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edited by
S. Fdida
G. Pujolle

MODELLING TECHNIQUES AND PERFORMANCE EVALUATION

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Paris, France, 9–11 March, 1987

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edited by

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PREFACE

Today's computer systems handle more functions for more users than ever before. As a result, they are becoming very complex. As users become increasingly reliant on computer systems, the demand for good performance grows, creating a need for better modelling techniques and tools in the performance area. In the last few years, significant advances and contributions in this area have been made. This workshop aims at being a meeting point where researchers and practicians can listen to each others' concerns and where the state of the art in both theoretical and practical aspects of systems performance evaluation can be presented.

The workshop focuses on modelling and measurement techniques and packages. It also covers the use of artificial intelligence tools in this area.

We would like to express our thanks to the members of the program committee and to the reviewers without whose careful work it would have been impossible to provide the standards of these proceedings. In addition, thanks are extended to the invited speakers and to the authors of submitted papers. Without the kind cooperation of these people this workshop could not have been realized.

We would also like to express our appreciation to AFCET for its substantial support, to ENST and MASI Lab for their help, and to the sponsors of this workshop.

We believe that this workshop provides a good opportunity to those interested in the future advances in the field of performance evaluation of computer systems.

Finally, we would like to thank the workshop attendees and the readers of these proceedings.

Guy PUJOLLE
Serge FDIDA

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TABLE OF CONTENTS

I

MODELLING TECHNIQUES

Modelling Techniques and Performance Evaluation
S. Fdida and G. Pujolle (Editors)

ON THE COMPLEXITY OF THE MATRIX-GEOMETRIC SOLUTION OF EXPONENTIAL OPEN QUEUEING NETWORKS WITH BLOCKING

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ABSTRACT

Open exponential queueing networks with blocking have a rate matrix which has a block tri-diagonal structure and, therefore, they can be analyzed using Neuts' matrix-geometric procedure. The essential step in the matrix-geometric procedure is the evaluation of the matrix R , the unique non-negative solution of a matrix quadratic equation. The matrix R is determined iteratively by successive approximations. In this paper, the time and storage complexity associated with the calculation of the matrix R for open exponential queueing networks with blocking is examined.

I. INTRODUCTION

Queueing network models consisting of arbitrarily linked service facilities with finite queues have been recently receiving increasing attention in the literature. This is primarily due to the recognition of the fact that these models can be very helpful in understanding the behavior of complex systems such as distributed systems, communication networks and manufacturing systems. Due to the limitations imposed on the queue sizes in these systems, the flow of units (customers, jobs, packets etc.) through a facility is liable to getting blocked when the capacity limitation of a destination facility is attained. Various types of blocking mechanisms have been considered in the literature so far and a comparison among them can be found in Onvural and Perros [6].

Queueing networks with finite queues have been mainly analyzed using analytic approximations, simulation techniques and numerical techniques. Exact closed form solutions have been obtained in some cases. A bibliography of papers in which analytic investigations (exact or approximate) or numerical investigations of queueing networks with blocking is given in Perros [7].

In this paper, we are concerned with matrix-geometric numerical solutions of queueing networks with blocking. Numerical investigations of such queueing systems have been reported in the literature. Hillier and Boling [2] studied finite capacity queues in tandem with exponential or Erlang service times. They used a modified version of Gauss-Seidel method in order to obtain the joint steady state probabilities of the number of units in the network. Latouche and Neuts [3] used the matrix-geometric approach in a two node system with a finite intermediate queue. Each node consisted of several servers with exponential service times. Shanthikumar and Tien [8] also used the matrix-geometric approach to calculate the performance measures in a two machine synchronized transfer line where machines are subject to failures. Muth and Yeralan [4] numerically computed the steady state probabilities in a two station transfer line utilizing the block diagonal structure of the underlying transition probability matrix. Altiok and Stidham [1] obtained performance measures in a three station transfer line using the power method.

In this paper, we show how the matrix-geometric procedure can be used to analyze queueing networks with blocking involving more than two nodes. Time and storage complexity issues are discussed. More specifically, we consider queueing networks with blocking consisting of nodes which are arbitrarily connected. Each node is represented by a finite queue served by a single exponential server. External arrivals at each node occur in a Poisson process. Customers in each queue are served in a FIFO discipline. Only one class of customer is considered. The topology of the queueing networks considered here is restricted to configurations which do not give rise to deadlock. The particular blocking mechanism used in connection with these queueing networks is as follows: A customer upon completion of its service at queue i attempts to enter destination queue j. If queue j is at that moment full, the customer is forced to wait in front of server i until it enters destination queue j. The server remains blocked for this period of time and it can not serve any other customer waiting in its queue. This blocking mechanism has been used to model systems such as production lines , disk I/O subsystems and telecommunication systems. In section II, we review briefly Neuts' matrix-geometric procedure. In section III, we discuss how this procedure can be employed to analyze queueing networks with blocking, with a particular emphasis on time and storage complexity aspects of this procedure. Conclusions are given in section IV.

II.THE MATRIX-GEOMETRIC PROCEDURE

In the Markovian analysis of the network defined above, the state vector has to keep track of the number of units at each facility and the status of the server (i.e. blocked or working). The resulting continuous time irreducible Markov process denoted by {X} will have an infinitesimal generator Q of the following form after the states are ordered lexicographically.

$$Q = \begin{vmatrix} B_1 & C & & & & \\ B_2 & A_1 & A_0 & & & \\ & A_2 & A_1 & A_0 & & \\ & & A_2 & A_1 & A_0 & \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot \quad \cdot \end{vmatrix}$$

where B_1 is a square matrix of order N, the square matrices A_0, A_1 and A_2 are of order M, and matrices B_2 and C are rectangular with appropriate dimensions. M and N are integer variables and are functions of the queue capacities and the topology of the network. We should note that only one of the nodes in the network is allowed to have an infinite capacity, otherwise the above sub-matrices will not be finite. The diagonal elements of Q are strictly negative,and the off-diagonal elements are non-negative. The stationary probability vector, if it exists, is the solution to the system $xQ=0$, $xe=1$, where the column vector e has all elements equal to one. Let the vector x be partitioned as $x=(y,x_1,x_2,\ldots)$ where y is an N-vector, x_i, $i=1,2,\ldots$, are M-vectors. Also let $A=A_0+A_1+A_2$ be the infinitesimal generator for a finite continuous parameter Markov Process, which is assumed to be irreducible. Its stationary probability vector is denoted by π: $\pi A=0$, $\pi e=1$. The following theorem is proved by Neuts [5].

THEOREM: The Markov Process {X} is positive recurrent if and only if $\pi A_2 e > \pi A_0 e$. If the Markov Process is positive recurrent, then there exists a non-negative matrix R with spectral radius less than one, such that

$$x_i = x_1 R^{i-1} \text{ for } i>1 \quad (1)$$

The vectors y and x_1 are uniquely determined by

$$yB_1 + x_1 B_2 = 0 \quad (2)$$

$$yC + x_1(A_1 + RA_2) = 0 \quad (3)$$

$$ye + x_1(I-R)^{-1}e = 1 \quad (4)$$

The matrix R is the unique non-negative solution, with spectral radius less than one, of the matrix-quadratic equation

$$A_0 + RA_1 + R^2 A_2 = 0 \quad (5)$$

The matrix R is the limit of the monotonically increasing sequence of matrices $\{R_n, n>0\}$

$$R_0 = 0,\ R_{n+1} = A_0(-A_1)^{-1} + R_n^2 A_2(-A_1)^{-1},\ n>0\ . \quad (6)$$

It can be numerically determined using the recurrence relation (6).

III. COMPUTATIONAL CONSIDERATIONS

In this section, we will discuss the computational performance of the matrix-geometric procedure in three, four and five node networks. Let us first consider the three node triangular network shown in figure 1.

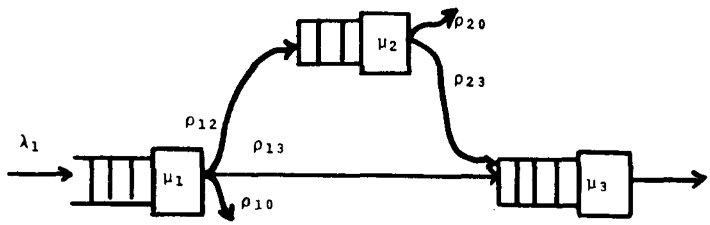

Figure 1: Triangle Configuration

As mentioned earlier, in the Markovian analysis of the triangular network, the state vector has to keep track of the number of units in each facility and the status of each server. Hence, the state of the network can be fully described by the vector (n_1, n_2, n_3), where $n_1 = 0,\ldots,N_1$, $n_2 = 0,\ldots,N_2+1$, and $n_3 = 0,\ldots,N_3+4$. N_1 can be finite or infinite. The random variable n_i , i=1,2,3, simply indicates the number of units in facility i which may vary from 0 to N_i. If N_1 is infinite then n_i may take any non-negative value. The additional values associated with random variables n_2 and n_3 are used to describe the various states associated with blocking. In particular, $n_2=N_2+1$ denotes that facility 1 is blocked by facility 2, $n_3=N_3+1$ denotes that facility 1 is blocked by facility 3, $n_3=N_3+2$ denotes that facility 2 is blocked by facility 3, $n_3=N_3+3$ denotes that facilities 1 and 2 are blocked by facility 3 in that order, $n_3=N_3+4$ denotes that facilities 2 and 1 are blocked by facility 3 in that order.

The numerical investigation of a queueing model involves the following stages: a) generation of the states, b) generation of the rate matrix and c) solving for the stationary probability vector.

The states can be generated in a lexicographical order using the following algorithm:

```
for i:=0 to N1 do
  for j:=0 to N2+1 do
    for k:=0 to N3+4 do
      if (i,j,k) is feasible then
          begin
            states(ind):=(i,j,k)
            ind:=ind+1
          end;
```

The states for the triangular network for the case $N_2=1$ and $N_3=2$ are as follows (i >1):

(0,0,0)	(i,0,0)	(i,1,3)
(0,0,1)	(i,0,1)	(i,1,4)
(0,0,2)	(i,0,2)	(i,1,5)
(0,1,0)	(i,0,3)	(i,1,6)
(0,1,1)	(i,1,0)	(i,2,0)
(0,1,2)	(i,1,1)	(i,2,1)
(0,1,4)	(i,1,2)	(i,2,2)
		(i,2,4)

Table 1

The generation of the rate matrix can be accomplished easily by going through the list of states and for each state generate all feasible transitions. In fact, due to the structure of the rate matrix, we only need to generate submatrices B_1, B_2, C, A_0, A_1 and A_2. These six submatrices are given below for the triangular case when $N_2=1$ and $N_3=2$, where λ_j is the external arrival rate at node j, μ_j is the service rate at facility j and p_{ij} are the branching probabilities. In general, the order, M, of submatrix A_i ,i=0,1,2, for the triangular case is:

$$M=(N_2+1)(N_3+1)+4N_2+N_3+3 \quad (7)$$

where N_j is the capacity of node j.

$$C = \begin{vmatrix}
\lambda_1 & & & & & & & & & & & \\
 & \lambda_1 & & & & & & & & & & \\
 & & \lambda_1 & & & & & & & & & \\
 & & & & \lambda_1 & & & & & & & \\
 & & & & & \lambda_1 & & & & & & \\
 & & & & & & \lambda_1 & & & & & \\
 & & & & & & & & \lambda_1 & & &
\end{vmatrix}$$

$$B_1 = \begin{vmatrix}
\mu_3 & & & & & \\
 & \mu_3 & & & & \\
\rho_{20}\mu_2 & \rho_{23}\mu_2 & & & & \\
 & \rho_{20}\mu_2 & \rho_{23}\mu_2 & \mu_3 & & \\
 & & \rho_{20}\mu_2 & & \mu_3 & \rho_{23}\mu_2 \\
 & & \mu_3 & & &
\end{vmatrix}
\qquad
B_2 = \begin{vmatrix}
\rho_{10}\mu_1 & \rho_{13}\mu_1 & & \rho_{12}\mu_1 & & & \\
 & \rho_{10}\mu_1 & \rho_{13}\mu_1 & & \rho_{12}\mu_1 & & \\
 & & \rho_{10}\mu_1 & & & \rho_{12}\mu_1 & \\
 & & \mu_3 & & & & \\
 & & & \rho_{10}\mu_1 & \rho_{13}\mu_1 & & \\
 & & & & \rho_{10}\mu_1 & \rho_{13}\mu_1 & \\
 & & & & & \rho_{10}\mu_1 & \\
 & & & & & \mu_3 & \\
 & & & & & & \rho_{10}\mu_1 \\
 & & & & & & \mu_3 \\
 & & & & & & \\
 & & & \rho_{23}\mu_2 & & & \\
 & & & & \rho_{20}\mu_2 & \rho_{23}\mu_2 & \\
 & & & & & \rho_{20}\mu_2 & \\
 & & & & & \mu_3 &
\end{vmatrix}$$

$$A_0 = \begin{vmatrix}
\lambda_1 & & & & & & & & & & & & & & \\
 & \lambda_1 & & & & & & & & & & & & & \\
 & & \lambda_1 & & & & & & & & & & & & \\
 & & & \lambda_1 & & & & & & & & & & & \\
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 & & & & & & & & & & & \lambda_1 & & & \\
 & & & & & & & & & & & & \lambda_1 & & \\
 & & & & & & & & & & & & & \lambda_1 & \\
 & & & & & & & & & & & & & & \lambda_1
\end{vmatrix}$$

$$A_1 = \begin{bmatrix}
-(\lambda_1+\mu_1) & & & & & & & & & & & & & & \\
\mu_3 & -(\lambda_1+\mu_1+\mu_3) & & & & & & & & & & & & & \\
 & \mu_3 & -(\lambda_1+\mu_1+\mu_3) & p_{13}\mu_1 & & & & & & & & & & & \\
 & & & -(\lambda_1+\mu_3) & & & & & & & & & & & \\
p_{20}\mu_2 & p_{23}\mu_2 & & & -(\lambda_1+\mu_1+\mu_2) & & & & & & & p_{12}\mu_1 & & & \\
 & & & & & -(\lambda_1+\mu_1) & & & & & & & p_{12}\mu_1 & & \\
 & & p_{20}\mu_2 & & & \mu_3 & -(\lambda_1+\mu_2+\mu_3+\mu_1) & p_{13}\mu_1 & p_{23}\mu_2 & & & & & p_{12}\mu_1 & \\
 & & & p_{20}\mu_2 & & & & -(\lambda_1+\mu_1+\mu_2) & & p_{23}\mu_2 & & & & & \\
 & & \mu_3 & & & & & & -(\lambda_1+\mu_1+\mu_3) & & & & & & p_{12}\mu_1 \\
 & & & & & & & & & -(\lambda_1+\mu_3) & p_{13}\mu_1 & & & & \\
 & & & \mu_3 & & & & & & & -(\lambda_1+\mu_3) & & & & \\
 & & & & & p_{23}\mu_2 & & & & & & -(\lambda_1+\mu_2) & & & \\
 & & & & & & & & & & & \mu_3 & -(\lambda_1+\mu_2+\mu_3) & & \\
 & & & & & & & & & & & & \mu_3 & -(\lambda_1+\mu_2+\mu_3) & p_{23}\mu_2 \\
 & & & & & & & & & & & & & & -(\lambda_1+\mu_3)
\end{bmatrix}$$

$$A_2 = \begin{bmatrix}
p_{10}\mu_1 & p_{13}\mu_1 & & p_{12}\mu_1 & & & & \\
 & p_{10}\mu_1 & p_{13}\mu_1 & & p_{12}\mu_1 & & & \\
 & & p_{10}\mu_1 & & & p_{12}\mu_1 & & \\
 & & \mu_3 & & & & & \\
 & & & p_{10}\mu_1 & p_{13}\mu_1 & & & \\
 & & & & p_{10}\mu_1 & p_{13}\mu_1 & & \\
 & & & & & p_{10}\mu_1 & & \\
 & & & & & \mu_3 & & \\
 & & & & & & & p_{10}\mu_1 \\
 & & & & & & & \mu_3 \\
 & & & & & & & \\
 & & & p_{20}\mu_2 & & & & \\
 & & & & p_{20}\mu_2 & p_{23}\mu_2 & & \\
 & & & & & p_{20}\mu_2 & & \\
 & & & & & \mu_3 & &
\end{bmatrix}$$