ANALOG AND DIGITAL FILTERS: DESIGN AND REALIZATION

HARRY Y-F. LAM

ANALOG AND DIGITAL FILTERS: DESIGN AND REALIZATION

HARRY Y-F. LAM

Bell Telephone Laboratories, Inc.

Formerly with University of California, Berkeley

PRENTICE-HALL, INC., *Englewood Cliffs, New Jersey 07632*

Library of Congress Cataloging in Publication Data

Lam, Harry Y-F. (date)
Analog and digital filters.

(Prentice-Hall series in electrical and computer engineering)
Includes bibliographical references and index.
1. Electric filters. I. Title.
TK7872.F5L26 621.3815'32 78-6434
ISBN 0-13-032755-7

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

PREFACE

The basic concept of a filter was originally introduced by G. Campbell and K. Wagner independently in 1915 in relation with their work on transmission lines and vibrating systems. Since then, the development of filter knowledge and filter technologies has been and is still expanding. Today, filters have permeated the electronic technology so much that it is difficult to think of any moderately complex system or device that does not employ a filter in one form or another.

This book is a result of a junior/senior filter design course organized and taught by the author at the University of California, Berkeley. The course was developed with two purposes in mind. One is to give students some of the basic knowledge and the tools of filter design, with the modest aim that students will be able to do some simple (analog and digital) filter design work after completion of the course. The other purpose is to provide students with a solid background for more advanced courses in analog and digital filters. The prerequisite of the course is the two-quarter course sequence on the book *Basic Circuit Theory*, by C. A. Desoer and E. S. Kuh.

The basic approach of this book is practical. The treatment is simple and yet brings out the substance of the subject matter. Intuitive (and theoretically sound) arguments are used to explain the theory with extensive examples to illustrate the design techniques and procedures. The book leads the students step-by-step from the elementary to the fairly advanced topics. When the level of the material is beyond that assumed, the author gives references to relevant literature. As a result, the book should serve equally well as a text for a filter design course for junior/senior students and as a guide on filters to the practicing engineers who desire a good solid introduction to the field. It is the intention of the author that the book may be readily understood by a reader who has had a first course (or two courses) in circuit theory.

The book is closely coordinated to give students and readers a maximum amount of exposure to the many subject matters in the field. In Chapters 1 through 4, the author develops the fundamentals of analog filter design. Chapter 2 covers the building blocks of both passive and active filters. Chapter 3 introduces the properties of network functions. The implications of Hilbert transform, the concept of minimum phase functions, and the various procedures to construct network functions are discussed. Chapter 4 deals with Hurwitz polynomials and positive real functions, which form a mathematical foundation for passive networks.

In Chapters 5 through 7, the author considers the problem of passive circuit realizations. With the concept of positive real functions established in Chapter 4, the author examines the properties associated with RC and LC driving-point functions in Chapters 5 and 6. Based on these properties, realization techniques for RC and LC driving-point functions are derived. Chapter 7 applies these techniques to realize various classes of transfer functions. In particular, RC, LC, and Darlington ladder circuits are developed to synthesize transfer functions of low-pass, bandpass, and high-pass types; lattice circuits are used to realize all-pass transfer functions.

Chapter 8 examines the problem of finding appropriate transfer functions. The magnitude-selective filters of Butterworth and Chebyshev and the group delay Bessel filters are discussed in great detail. Supplementary graphs and tables as design aids are also included. Chapter 9 introduces the concept of sensitivity. Chapter 10 deals with active filters. Two basic approaches are considered. The direct approach involves the realization of passive RC 1-ports and 2-ports. The indirect approach is concerned primarily with second-order active-filter realizations. Both single-amplifier and multiple-amplifier techniques are considered. The advantages and disadvantages of each technique are discussed. Chapter 10 also examines the effect of nonideal operational amplifiers on circuit performance. Finally, a class of active circuits containing only operational amplifiers and resistors (called active R circuits) is introduced. This class of circuits is shown to be versatile in high-frequency applications.

Digital filters are discussed in Chapters 11 through 13. Chapter 11 presents the background material for digital filters including *z*-transforms, inverse *z*-transofrms, discrete Fourier transforms, frequency responses, sampling theorems, and the building blocks for digital filters. Chapter 12 develops techniques to obtain appropriate digital transfer functions. (A good understanding of Chapter 8 is required here.) The impulse invariance method and the bilinear transformation method are examined in detail. Chapter 13 deals with the realization of digital filters. A technique to eliminate delay-free loops is also presented.

A full set of problems, designed to enhance and to extend the presentation, is included at the end of each chapter. Most of these problems have been class-tested to ensure that their levels of difficulty and their degrees of complexity are proper for the students. To avoid assigning the same problems over successive years, every exercise contains problems having similar parts that differ only in their numerical parameters and in other trivial details.

The author would like to thank Professors J.D. McPherson of University of Wisconsin, Milwaukee, and K.A. Stromsmoe of University of Alberta for reading the complete final manuscript of this book. He also would like to acknowledge the contributions of the many students who took the course in 1974–1976; their enthusiastic class participation and feedback are invaluable information for the revisions and improvements made on the earlier versions. It is the pleasure of the author to express his appreciation to Professors L.O. Chua, C.A. Desoer, and E.S. Kuh of University of California, Berkeley, and Messrs. F.J. Witt, C.F. Kurth, and R.P. Snicer of Bell Telephone Laboratories, North Andover, Massachusetts, for their encouragement, constructive criticism, and moral support. Thanks are also due to the Department of Electrical Engineering and Computer Science of University of California, Berkeley, for providing a wonderful environment under which the work on this book was substantially completed.

Finally, the author would like to express his gratitude and appreciation to his wife Alice, who copied the first two versions of the manuscript (for students' use) and who maintained peace and quiet in their home.

HARRY Y-F. LAM

CONTENTS

7 PASSIVE REALIZATION OF TRANSFER FUNCTIONS 162

8 FILTER APPROXIMATION 220

9 SENSITIVITY 321

10 ACTIVE FILTERS 343

1

INTRODUCTION

In the most general sense, a "filter" is a device or a system that alters in a prescribed way the input that passes through it. In essence, a filter converts inputs into outputs in such a fashion that certain desirable features of the inputs are retained in the outputs while undesirable features are suppressed.

There are many kinds of filters; only a few examples are given here. In automobiles, the oil filter removes unwanted particles that are suspended in the oil passing through the filter; the air filter passes air but prevents dirt and dust from reaching the carburetor. Colored glass may be used as an optical filter to absorb light of certain wavelengths, thus altering the light that reaches the sensitized film in a camera.

An electrical filter is designed to separate and pass a desired signal from a mixture of desired and undesired signals. Typical examples of complex electrical filters are televisions and radios. More specifically, when a television is turned to a particular channel, say Channel 2, it will pass those signals (audio and visual) transmitted by Channel 2 and block out all other signals. On a smaller scale, filters are basic electronic components in many communication systems such as the telephone, television, radio, radar, and sonar. Electrical filters can also be found in power conversion circuits and power systems in general. In fact, electrical filters permeate modern technology so much that it is difficult to think of any moderately complex electronic device that does not employ a filter in one form or another.

Electrical filters may be classified in a number of ways. *Analog* filters are used to process analog or continuous-time signals; *digital* filters are used to process digital signals (discrete-time signals with quantized magnitude levels).[1] Analog filters may be classified as *lumped* or *distributed* depending on the frequency ranges for which they are designed.[2] Finally, analog filters may also be classified as *passive* or *active* depending on the type of elements used in their realizations.

In more abstract terms, a filter is a system characterized by a set of *input–output* pairs or *excitation–response* pairs, as shown in Fig. 1-1, where

$$y(t) = \int_0^\infty h(t - \tau)\, x(\tau)\, d\tau \tag{1-1}$$

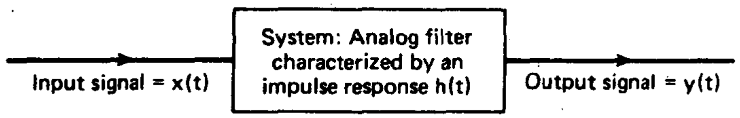

Fig. 1-1 A filter is a system with a set of prescribed input–output properties.

In writing (1-1), we assume that the single-input–single-output analog filter under consideration is *causal, linear, lumped,* and *time-invariant* and that $h(t)$ is the *impulse response* of the filter. The Laplace transform of (1-1) gives

$$Y(s) = H(s)X(s) \tag{1-2}$$

where $Y(s)$, $H(s)$, and $X(s)$ are respectively the Laplace transforms of $y(t)$, $h(t)$, and $x(t)$. Here, the filter is characterized by $H(s)$, the transfer function (or the frequency response function when $s = j\omega$) of the filter.[3] Because either s or $j\omega$ is a complex variable, $H(s)$ or $H(j\omega)$ is a complex quantity. That is, $H(j\omega)$ has a *real part* $\mathrm{Re}\,[H(j\omega)]$ and an *imaginary part* $\mathrm{Im}\,[H(j\omega)]$, and

$$H(j\omega) = \mathrm{Re}\,[H(j\omega)] + j\,\mathrm{Im}\,[H(j\omega)] \tag{1-3}$$

In terms of polar representation, we can write

$$H(j\omega) = |H(j\omega)|\, e^{j\underline{/H(j\omega)}} \tag{1-4}$$

where $|H(j\omega)|$ and $\underline{/H(j\omega)}$ denote respectively the *magnitude* and the *phase lead angle* of $H(j\omega)$, with

$$\begin{aligned} |H(j\omega)|^2 &= \{\mathrm{Re}\,[H(j\omega)]\}^2 + \{\mathrm{Im}\,[H(j\omega)]\}^2 \\ &= H(j\omega)H(-j\omega) \end{aligned} \tag{1-5}$$

$$\underline{/H(j\omega)} = \tan^{-1} \frac{\mathrm{Im}\,[H(j\omega)]}{\mathrm{Re}\,[H(j\omega)]} \tag{1-6}$$

---

[1] We discuss digital filters in Chapter 11. In the following discussion, we deal with analog filters and continuous-time systems only.

[2] In this book, we consider lumped filters only.

[3] In sinusoidal steady-state analysis, we let $s = j\omega$.

$$\text{Re}\,[H(j\omega)] = |H(j\omega)| \cos \underline{/H(j\omega)} \tag{1-7}$$

$$\text{Im}\,[H(j\omega)] = |H(j\omega)| \sin \underline{/H(j\omega)} \tag{1-8}$$

Note that the last equality of (1-5) holds because all coefficients of $H(s)$ are assumed to be real.

1-1 MAGNITUDE FUNCTION

As mentioned previously, the general purpose of an electrical filter is to separate and pass a desired signal from a mixture of desired and undesired signals. In the case of a radio receiver, the signal going into the receiver is a mixture of electrical noise and signals from all the radio stations in the area including the desired station. By tuning the radio receiver to a particular frequency setting, we filter out "all" the signals from the undesired stations and pass the signal transmitted by the desired station. Because of the limitations of causal systems, we can neither build a receiver that will pass one particular frequency, ω_p, and reject all other frequencies, nor can we build a broadcasting station that will broadcast at ω_p exactly. Consequently, we build a filter that will pass signals within an interval of frequencies $(\omega_{p_1}, \omega_{p_2})$ containing ω_p and reject all others, where the words "pass" and "reject" are used in a relative sense rather than in an absolute sense.

From (1-2), we have

$$|Y(j\omega)| = |H(j\omega)|\,|X(j\omega)| \tag{1-9}$$

$$\underline{/Y(j\omega)} = \underline{/H(j\omega)} + \underline{/X(j\omega)} \tag{1-10}$$

Equation (1-9) says that the magnitude of the output signal is the product of the magnitudes of the input signal and the frequency response function of the filter. This means that if a filter has a magnitude function $|H(j\omega)|$ equal to *zero* (or approximately equal to zero) for a certain frequency range, say between ω_{s_1} and ω_{s_2}, then the output signal will have a zero (or an approximately zero) magnitude if the frequency of the input signal is within this frequency band of $(\omega_{s_1}, \omega_{s_2})$. Thus, the interval $(\omega_{s_1}, \omega_{s_2})$ is called the *stopband* of the filter. Similarly, if the magnitude function $|H(j\omega)|$ is greater than or equal to some number close to *one* within the frequency band $(\omega_{p_1}, \omega_{p_2})$, then $(\omega_{p_1}, \omega_{p_2})$ is called the *passband* of the filter.[4] This name is given because if the input frequency is within $(\omega_{p_1}, \omega_{p_2})$, then the output signal is an enhanced or at worst a slightly attenuated version of the input signal. In addition, we define a *transitional band* as a band of frequencies between a passband and a stopband. A specification on the magnitude of the frequency

[4]The *one* here may be interpreted as a *unit* normalized with respect to a magnitude reference.

response function of a filter may include specifications on passbands and stopbands as well as transitional bands.

Based on (1-9), we can define the following five basic types of frequency selective filters:

1. *Low-Pass* filter—A filter whose passband is from 0 to some frequency ω_p and whose stopband extends from some frequency ω_s to infinity, where $\omega_p < \omega_s$.
2. *High-Pass* filter—A filter whose passband is from some frequency ω_p to infinity and whose stop band is from 0 to ω_s, where $\omega_s < \omega_p$.
3. *Bandpass* filter—A filter whose passband is from some frequency ω_{p_1} to some other frequency ω_{p_2} and whose stopbands are from 0 to ω_{s_1} and from ω_{s_2} to ∞, where $\omega_{s_1} < \omega_{p_1} < \omega_{p_2} < \omega_{s_2}$.
4. *Band-Reject* filter—A filter whose passbands are from 0 to ω_{p_1} and from ω_{p_2} to ∞ and whose stopband is from ω_{s_1} to ω_{s_2}, where $\omega_{p_1} < \omega_{s_1} < \omega_{s_2} < \omega_{p_2}$.
5. *All-Pass* filter—A filter whose magnitude is 1 for all frequencies (i.e., whose passband is from 0 to ∞). This type of filter is used mainly for phase compensation and phase shifting purposes.

These five basic types of frequency selective filters are illustrated in Fig. 1-2. Of course, there are filters that do not belong to any one of these five

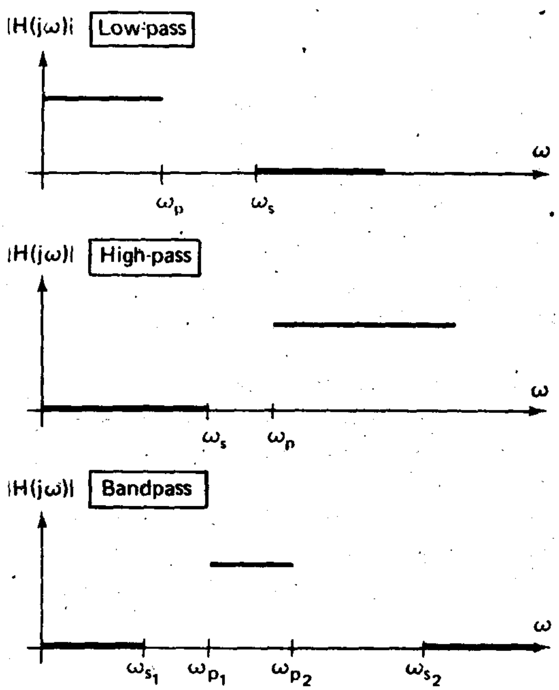

Fig. 1-2 Five basic types of frequency-selective filters.

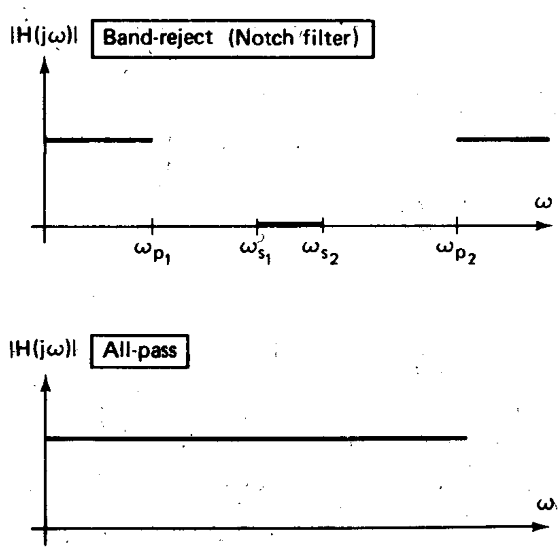

Fig. 1-2 (Continued)

types. In most cases of interest, the magnitude specifications of filters will fall into one of these five basic categories or a combination of these five types. A case in point is a filter whose magnitude specification is given in Fig. 1-3(a). This filter can be considered a combination of a low-pass and four bandpass filters, as shown in Fig. 1-3(b).

To illustrate some uses of some of these filters, let us consider the following two examples:

1. In transmitting a low-frequency signal $X_0(t)$, such as a voice signal, over a distance, it is imperative to modulate this low-frequency signal with a high-frequency signal carrier before transmitting. There are a number of ways to modulate a signal. Figure 1-4 is a schematic diagram of a double sideband amplitude modulation. At the receiver, the transmitted signal $X_1(t)$ goes through a mixer where the transmitted signal is multiplied by a signal at the modulating frequency. In order to recover the desired low-frequency signal $X_0(t)$, the output signal of the mixer $X_2(t)$ is passed through a low-pass filter with a passband containing $[0, \omega_L]$ and a stopband containing the frequencies from $(2\omega_H - \omega_L)$ to infinity.
2. In long distance communication, a line carries many signals simultaneously. This is accomplished by employing frequency multiplexing—each of the low-frequency input signals are frequency translated to a different center frequency, as shown in Fig. 1-5(a), where ω_i is the center frequency of the ith low-frequency signal. At the receiving end, the transmitted signal is fed through a band of parallel bandpass filters to corresponding message receivers, as shown in Figure 1-5(b).

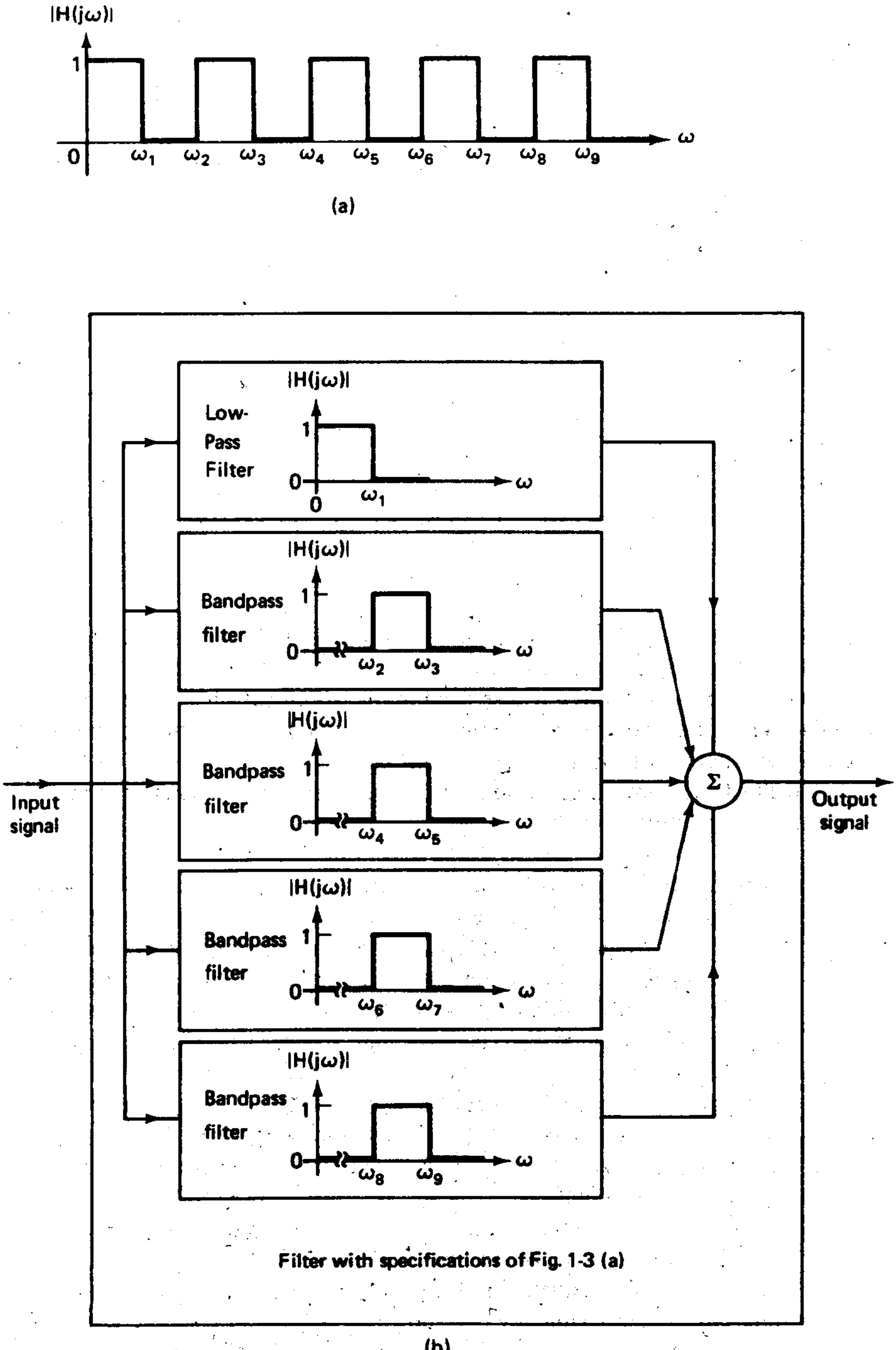

Fig. 1-3 An example of a decomposable filter.