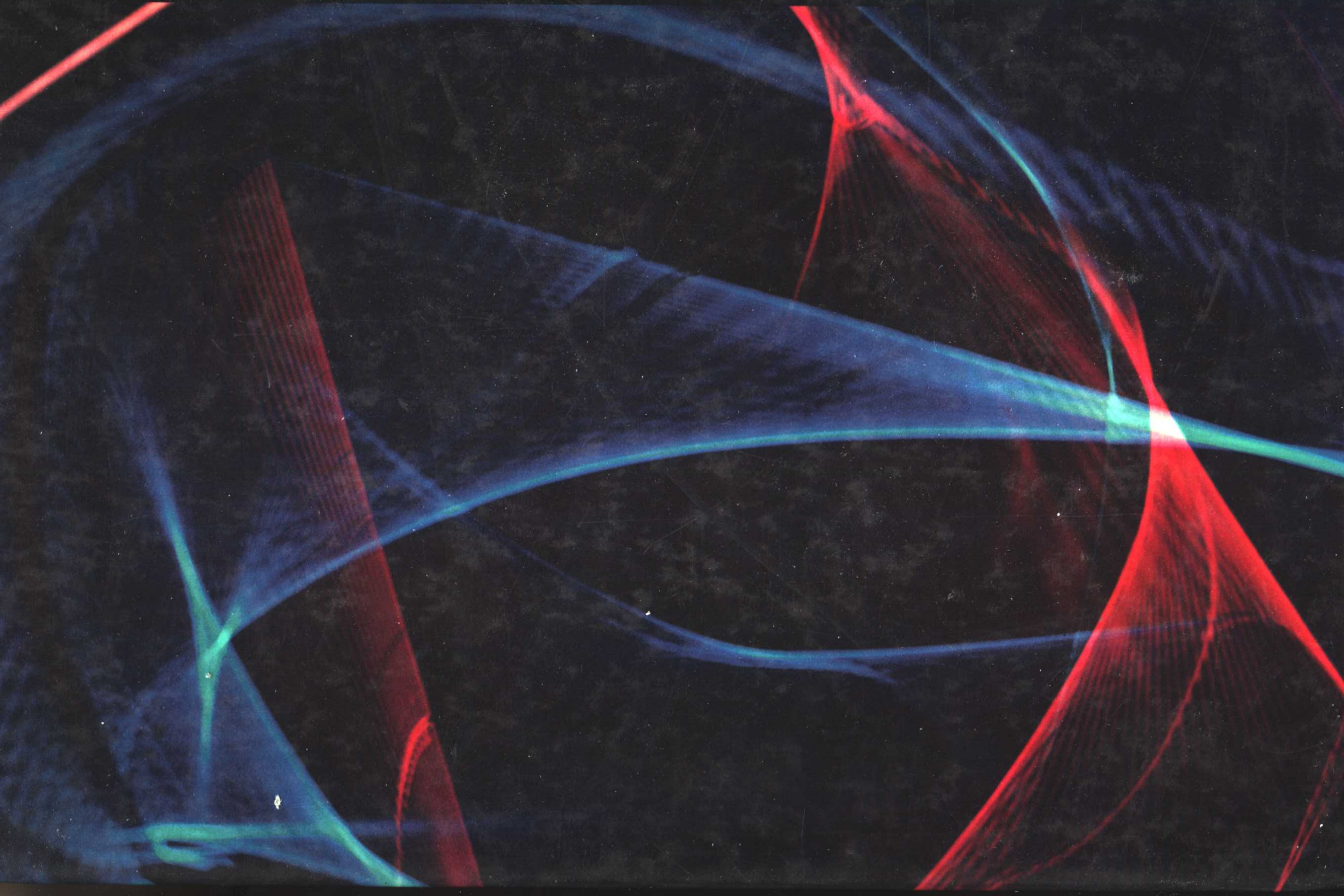
ALGEBRA AND TRIGONOMETRY

THIRD EDITION

RICE & STRANGE



Algebra and Trigonometry

THIRD EDITION

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Preface

The third edition of *Algebra and Trigonometry* is a result of our efforts to bring the calculator into proper perspective in precalculus mathematics. In this edition, the calculator is used as the primary tool for evaluating trigonometric, exponential, and logarithmic functions. This change from use of tables to the use of the calculator was motivated by our belief that the calculator is the most efficient approach and by the fact that students are now using calculators at all levels of mathematics education. Although we give the calculator and its uses in trigonometry and logarithms special attention, tables of these functions are still included in the book as optional material.

This edition of *Algebra and Trigonometry* maintains the three major features that made the first two editions successful: the functional concept remains the essential theme of the book; wherever possible the mathematics is presented in the context of real-world situations; and the book contains an abundance of well-graded exercises with over 500 worked out examples. More application-oriented problems are included in this edition, and steps have been added to the explanations of many of the existing examples so that they are more easily followed by the student.

As with the first two editions, this edition assumes that the student has obtained some ability in algebra, from either a high school or a post-high school elementary algebra course. However, we have included review of topics such as special products, factoring, exponents, and radicals. Heavy emphasis is placed on graphing throughout the text. Each chapter concludes with two chapter tests, and, new to this edition, a complete set of review exercises.

In this edition we have streamlined the discussions wherever possible by eliminating extraneous material and by combining some sections. These changes permit more efficient coverage of the material in one-semester and one-quarter precalculus courses.

Some of our specific changes to this edition include the following:

- Added emphasis is given to fractional expressions in Chapter 1, particularly to division of such expressions.

- The section on inverse functions is reorganized so that composite functions precede the introduction of the inverse function.
- The polynomial section is combined with the section on translation and reflection. This makes the latter discussion far less abstract and provides a meaningful vehicle for the graphing of most important polynomial functions.
- The chapter on exponential and logarithmic functions is reordered to emphasize the functional and graphical concepts needed in later work; the sections on common logarithms, interpolation and computation are at the end of the chapter and are clearly optional.
- The definitions of the trigonometric functions are made in terms of ratios of coordinates and the calculator supplants the tables as the principal means of computing the trigonometric functional values. Triangles are covered in a separate chapter. All of these changes make the coverage of trigonometry more compact.
- The material on vectors has been moved from triangle trigonometry to the chapter on vectors and complex numbers.
- The material on trigonometry is repositioned before the chapter on systems of equations, thus permitting earlier coverage.

In addition to the above changes, we have frequently included WARNING and COMMENT labels designed to highlight a common error or some pertinent explanation.

Since, as with most algebra and trigonometry texts, more topics are included than can be covered in a one-semester or one-quarter course, the book is structured to permit omission of certain topics without affecting the continuity of discussion. This feature makes the book appropriate for use in a variety of courses. For example, in a standard precalculus course we suggest the following procedure:

Chapters 1 and 2: The topics here can be emphasized as deemed necessary, taking into account the background of the student. The notion of *completing the square* and the coverage of the *quadratic formula* are important to the subsequent chapters.

Chapter 3: Coverage of Section 3.5 on *inverse functions* can be delayed until the concept is needed in Chapter 5.

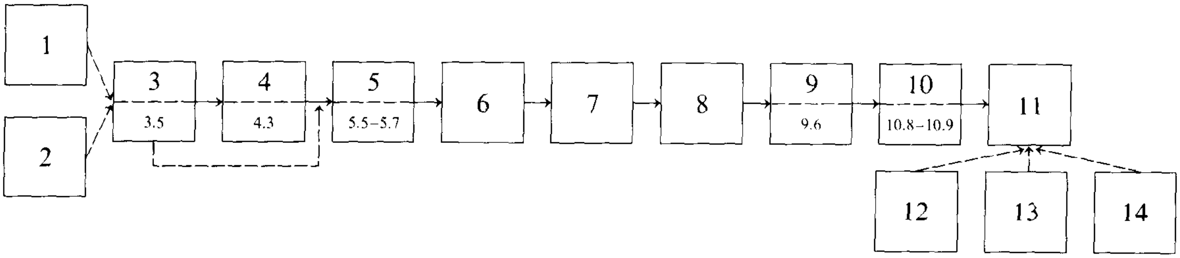
Chapter 4: The *graphing of polynomials* is not needed in the remainder of the book but is important to many precalculus courses.

Chapter 5: The last three sections may be omitted at the option of the instructor.

Chapters 6–9: This *trigonometry* material would normally be covered as an integral part of any precalculus course. Certain applications may, of course, be omitted and less attention may be given to the *solutions of triangles*. In a precalculus setting, the abundance of material on trigonometry is valuable for its continual sharpening of algebraic manipulative skills.

Chapter 10: The last sections, on *linear inequalities* and *linear programming*, are optional.

Chapters 12–14: These chapters present material that is usually not considered a specific prerequisite for calculus and may be covered at the option of the instructor.



Answers to the odd-numbered exercises are included in the back of the book. These answers have generally been worked out with the use of a calculator; thus some of your answers may differ slightly since decimal approximations often vary in accuracy.

This third edition of *Algebra and Trigonometry* has benefited from critical review and comments. Our thanks to the following: Robert B. Dressel, Kent State University; James C. Runyon, Rochester Institute of Technology; Linda Sons, Northern Illinois University; John S. Cross, University of Northern Iowa; Ahmad Abu-Said, Southern Technical Institute; and John Spellmann, Southwest Texas State University.

Finally, it is our pleasure to acknowledge the fine cooperation of the staff of Brooks/Cole, particularly our editor Craig Barth, production editor Gay Orr, art coordinator Rebecca Tait, and designer Jamie Brooks.

Bernard J. Rice
Jerry D. Strange

Contents

CHAPTER 1

Basic Algebra

Algebra is the branch of mathematics that uses numbers and symbols to express and analyze relationships between known and unknown quantities. In its most elementary form, algebra is an extension of arithmetic.

The word **algebra** comes from the Arabic word **al-jabr,** which was included in the title of a ninth-century work by Mohammed ibn Mûsâ al-Khowârizmî, *Hisâb al-jabr w'al-muquâbalah.* This text first explained some of the basic concepts used in working with known and unknown numbers. Latin translations later introduced European mathematicians to its contents, and in the process made the word "algebra" synonymous with the science of solving equations. Today, algebra means much more than equation solving, but it is always concerned with numbers.

1.1 / Real Numbers

Numbers are the central theme of algebra. Consequently, you must be familiar with the terminology of numbers. We begin with a discussion of the real number system: three important subsets of the real numbers are integer numbers, rational numbers, and irrational numbers.

The most familiar subset of the real numbers is the set of counting numbers 1, 2, 3, 4, 5, . . . , also called the **positive integers** or the natural numbers and denoted by N. The **negative integers** $-1, -2, -3, -4, -5, \ldots$, together with the positive integers and the number 0, make up the set of **integers** I.

A real number is said to be **rational** if it can be represented as a quotient a/b, where a is any integer and b is any nonzero integer. Numbers such as $-\frac{3}{2}$, $\frac{2}{7}$, 3, and $\frac{17}{11}$ are examples of rational numbers. The set of all rational numbers is denoted by Q. Since each integer, n, can be written as $n/1$, I is a proper subset of Q, as shown graphically in Figure 1.1.

Rational numbers can also be written in *decimal form,* as:

(1) **Terminating decimals;** for example,

$$\frac{1}{2} = 0.5, \qquad \frac{25}{4} = 6.25, \qquad \frac{19}{8} = 2.375, \text{ or}$$

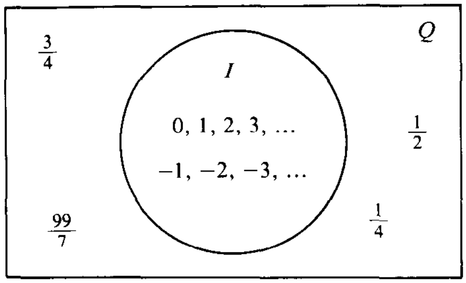


Figure 1.1

(2) **Infinite repeating decimals,** where infinitely many decimal places are necessary, but a block of digits continually repeats itself; for example,

$$\frac{1}{6} = 0.166666\ldots, \qquad \frac{11}{7} = 1.571428571428\ldots.$$

Instead of the three dots, a bar, called a **vinculum,** is often placed over the repeating block so that $\frac{1}{6} = 0.1\overline{6}$ and $\frac{11}{7} = 1.\overline{571428}$.

Some real numbers are not rational. For example, there is no rational number whose square is 2. (The real number whose square is 2 is denoted by $\sqrt{2}$, read "radical two.") Real numbers that cannot be expressed as the ratio of two integers are called **irrational.** Well-known examples are $\sqrt{2}$ and π.

Irrational numbers have nonterminating, nonrepeating decimal representations. (When we write $\sqrt{2} = 1.4142\ldots$ or $\pi = 3.14159\ldots$, it is understood that the decimal is nonterminating and that no block of digits repeats itself.) Thus, irrational numbers may be thought of either as

(1) Numbers that *cannot* be expressed as the ratio of two integers, or
(2) Numbers whose decimal representation is *not* terminating and *not* infinite repeating; for example, 0.1001000100001 . . . .

The rational and irrational numbers together make up the real numbers. Rational numbers and irrational numbers are mutually exclusive; that is, no rational number is irrational and, conversely, no irrational number is rational. Figure 1.2 graphically displays the hierarchy of real numbers.

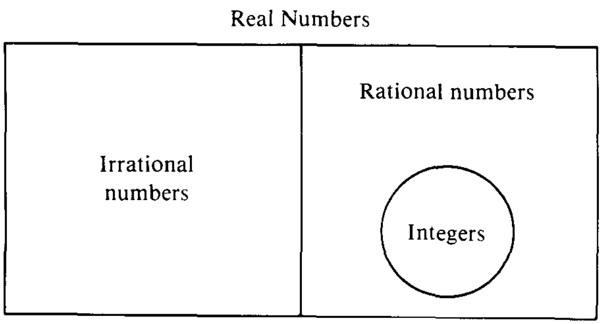


Figure 1.2

You should know the various kinds of real numbers and how they fit in the hierarchy of the real number system. For instance, 3 is a natural number, an integer, a rational number, and a real number; $\frac{1}{2}$ is *both* rational and real, but not an integer or an irrational number; and π is *both* irrational and real, but not an integer or a rational number.

Computers and calculators use ***truncated*** *numbers; that is, numbers rounded off after several decimal places. The number of digits and the method of representing truncated numbers vary with the particular computer or calculator; some calculators truncating to seven decimal places will display* $\frac{2}{3}$ *as* 0.6666667, *some as* 0.6666666. *In* 0.6666667, *the number is said to be* ***rounded up.*** *Most calculators round up if the truncated digit is* 5 *or greater.*

Throughout the text you will find special exercise problems to be solved using a calculator. Of course, a calculator can be used for any arithmetic operation, and we advise you to do so. However, a calculator will not reason for you. You must supply the mathematical reasoning and precise thinking.

In this text we do not tell you how to use a calculator, except in the most general terms, but you should become increasingly more skilled at using your calculator as you progress through the text and increase your mathematical knowledge. Since calculators differ in their method of operation, consult the operations manual of your calculator for its peculiarities.

1.2 / Operations with Numbers

Symbols of Grouping and Order of Operation

Parentheses, brackets, and braces are used to group numbers and indicate the precise order in which arithmetic operations are to be performed. For instance, $5 + (2 \cdot 3)$ indicates that the multiplication $2 \cdot 3$ is performed first and then added to 5 to give 11, while $(5 + 2) \cdot 3$ means that 5 is added to 2 before multiplying by 3 to give 21. Often the multiplication sign is omitted next to a grouping symbol. Thus, the expressions $4 \cdot (3 + 8)$ and $4(3 + 8)$ mean the same thing.

Confusion can arise if grouping symbols are omitted or if multiple grouping symbols are used. Therefore, we adopt the following conventions for sequences of arithmetic operations.* These conventions are also used in most computers.

(1) Perform all operations within any grouping symbol before performing other operations. If grouping symbols are contained within one another, begin with the innermost pair.

Example (a)
$$(7 \cdot 4) + (8 - 5) - (16 - 4) + (42 \div 6)$$
$$= 28 + 3 - 12 + 7 = 26$$

(b)
$$3 + \{6 + (2 + [7 - 2])\}$$
$$= 3 + \{6 + (2 + 5)\}$$
$$= 3 + \{6 + 7\}$$
$$= 3 + 13 = 16$$

* Note the distinction between a "law" and a "convention" governing mathematical operations. A law is a direct consequence of the nature of the operation. A convention is merely a convenient widespread usage of the operation.

(2) In a sequence of multiplications and divisions, perform the operations in the order in which they occur from left to right.

Example (a) $3 \cdot 18 \div 9 = 54 \div 9 = 6$

(b) $24 \div 8 \cdot 5 \div 15 = 3 \cdot 5 \div 15 = 15 \div 15 = 1$

(3) In a sequence of additions, subtractions, multiplications, and divisions, perform the multiplications and divisions first and then perform the additions and subtractions. Multiplication and division are said to be **higher priority** operations than addition and subtraction.

Example $5 \cdot 6 - 3 \cdot 7 + 24 \div 8 = 30 - 21 + 3 = 12$

Most scientific calculators can group operations together by a [( ] *key and a* [ )] *key. Keystrokes performed after pushing the* [( ] *key and before pushing the* [ )] *key are separated from the sequence of operations outside the grouping symbol. For example,* $3 \cdot (6 + 4) + 7$ *is evaluated by the following sequence:*

3 [×] [( ] 6 [+] 4 [ )] [+] 7 [=] 37.

If the grouping keystrokes are not included, the display will show 29 *since* $3 \cdot 6$ *is performed first.*

Fundamental Laws

Five basic laws govern the operations of addition and multiplication. Although you may not be aware of their specific nature, you already use these laws every day. For instance, we know that the sum of two numbers is independent of the order of the numbers. Thus, $2 + 7 = 7 + 2$. This property, which is called the **commutative law for addition,** is valid for all real numbers. Another addition law, called the **associative law for addition,** states that the sum of three or more numbers is the same regardless of how they are grouped for addition; that is, $2 + (9 + 1) = (2 + 9) + 1$. We use x, y, and z to represent real numbers and state the laws that govern addition and multiplication for your reference.

- **Commutative Law of Addition: $x + y = y + x$**
  For example, $5 + 2 = 2 + 5$.
- **Commutative Law of Multiplication: $x \cdot y = y \cdot x$**
  For example, $2 \cdot 7 = 7 \cdot 2$.
- **Associative Law of Addition: $(x + y) + z = x + (y + z)$**
  For example, $(7 + 3) + 5 = 7 + (3 + 5)$.
- **Associative Law of Multiplication: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$**
  For example, $2 \cdot (7 \cdot \pi) = (2 \cdot 7) \cdot \pi$.
- **Distributive Law of Multiplication over Addition: $x \cdot (y + z) = x \cdot y + x \cdot z$**
  For example, $2 \cdot (\pi + 7) = 2 \cdot \pi + 2 \cdot 7$.

The distributive law tells us that the grouping symbols can be removed from an expression of the form $x \cdot (y + z)$ by simply multiplying each of the numbers within the parentheses by x.

EXAMPLE 1 (a) $x(2 + \sqrt{3}) = 2x + x\sqrt{3}$

(b) $5(x + y + z) = 5x + 5y + 5z$

(c) $2(3 + a) = 6 + 2a$ ∎

Warning: There is a commutative law and an associative law for multiplication but not for subtraction or division. For example, since $8 \div 4 = 2$ and $4 \div 8 = \frac{1}{2}$, it is clear that division is not commutative. Likewise, the fact that $8 \div (4 \div 2) = 4$ and $(8 \div 4) \div 2 = 1$ shows that division is not associative.

Signed Numbers

We will now review the basic rules for operating with positive and negative numbers. You must be careful since the negative sign designates *both* the operation of subtraction *and* the sign of the number.

Grouping Symbols

- If there are grouping symbols around a single number, positive or negative, simply remove the grouping symbols. For instance, $(7) = 7$ and $(-7) = -7$.
- If the grouping symbol is preceded by a negative sign, remove the grouping symbol by changing the sign of the number. For instance, $-(+2) = -2$ and $-(-2) = 2$.

The magnitude of a real number, independent of its sign, is called the **absolute value** of the number. For instance, the absolute value of -2 is 2; of $-\sqrt{3}$ is $\sqrt{3}$; and of 5 is 5. An algebraic definition of the absolute value of a number is given in Section 1.3.

Addition

- The sum of two real numbers with *like* signs is the sum of the absolute values of the two numbers preceded by their common sign. For example,

$$(+3) + (+5) = +(3 + 5) = +8$$
$$(-5) + (-7) = -(5 + 7) = -12.$$

- To add two real numbers with *unlike* signs, subtract the smaller absolute value from the larger. The sum will have the sign of the one with the larger absolute value. Thus,

$$(-4) + (+9) = +(9 - 4) = +5 \quad \text{and} \quad (-8) + (+2) = -(8 - 2) = -6.$$

Subtraction

To subtract two real numbers, change the sign of the number being subtracted and then follow the rules for addition. For example,

$$(+4) - (+9) = (+4) + (-9) = -(9 - 4) = -5$$
$$(-3) - (-7) = (-3) + (+7) = +(7 - 3) = +4.$$