MICHAEL C. KELLEY BENJAMIN NICHOLS

INTRODUCTORY
LINEAR
ELECTRICAL
CIRCUITS
AND
ELECTRONICS

Introductory
Linear
Electrical
Circuits
and
Electronics

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Preface

The study of electrical engineering is often described under two major headings: *electrophysics* and *electrosystems*. An electrical system in its most general definition consists of physical devices connected together to perform a particular function. A detailed understanding of such a system requires knowledge of the physical principles that underlie the properties of the devices. These include the elements of electrodynamic theory, material science, solid state and quantum physics, to name just a few. The understanding also requires knowledge of the principles that govern the interconnections of these devices into a system and the response of the system to certain inputs ("signals"). These include the mathematical tools required to solve the equations of the system and the choice and description of the particular time functions to represent the signals.

In this introductory text the emphasis is on the system approach, and no previous study of electrical circuits is assumed. It is expected, however, that students have taken (or are taking concurrently) introductory physics courses including electrodynamics. Knowledge of introductory calculus is also assumed. Since the text is primarily geared toward the normal sophomore level, some mathematical methods are developed as needed. An example is the algebra of complex numbers which the typical student may not yet fully understand.

At the end of each chapter there is a section entitled "Practice Problems and Illustrative Examples." These sections are an integral part of the text. Engineering is the process of investigating how to solve problems. Consequently, understanding of the concepts is developed by the practice of applying the concepts to the solution of problems. The "Illustrative Examples" demonstrate that application and bring out some of the features of problem solution that are not discussed specifically in the body of the chapter. There are also a number of examples worked out within the text itself which are meant to reinforce the material as students encounter it. Some sections are marked with asterisks, which means that the material is not necessary for the logical flow of the text. They will provide students with some insights that may be valuable later.

The term "linear" in the title of the text is important. As with most fields of technical and scientific study, the systems with which we deal often display the property of linearity. One important consequence of this property is that if two solutions satisfy the equations which govern the system, then their sum will also satisfy the same equations. This leads to a number of simplifications in the mathematics and analysis. In fact, the mathematical benefits are so great that we create linear models of those devices we wish to describe in the text, but which are themselves inherently nonlinear. Elementary examples of these include diodes and transistors. We also make extensive use of an important integrated circuit element termed the *operational amplifier*. When a system includes such devices as diodes, transistors, and operational amplifiers, the system is usually called "electronic." It makes sense therefore, to begin the study of electrical engineering with an introduction to linear electrical circuits and electronics.

In Part A we present the fundamental elements of circuit theory. Although most students have had some contact with electrical circuits, we spend some time in Chapter 1 on fundamental concepts to define the basis for the text material. The

standard circuit laws and theorems are developed and applied in Chapters 2 and 3, and Chapter 4 introduces dependent sources, which are used throughout the text. Two-port systems are also described in Chapter 4 and the ideal operational amplifier is introduced as a circuit element.

In Part B we discuss the concept of energy storage elements and show that their use in electrical circuits leads to the necessity of mathematical analysis using differential equations. The solutions to such equations are well known and lead in a straightforward manner to the introduction of the parameter s and the impedance function Z(s). The latter is used extensively in solutions to the natural and forced response of first-order electrical systems in Chapter 6. Emphasis is placed upon the concept that the natural response characterizes the response of a system when the energy sources are set equal to zero. Phantom sources of value zero are used to illustrate this and the pole-zero method then evolves very naturally. The circuit response to pulses and pulse trains is also presented.

In the last part of Chapter 6 a second order system is analyzed using the impedance method and it is found that an imaginary value for s arises. This leads to the use of complex algebra, which is reviewed in Chapter 7. This chapter is the first in a series of three chapters dealing with the response of electrical systems to signals characterized by a single frequency ω . Since electrical utilities are the most important example of such systems, they are studied in some detail in Chapters 8 and 9. Power factors, complex power, ideal transformers, generators, and three-phase power are all discussed and numerous examples presented in the text and the problem sets.

In Part D students can extend their capabilities to include analysis of the response of linear systems to signals which have a more complicated time dependence than that characterized by a single frequency. Fourier's theorem is at the heart of this analysis and is introduced as a fundamental mathematical concept. The transfer function plays an essential role in the development and is used to implement the analysis methods. The detailed study of Fourier-Laplace methods are beyond the scope and needs of the present text, but the fundamental role they play in the analysis is made clear and should provide a solid conceptual basis for upper-level courses. We show that the transfer function may be used to determine both the natural and the forced response. Again the pole-zero method arises in a natural fashion. We continue with a discussion of the frequency response of electrical systems. The db unit is introduced as are Bode plots. A general solution is found in terms of the poles and zeroes of the transfer function which is then specialized to the case of real poles and zeroes. The latter case is analytically tractable and many examples are discussed including the design of simple Op-Amp filters. In Chapter 12 we consider second-order systems and such concepts as critical damping and the quality factor are described. We also describe analytical and graphical solutions for the case of complex poles or zeroes and the resonant response of second-order systems.

The linear elements used up to this point are now supplemented with non-linear elements that are based upon the use of semiconductor materials. The response of a pn junction is crucial to the usefulness as well as the nonlinear behavior of such electronic devices. In Chapter 13 a simple description of the pn junction physics is provided, but emphasis is placed upon the I-V characteristics of the diode. Diode models and rectifier circuits are discussed. A linear model of the diode is described as background material for the more important linear models needed to understand transistor amplifiers. In Chapter 14 two generic transistors are introduced: the n

channel enhancement MOSFET and the *npn* bipolar junction transistor. There is no attempt to describe the entire range of transistor types available. Rather, two important and qualitatively different types are chosen and analyzed in detail. This basis should be sufficient to allow students to understand other devices as they are introduced in later courses or in the laboratory. Linear small signal models are introduced in Chapter 14 along with the various dc biasing methods used. The full frequency response of *RC* transistor amplifiers is presented in Chapter 15 including coupling and stray capacitors, as well as parallel capacitors used to deliberately limit the frequency response of the amplifier. A discussion of voltage followers completes the small signal amplifier discussion.

The material can be covered in one semester, but the pace is quite fast and it may be necessary to leave out some material. The three-phase power section of Chapter 9 is fairly self-contained and could be eliminated. Also the section in Chapter 12 on second-order systems could be presented as lecture material only, something of a breadth component, but material for which the student is not fully responsible. Similarly in the last chapter on practical *RC* transistor amplifiers, it simplifies matters if students are not required to analyze circuits with self-bias resistors. Once again the lecture material could include a discussion of the practical benefits of including this feature. Accompanying Introductory Linear Electrical Circuits and Electronics is a special SPICE supplement written by Joseph Tront from Virginia Polytechnic University. Please contact your local sales representatives for more details.

The authors gratefully acknowledge the efforts by the numerous students who struggled through the early version of this text. We are particularly indebted to Andrew Noel, Jon Schoenberg, and Dianne Umpiere who worked on the early text and Stephanie Berg who helped with the solutions manual. Special thanks are due Laurie Shelton whose cheerful competence is a marvel to behold and Ali Avcisoy whose skill and attention to detail made the artwork takk feasible.

Michael C. Kelley

Benjamin Nichols

Contents

SECTION A

CIRCUIT ANALYSIS / 1

Chapter 1

Fundamental Elements of Circuit Theory / 3

- 1.1 Voltage and Current / 3
- **1.2** Electrical Resistance / 7
- 1.3 Ideal Circuit Elements / 8
- 1.4 The Elemental Circuit / 11
- 1.5 Measuring Devices / 12
- 1.6 Flectrical Power / 14
- *1.7 A Simple Model for Electrical Resistance / 15

Chapter 2

The Circuit Laws / 29

- 2.1 Kirchhoff's Voltage Law / 29
- 2.2 Kirchhoff's Current Law / 31
- 2.3 Some Useful Consequences of Kirchhoff's Laws / 37
- 2.4 Applications of KVL and KCL to More Complicated Circuits: A Standardized Approach / 43
- 2.5 Time-Varying Voltages and Currents / 45

Chapter 3

Circuit Theorems and Analysis Methods / 59

- **3.1** Linearity and Superposition / 59
- 3.2 Thevenin and Norton Equivalent Circuits / 65
- **3.3** Applications of Equivalent Circuits—The Source Transformation Method and the Maximum Power Transfer Theorem / 72
- 3.4 Introduction to the Node Voltage and Loop Current Methods 73
- 3.5 Further Comments on the Node Voltage and Loop Current Methods / 77

Chapter 4

Controlled Sources and Two-Port Systems / 97

- **4.1** Controlled Sources / 97
- **4.2** Two-Port Systems / 103
- **4.3** Voltage Amplifiers / 105
- 4.4 Further Examples of Three Parameter Models / 108
- **4.5** The Operational Amplifier / 112
- *4.6 Negative Feedback and Op-Amp Circuits / 118

^{*}Sections marked with an asterisk are not required for the logical flow of the text.

SECTION B

CIRCUITS INCLUDING ENERGY STORAGE ELEMENTS / 139

Chapter 5

Inductors and Capacitors / 141

- **5.1** Inductance and Inductors / 141
- **5.2** Capacitance and Capacitors / 148
- **5.3** Energy Storage / 153
- **5.4** Applications of Kirchhoff's Laws / 155
- **5.5** Capacitors and Inductors in Op-Amp Circuits / 157
- 5.6 Summary Results for Inductors and Capacitors / 158
- *5.7 The Displacement Current / 159

Chapter 6

Circuits With Energy Storage Elements / 173

- **6.1** Natural Response / 173
- **6.2** General Solutions to the Differential Equations of Circuit Theory / 177
- **6.3** The Impedance Function / 179
- 6.4 Natural Response Using the Impedance Function / 182
- 6.5 Complete Response of First-Order Systems with DC Sources / 189
- **6.6** Circuit Response to Pulses / 195
- **6.7** Natural Response of a Second-Order System: Oscillatory Behavior / 198

SECTION C

SINGLE-FREQUENCY SYSTEM AND ELECTRICAL POWER / 219

Chapter 7

Complex Numbers and the Phasor Notation / 221

- 7.1 Complex Number Algebra and Sinusoidal Functions / 221
- 7.2 The Impedance Function Revisited / 226
- 7.3 Application of Complex Algebra to Oscillatory Signals / 230
- 7.4 Phasor Notation and Maximum Value Phasors / 237
- 7.5 Effective Value Phasors / 241
- **7.6** Summary / 245

Chapter 8

Forced Response to Sinusoidal Signals and Power Computation / 261

- 8.1 The Impedance Triangle and Phasor Diagrams / 261
- **8.2** Reactance / 266
- **8.3** Power Computation Using Phasors and Impedance / 270
- 8.4 The Power Factor and the Concept of Complex Power / 275
- 8.5 Summary of Power Computation Methods / 280
- 8.6 Measuring Instruments / 281

Chapter 9

Electric Power Systems / 295

- 9.1 Voltage Levels / 295
- 9.1 The Transformer / 296
- 9.3 Generator Action / 300
- 9.4 Three-Phase Systems / 304
- 9.5 Ratinas / 310

SECTION D

FREQUENCY RESPONSE OF ELECTRICAL SYSTEMS / 321

Chapter 10

Transfer Functions, Spectral Synthesis, and Complete Response Revisited / 323

- **10.1** The Transfer Function / 323
- 10.2 Fourier's Theorem / 328
- 10.3 Spectral Synthesis / 333
- 10.4 Complete Response Revisited: Sinusoidal Forcing / 338
- *10.5 A Note on the Laplace Transform / 345
- **10.6** Summary / 346

Chapter 11

Frequency Response—Analysis and Design / 365

- 11.1 Frequency Response—A Case Study / 365
- 11.2 Frequency Response—A General Formulation / 371
- **11.3** A General Approach to the Frequency Response For Real Poles and Zeros—The Bode Plots / 377
- 11.4 Frequency Response of Capacitor-Resistor Op-Amp Circuits / 385
- *11.5 Feedback Revisited / 396

Chapter 12

Second-Order Circuits / 413

- **12.1** Circuits with Multiple Energy Storage Elements / 413
- 12.2 Natural Response of Circuits with Inductors and Capacitors / 418
- **12.3** Complete Response of Second-Order Underdamped Systems / 425
- **12.4** Frequency Response of Second-Order Systems—Complex Poles and 7eros / 429
- 12.5 Second-Order Filters / 434

SECTION E

ELEMENTARY ELECTRONIC DEVICES / 451

Chapter 13

Semiconductor Diodes and Diode Circuits / 453

- **13.1** Semiconductor Materials / 453
- 13.2 Semiconductor Diodes / 455

- 13.3 Operation of Semiconductor Diodes / 457
- **13.4** Diode Models With Application to Rectifier Circuits / 463
- 13.5 Elements of Power Supply Design / 467
- **13.6** The Response of Diodes to Small Time-Varying Signals / 472

Chapter 14

Transistors and Elementary Transistor Amplifiers / 493

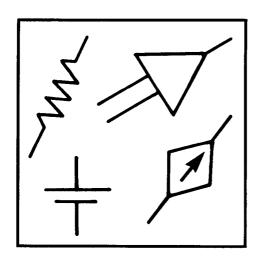
- **14.1** MOSFET Transistors / 493
- 14.2 A Linear Model for the MOSFET Transistor / 501
- **14.3** Bipolar Junction Transistors / 506
- 14.4 Linear Model for the BJT Amplifier / 512
- **14.5** Transistor Biasing / 514
- **14.6** Bias Stabilization Circuit: MOSFET / 520
- 14.7 Bias Stabilization Circuit: Bipolar Junction Transistor / 524
- *14.8 The h-Parameter Model / 527
- **14.9** Summary / 529

Chapter 15

An Introduction to Realistic Transistor Amplifiers / 547

- 15.1 Frequency Response of Transistor Amplifiers: Coupling Capacitors / 547
- **15.2** High-Frequency Response of Transistor Amplifiers: Stray and Parallel Capacitors / 552
- **15.3** A Complete Frequency Response Analysis of *R*–C Transistor Amplifiers / 555
- 15.4 Bias Stabilization Revisited / 561
- **15.5** The Emitter Follower / 565

Index / 583



SECTION A CIRCUIT ANALYSIS

The field of electrical engineering uses concepts from physics, mathematics, material science, and a number of other disciplines. In this section we begin by relating the two fundamental parameters used by electrical engineers, namely, voltage and current, to principles from electromagnetic theory. The physical and mathematical properties of a number of circuit elements are then defined based upon their current and voltage characteristics. These are enough to begin our study and, as the text progresses, we shall add new circuit elements as they naturally arise. Conservation principles from physics, along with the mathematical properties of the equations that describe electrical phenomena, are then used to derive several laws and theorems which may be used to analyze circuits. These include Ohm's and Kirchhoff's laws, the principles of linearity and superposition, Norton's and Thevenin's theorems, and so on. In Chapter 4 dependent sources and ideal operational amplifiers are added to the circuit elements with which we deal, and further practice is given in circuit analysis methods. We also introduce the important electrical system concept of a two-port, which accepts some voltage or current at the input, modifies it in some way, and presents a modified parameter at the output.



Chapter 1

Fundamental Elements of Circuit Theory

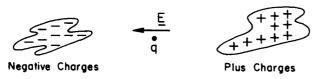
In this chapter we review elements of electrostatics and the response of charged particles to forces. The concepts of voltage, current, electrical energy, and electrical resistance are introduced and analogies are drawn to other physical systems. Various symbols are introduced, and so are the systems of units used in the text. Ideal voltage and current sources are defined, as are idealized instruments used to measure voltage and current.

Although the principles in this chapter are elementary, their mastery is very important for the subsequent material. Considerable care has been taken in the definition of the various parameters, particularly with respect to the sign conventions used in analysis. Rationalized MKS units are used throughout the text.

1.1 VOLTAGE AND CURRENT

Electric charges have units of coulombs and can be positive or negative. Charged particles exert forces on each other which are conveniently described by introduction of the electric field, \mathbf{E} . The boldface denotes that \mathbf{E} is a vector quantity and has a magnitude and direction. The force on a charged "test" particle of charge q is given by $\mathbf{F} = q\mathbf{E}$ (Fig. 1.1). Inasmuch as there are particles with both positive and negative charge values, the particle motion can either be parallel or antiparallel to \mathbf{E} . The units of \mathbf{E} are newtons per coulomb. A close analogy exists with the gravitational force, which is given by $\mathbf{F} = M\mathbf{G}$, where M is the particle mass and \mathbf{G}

FIGURE 1.1. Separated charges produce an electric field pointing from positive toward negative charge. The units for \boldsymbol{E} are newtons/coulomb.



F = qE

is the gravitational field. As far as we know there are no negative mass particles, hence the motion due to a gravitational force is always parallel to G. The units of G are newtons per kilogram.

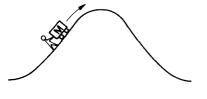
In dealing with electrical circuits it is more convenient to work with an energy parameter, the voltage, rather than the electric field itself. This parameter is measured in joules per coulomb, which corresponds to energy per unit charge. The unit is the volt. In the case of gravity it is easy to see how energy may be stored or released. When a cart is pushed up a hill, gravitational energy is stored (Fig. 1.2). This stored energy may then be released during the descent. Note that energy is stored by moving the cart against the gravitational field and is released when the cart is allowed to fall down with a component of velocity parallel to G. For any reasonable-size hill of height h, the earth's gravitational field may be considered to be a constant value g, and the gravitational potential energy is given by Mgh. If we divide this quantity by M, we obtain the gravitational potential gh, which is equal to the potential energy per unit mass, in joules per kilogram.

Likewise, in an electrical system, energy is stored when charges are caused to move against the electrical force exerted upon them. The agent that accomplishes this might be chemical, as in a battery, or it might be supplied by photons as in a solar cell. Quantitatively, the amount of electrical energy given to a charge of q when it is moved against a constant electric field from point b to point a through a distance l is given by force times distance, or

$$W = qEl$$
 (joules) (1.1)

This expression is analogous to the gravitational energy Mgh discussed above. If we divide by the charge q, we obtain a quantity analogous to the gravitational potential, which we call the electrical voltage:

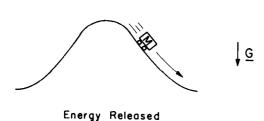
$$V = \frac{W}{q} = El$$
 (joules/coulomb) (1.2)



Energy Stored

FIGURE 1.2.

Gravitational energy is stored when the cart is pushed up the hill and released when it rolls down.



Unlike mass, electric charge comes in two forms, which we term positive and negative. This means we must pay particular attention to sign conventions in defining the voltage. Consider the simple representation of an electrical battery in Fig. 1.3. The figure carrying a positive charge q represents chemical forces inside the battery which transport charges in the direction opposite to the electrical forces to which they are subjected. This is analogous to the figure in Fig. 1.2 pushing the mass against gravity. The electrical potential at point a with respect to point b we therefore define as positive, just as we consider the gravitational potential to be positive at the top of a hill. The voltage difference across the battery then is $\Delta V = V_a - V_b$.

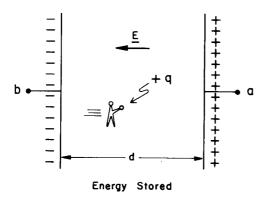
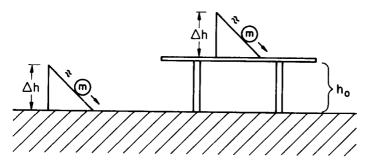


FIGURE 1.3. Electrical energy is stored either when positive charges are moved in the direction opposite to an electric field or when negative charges are moved parallel to an electric field.

It is important to realize that only voltage differences have physical meaning. This can be understood again with reference to a gravitational analogy such as the one illustrated in Fig. 1.4. The inclined plane and the ball of mass m have the same properties in both halves of the figure, but the entire unit on the right-hand side has been elevated by the distance h_0 . The energy available after the ball rolls down the plane, $mg\Delta h$, is the same in both cases and only depends upon the height, Δh , of the inclined plane. The extra energy (mgh_0) of the unit does not contribute at all to the velocity of the ball. It is the energy difference due to the height of the inclined

FIGURE 1.4. An elevated massless inclined plane yields an identical energy output, independent of the "reference voltage" aho.



plane which is analogous to the voltage difference ΔV across a battery. An arbitrary additive constant V_0 can therefore be added to every voltage in an electrical system without changing the properties of the circuit.

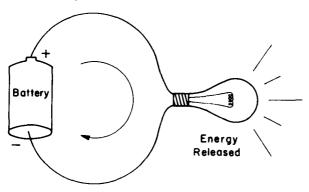
This feature may be used to advantage in circuit analysis since it allows us to arbitrarily define the voltage at one point equal to zero volts. This reference point is usually termed ground potential because it is often physically connected by a wire to the earth's surface. Once such a reference point, b, is defined in a circuit we often refer to the voltage at some other point, a, in the circuit such as V_a . By this we mean the voltage $\Delta V = V_{ab} = V_a - V_b$, which just equals V_a since the voltage at b is defined to be zero. This is such a common practice in circuit design that henceforth we will often discuss the "voltage at a point" with the tacit understanding that we mean the difference in potential between that point and the zero-volt reference.

The flow of charge, the electrical current *I*, is the second important parameter of electrical engineering. The units of *I* are coulombs per second, that is, the amount of charges per unit time that flow past a given point. One coulomb per second is called an ampere, which is abbreviated A. In principle, current may be carried by charges of either sign. In most materials negatively charged electrons carry the current because they are very light and move much faster than heavy positively charged particles.

In Fig. 1.5 a circuit is shown in which energy is released in the form of heat and light when an external path allows current to flow from a battery through a light bulb. The positive and negative signs on the battery indicate that a positive charge would give up electrical energy in moving from the positive to the negative terminal along an external path. Likewise, a negative charge would give up its stored electrical energy in moving from the negative terminal to the positive terminal along an external path. In this example the electrical energy drawn from the battery appears as light and heat from the light bulb. Note that if the battery voltage is to remain constant, current must also flow through the battery to maintain the charge separation at the terminals of the battery. This current must be driven against the electric field inside the battery by the chemical reactions in the battery. Commonly used voltage and current values are listed in Table 1.1.

FIGURE 1.5.

Charges falling off the "electrical hill" can release stored electrical energy and, for example, light a bulb.



Common ronago ana carron como					
picovolt	pV	10 ⁻¹² Volt	picoamp	pA	10 ⁻¹² Ampere
nanovolt	nV	10 ⁻⁹ Volt	nanoamp	nΑ	10 ⁻⁹ Ampere
microvolt	μV	10 ⁻⁶ Volt	microamp	μ A	10 ^{−6} Ampere
millivolt	mV	10^{-3} Volt	milliamp	mA	10 ^{−3} Ampere
volt	V	1 Volt	amps	Α	1 Ampere
kilovolt	kV	10 ³ Volt	kiloamp	kA	10 ³ Amperes
megavolt	MV	10 ⁶ Volt	megamp	MA	10 ⁶ Amperes

TABLE 1.1 **Common Voltage and Current Units**

1.2 **ELECTRICAL RESISTANCE**

To use stored electrical energy in a sensible way, the engineer must be able to control the rate at which current flows from an energy source such as a battery. In the gravitational analogy the mass current down the hill is determined not only by the height of the hill, but by friction in the wheels and by the resistance of the air. Likewise, the current flow from a battery can be limited by the friction or resistance to current flow of the elements attached to the battery. Elements used to control the flow of electric charge are termed resistors.

Materials can be divided into categories depending upon how easily they allow the passage of electric charges. If a material is not a good conduit for charged particles it is called an insulator, while if it easily passes current it is labeled a conductor. For example, metals such as copper and silver are quite good conductors, while rubber and many plastics are good insulators. The relative conductivities of common materials differ by 22 orders of magnitude. This enormous range is indicative of the great control an engineer can have over the flow of current.

Experiments show that most materials behave in such a way that the current through them is directly proportional to the voltage across them. This property of materials is most often expressed as Ohm's law,

$$V = IR \tag{1.3}$$

where R is termed the resistance of the material. In Section 1.7 at the end of this chapter a very simple mathematical model is constructed to represent a crystalline material. For this model we are able to derive Eq. 1.3 and to relate it to the geometry and internal makeup of the material. Section 1.7 is marked with an asterisk to indicate that it need not be considered as required reading but rather as background material for the rest of the book. Such sections appear throughout the text and act as a linkage with physics, materials science, mathematics, or other course work.

Equation 1.3 is a linear relationship between V and I. The expression may also be written in the form

$$I = \frac{V}{R} \tag{1.4}$$

in which the current I may be considered to be a function of the voltage V. If we plot the current as a function of the voltage for a resistor, the result is a straight line such as the one illustrated in Fig. 1.6. Such a plot is called the I-V curve for the