraviolet end of the spectrum, and possibly in the x-ray region. More efficient lasers may be invented, and lower-cost lasers will be produced. Efforts will continue to modify the output of existing lasers, resulting in new applications. For the present, some basic facts are needed for an understanding of the laser and why it is so valuable to modern technology.

A NOTE ON LASER SAFETY

Although the output of a laser can be a thing of beauty, one must always maintain caution while using these devices. Even the lowly helium-neon laser, used in many laser demonstrations, must be treated with respect and care. Never look into the beam of any laser. Reflections from polished surfaces are as dangerous as the raw beam itself. Do not point a laser at anyone. Always be aware of the beam path and be aware of others when adjusting the path. Do not operate a laser without proper electrical shielding — operating voltages are generally lethal.

The Bureau of Radiological Health of the United States Food and Drug Administration has classified lasers operating in the visible part of the spectrum into four categories, each with specific warning labels and safety requirements. Lasers with outputs in the invisible part of the spectrum must be operated with special care. Copies of Bureau of Radiological Health regulations for laser use should be obtained by anyone using a laser. The sources are given in the references at the end of Chapter 1.

REFERENCES

- 1.1 M. Ross (ed.) (1972), *Laser Applications, Vol. 1*. New York: Academic Press. Besides the applications covered in this text, the use of the laser as a gyroscope and in ranging (measuring over long distances) and geodesy are discussed.
- 1.2 S. S. Charschan (ed.) (1972), *Lasers in Industry*. New York: Van Nostrand-Reinhold. Probably the best applications book available to date. Chapter 1 covers much the same material as the first five chapters of this text, but in less detail.

The magazines below report current laser applications. Some may be carried by libraries; others are available only through individuals with complimentary subscriptions. They are indispensable to someone trying to keep up with current applications and products. There is a good deal of overlap in the material covered by all of them.

- 1.3 *Laser Focus*, Advanced Technology Publications, Newton, Massachusetts. This magazine offers the most comprehensive treatment of laser news. The applications section is particularly good. The magazine is a controlled circulation publication—complimentary subscriptions are available to persons working in laser-related fields.
- 1.4 *Electro-Optical Systems Design*, Kiver Publications, Inc., Chicago, Illinois. Covers the optical field in general. Large amount of material on laser applications, but also includes optical data processing. Controlled circulation publication exclusively.
- 1.5 *Optical Spectra*, The Optical Publishing Company, Pittsfield, Massachusetts. Somewhat more emphasis on spectroscopy than the above publications. Another controlled circulation magazine.

The following references relate to laser safety.

- 1.6 H. Weichel, W. A. D. Danne, and L. S. Pedrotti (1974), "Laser Safety in the Laboratory," *American Journal of Physics* **42**, 1006–1013. An excellent discussion of potential hazards associated with lasers.
- 1.7 *The ANSI Laser Safety Guide*, ANSI; Z136.1–1976. Published by the American National Standards Institute, 1430 Broadway, New York, NY 10018. Lists maximum permissible exposure levels and safety measures to be employed in using lasers. (\$9.00)
- 1.8 Copies of the federal regulations on lasers may be obtained by writing to the Director (HFX-400), Division of Compliance, Bureau of Radiological Health, 5600 Fishers Lane, Rockville, MD 20852, and asking for HEW (FDA) Pub. 76–803.

Laser Light

The great impact of the laser as an industrial and research tool is the consequence of the extraordinary properties of laser light—extraordinary in the sense that they are present to a far greater degree in the output of a laser than in the light from any other source, natural or man-made. The most striking properties of laser light are its extreme *brightness* and *directionality*. Light from even the most modest of lasers dazzles the eye; the threadlike beam seems never to grow in size. Another unusual characteristic soon evident to even the casual observer is the speckled or scintillating appearance of laser light reflected from a rough surface. This *laser speckle* is a consequence of the extremely high degree of *coherence* of the light from a laser. With proper design, lasers may also exhibit extreme *monochromaticity* and a high degree of *polarization*. Some properties of laser light, such as brightness and directionality, are easily understood, at least qualitatively, since they are evident to any observer. The properties of monochromaticity and coherence offer more difficulty, since one must rely on an instrument other than the eye to evaluate them.

2.1 LIGHT WAVES

To understand the properties of laser light and to understand the laser itself with its many applications, we must first understand the wave nature of light.

The light by which you read these words is wavelike in nature. It is characterized by a combination of time-varying electric and magnetic fields propagating through space. The frequency at which these fields oscillate, ν , and their wavelength in a vacuum, λ , are related by

$$\lambda \nu = c \quad (2.1)$$

where c is the speed of light in a vacuum, approximately 3×10^8 m/sec. In any other medium, $\lambda v = c/n = v$, where n is the refractive index of the medium and v is the velocity of light in the medium. Table 2.1 lists the various units common to the measurement of optical frequencies and wavelengths.

Table 2.1 Optical units.

Wavelength (λ)			
Unit	Designation	Value in meters	Range of common use
Micrometer	μm	10^{-6} m	Infrared
Nanometer	nm	10^{-9} m	Visible and UV
Angstrom*	$\text{\AA}$	10^{-10} m	Visible and UV
Frequency (ν) is expressed in hertz (Hz). (1 Hz = 1 cps)			
Unit	Designation	Value in hertz	Range of common use
Megahertz	MHz	10^6 Hz	Radio
Gigahertz	GHz	10^9 Hz	Microwaves, infared
Terahertz	THz	10^{12} Hz	Infrared, visible

* The *angstrom* ($\text{\AA}$) is still in use in the optical literature; however, by international agreement, quantities that in the past have been expressed in angstroms are now expressed in *nanometers* (nm).

Visible light is not the only kind of electromagnetic wave. The spectrum of electromagnetic radiation extends from the long-wavelength radio waves to x-rays and gamma rays at the shortest wavelengths, as illustrated in Fig. 2.1. Throughout this text, we shall confine our discussion of the properties of laser light to optical wavelengths, i.e., that part of the spectrum extending from the near infrared, through the visible, to the ultraviolet. The visible region of the spectrum (where

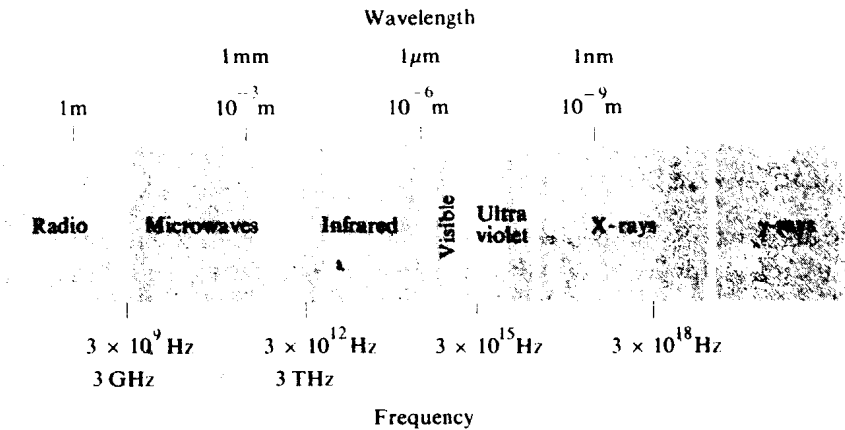


Fig. 2.1 The electromagnetic spectrum.

the human eye is sensitive) encompasses wavelengths from 400 nm (violet) to 700 nm (red). Below 400 nm and extending to 10 nm are the ultraviolet wavelengths; above 700 nm out to about 20 μm are the near-infrared wavelengths.

A discussion of the properties of light waves is greatly aided by a mathematical representation of a wave. In the late nineteenth century, James Clerk Maxwell showed that light waves in free space could be represented by a mathematical expression describing either the electric field or the magnetic field of the wave. Both are not necessary, as their behavior is complementary. Perhaps the simplest electromagnetic wave we can examine is the *monochromatic plane wave*, which is a sinusoidal wave of infinite extent that propagates in a single direction. If we choose the positive z -axis as the direction of wave propagation, the variation of the electric field of the wave in time and space, $E(x, y, z, t)$, is described mathematically by the expression

$$E(x, y, z, t) = A \cos \left[2\pi \left(vt - \frac{z}{\lambda} \right) + \phi \right] \quad (2.2)$$

where E is the value of the electric field at time t and at spatial coordinates (x, y, z) ; A is the amplitude of the wave; v is the frequency in hertz; λ is the wavelength (nm, μm , etc.); and ϕ is the phase constant. The term in brackets, called the *phase* of the wave, varies both as a function of time and as a function of the distance from the origin in the z -direction. By examining this expression at one instant in time and then again at one point in space, we can obtain a better understanding of the characteristics of the wave. In Fig. 2.2 we have plotted $E(x, y, z, 0)$ to show how E varies along the z -axis at the particular instant $t = 0$, assuming that $\phi = 0$. The corresponding mathematical description, from Eq. (2.2), is

$$E(x, y, z, 0) = A \cos \frac{2\pi z}{\lambda}. \quad (2.3)$$

In the plane $z = 0$, and in all parallel planes an integer number of wavelengths from this plane, the electric field is a maximum. In intermediate planes, the field

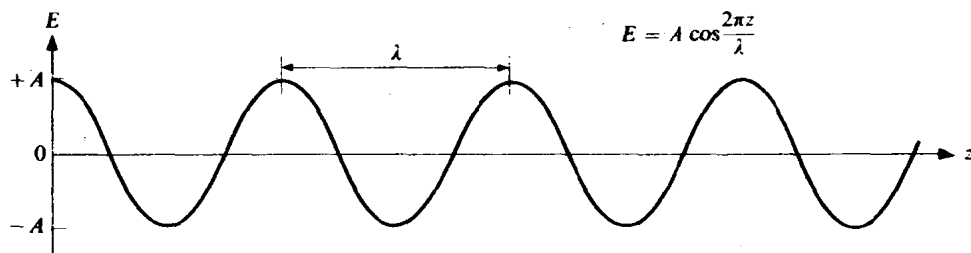


Fig. 2.2 Amplitude of an electromagnetic wave as a function of distance at one instant of time ($t = 0$). The separation between two maxima is the wavelength λ of the wave.

will be smaller in magnitude or negative, depending on the argument of the cosine. For example, any plane where $(2\pi z)/\lambda$ equals an odd multiple of 180° , $\cos(2\pi z)/\lambda$ is equal to -1 , and the electric field has its maximum negative value. The quantity $1/\lambda$ is appropriately referred to as the *spatial frequency* (the number of periods per unit distance) of the wave. We can simplify Eq. (2.3) by substituting the symbol k for $2\pi/\lambda$; k is simply the angular spatial frequency measured in radians per unit distance. With this substitution, the spatial dependence of the light wave at $t = 0$ becomes

$$E(x, y, z, 0) = A \cos kz, \quad k = \frac{2\pi}{\lambda}. \quad (2.4)$$

If we now examine the wave as a function of time as it passes the plane $z = 0$, as shown in Fig. 2.3, E is given by

$$E(x, y, 0, t) = A \cos 2\pi vt. \quad (2.5)$$

The number of oscillations of the wave per second at a fixed location equals the frequency, ν . Another convenient parameter is $\omega = 2\pi\nu$, which is the *angular temporal frequency* in radians per second. Expressed in terms of ω , the electric field amplitude of the wave at the plane $z = 0$ becomes

$$E(x, y, 0, t) = A \cos \omega t, \quad \omega = 2\pi\nu. \quad (2.6)$$

Allowing both t and z to vary again and using the radian measures of temporal and spatial frequency, we can describe the wave in the compact form

$$E(x, y, z, t) = A \cos (\omega t - kz). \quad (2.7)$$

A more general description includes the arbitrary phase constant in the argument,

$$E(x, y, z, t) = A \cos (\omega t - kz + \varphi) \quad (2.8)$$

which shifts the locations of the wave maxima and minima in time and space.

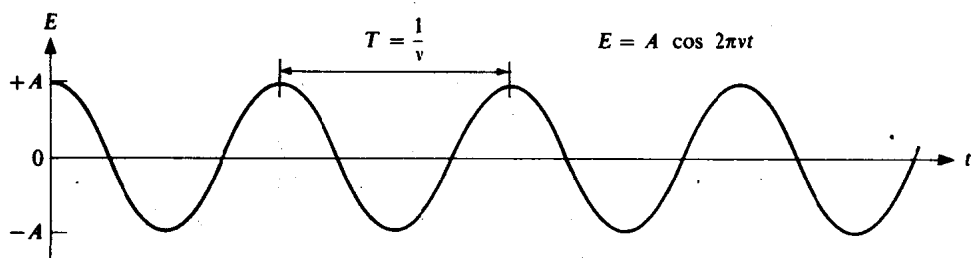


Fig. 2.3 Amplitude of an electromagnetic wave as a function of time at a point in space ($z = 0$). The period T , the time for one cycle, is the reciprocal of the frequency ν of the wave.

As time passes, the surfaces of constant phase move through space. These surfaces, or *wavefronts*, are planes in this simple case. Although E is written as a function of x and y as well as of z and t in Eq. (2.8), the only spatial dependence is in the z -direction. Thus the expression describes *plane waves* of infinite extent traveling in the z -direction.

At first, the notion of an infinite plane wave might be difficult to comprehend, it being a mathematical abstraction. In time, however, one comes to recognize the plane wave as the simplest expression we have for a propagating light wave. The simple cosine expression and the idea of an electric field having the same value across large surfaces often serves as a good approximation to real waves.

We can generalize our mathematical description to include a plane wave traveling in an arbitrary direction. Such a wave is characterized by a *propagation vector*, or *wave vector*, $\mathbf{k}$. Propagation vector $\mathbf{k}$ has a length equal to $2\pi/\lambda$, namely

$$k = |\mathbf{k}| = \sqrt{\mathbf{k} \cdot \mathbf{k}} = \frac{2\pi}{\lambda}. \quad (2.9)$$

The direction of $\mathbf{k}$ is the same as the direction of propagation of the plane wave. Under these circumstances, the expression for $E(x, y, z, t)$ becomes

$$E(x, y, z, t) = A \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi) \quad (2.10)$$

where $\mathbf{r}$ is the vector from the origin to the point (x, y, z) . The product $\mathbf{k} \cdot \mathbf{r}$ can be expressed in terms of the vector components of $\mathbf{k}$ and $\mathbf{r}$. For example, if the wave-propagation vector lies in the x - z plane making an angle θ with the positive z -axis, as shown in Fig. 2.4, then

$$\mathbf{k} = k_x \hat{e}_x + k_z \hat{e}_z = k \hat{e}_x \sin \theta + k \hat{e}_z \cos \theta \quad (2.11)$$

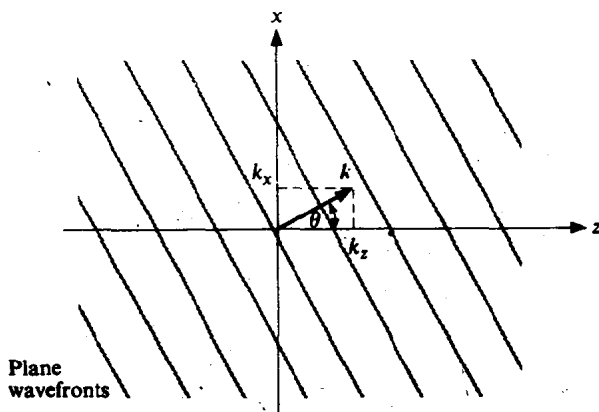


Fig. 2.4 Plane wave with propagation vector $\mathbf{k}$ in the x - z plane (the y -axis is perpendicular to the page). The components of the propagation vector, $k_x = |\mathbf{k}| \sin \theta$ and $k_z = |\mathbf{k}| \cos \theta$, are shown.