

INTRODUCTION TO
MECHANICS
OF MATERIALS

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WILEY

JOHN WILEY & SONS New York Chichester Brisbane Toronto Singapore

Cover: Frank Stella, *Damascus Gate Variation I*, 1969;
Private collection, Courtesy of M. Knoedler & Co., Inc., New York

Text and cover design by Madelyn Lesure
Production supervised by Lucille Buonocore
Illustrations supervised by John Balbalis
Copy editing supervised by Richard Koreto

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Library of Congress Cataloging in Publication Data:

Riley, William F. (William Franklin), 1925-

Introduction to mechanics of materials/by William F. Riley and
Loren W. Zachary.

p. cm.

Includes index.

ISBN 0-471-84933-2

1. Strength of materials. 2. Structures, Theory of. I. Zachary,
Loren W. II. Title.

TA405.R54 1989

620.1'12—dc 19

88-21587

CIP

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

This textbook is an outgrowth of the book *Mechanics of Materials* by A. Higdon, E.H. Ohlsen, and W.B. Stiles that was first published in 1960. This abbreviated edition retains many of the features of the previous four editions of this earlier work.

The primary objectives of a course in mechanics of materials are:

1. to develop a working knowledge of the relations between the loads applied to a nonrigid body made of a given material and the resulting deformations of the body
2. to develop a thorough understanding of the relations between the loads applied to a nonrigid body and the stresses produced in the body
3. to develop a clear insight into the relations between stresses and strains for a wide variety of conditions and materials, and
4. to develop adequate procedures for finding the required dimensions of a member of a specified material to carry a given load subject to stated specifications of stress and deflection.

These objectives involve the concepts and skills that form the foundation of all structural analysis and machine design.

The principles and methods used to meet the objectives are drawn largely from prerequisite courses in mechanics, physics, and mathematics. Some basic concepts of the theory of elasticity and some knowledge of the mechanical properties of engineering materials are also used. This book is designed to emphasize fundamental principles. Instead of deriving numerous formulas for specialized states of stress, we emphasize use of free-body diagrams, application of the equations of equilibrium, visualization and use of the geometry of the deformed body, and use of the relations between stresses and strains for the material being used. This approach makes it possible to develop all of the commonly used *Mechanics of Materials* formulas in a rational and logical manner, and to clearly indicate the conditions under which they may be safely applied to the analysis and design of actual engineering structures and machine components. Since extensive use is made of course material from statics and calculus, a working knowledge of these

subjects is considered an essential prerequisite to a successful study of mechanics of materials as presented in this book.

This abbreviated edition is designed for a first course in mechanics of deformable bodies. The extensive subdivision of the book into different topics will provide flexibility in the choice of assignments to cover courses of different length and content. The topics progress logically from a study of stresses and deformations in members subjected to simple tension, compression, or shear, to a study of stresses and deformations in shafts under torsion, beams under bending, and structural members and machine components under combined loadings. Statically indeterminate members are introduced throughout the text in connection with those topics for which the subject is appropriate. The variation of stress (and strain) at a point is presented at a place in the book where the topic can be treated fully and correctly and where students have sufficient understanding of the subject to appreciate the concepts of principal stress and maximum shearing stress at a point.

Throughout the book, strong emphasis is placed on the engineering significance of the subject, and not merely on mathematical methods of analysis. The course aims to develop in students the ability to analyze a given problem in a simple and logical manner, to apply a few fundamental principles to its solution, and to present results in a clear, logical, and neat manner. We believe that students gain mastery of a subject once they find that they can apply the basic theory to the solution of a problem that appears somewhat difficult. They learn less by solving a large number of simple but similar problems. A conscientious effort has been made to present the material in a simple and direct manner, with a student's point of view constantly in mind.

Students are usually more enthusiastic about a subject if they can see and appreciate its value as they proceed into the subject. The practical side of design is shown wherever possible, and there are frequent reminders that safety is a prime consideration in all engineering design. To help students acquire an intuitive feeling for the sizes, shapes, proportions, and load-carrying capacities of real machine components and structural members, we have chosen only realistic sizes and loads for the members used in illustrative examples and homework problems. Also, the materials used in the fabrication of the members are chosen from those materials that are commonly used in engineering practice, and the stresses to which the materials are subjected are realistic.

Many illustrative example problems have been integrated into the main body of the text at places where the presentation of a method can best be reinforced by the immediate use of the method in an illustrative example problem. The illustrative examples and homework problems have been carefully selected, and in general, require an understanding of the principles of mechanics of materials without demanding excessive time for computational work.

The answers to about half the problems are included at the end of the book. We feel that the first assignment on a given topic should include some problems for which the answers are given. Since the simpler problems are usually reserved for this first assignment, the answers are provided for the first four or five problems of each article and thereafter are given in general for alternate problems. We feel that answers for certain problems should not be given; hence, we have not followed the flat rule of answers for all even-numbered problems. Since the convenient designation of problems for which answers are provided is of great value to those who make up assignment sheets, the problems for which answers are provided are indicated by means of an asterisk after the number.

In general, all given data are assumed to be composed of three significant figures regardless of the number of figures shown. Answers are therefore given to three significant figures, unless the number lies between 1 and 2 or any decimal multiple thereof, in which case four significant figures are reported. Some of the problems involve material properties, which are quite variable, and possibly stress concentration factors, which are difficult to obtain to more than two significant figures from graphs. We hesitate to give results to more than two significant figures in these instances; however, in order to be consistent and to avoid confusion, the answers will be given to three significant figures.

Use of the International System of Units (SI) is slowly gaining acceptance in the United States, and as a result, engineers must be proficient during the transition period in both the SI system and the U.S. Customary System (USCS) in common use today. In response to this need, both U.S. customary units and SI units are used in approximately equal proportions in the text for both illustrative examples and homework problems. Since a large number of homework problems are provided, the instructor can assign homework problems in whatever proportion he finds desirable for his class. A discussion of the SI system together with a table of conversion factors is presented in an appendix of the book.

We are grateful for comments and suggestions received from colleagues and from users of the earlier editions of this book. We appreciate also the careful appraisal made by the publisher's consultant and have incorporated those recommended changes that we considered appropriate. We will be pleased to receive comments from readers of this edition of the book and will attempt to acknowledge all such communications.

William F. Riley
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ABOUT THE AUTHORS

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In 1972, Dr. Zachary returned to Iowa State and received his M.S. and Ph.D. degrees in engineering mechanics. He has been active in experimental stress analysis and nondestructive evaluation. He has authored and coauthored several journal papers in these areas. He is a member of the Society for Experimental Mechanics and is a licensed professional engineer.

SYMBOLS AND ABBREVIATIONS*

<i>A</i>	area
avg.	average
<i>b</i>	breadth, width
<i>C</i>	compression, Celsius
<i>c</i>	distance from neutral axis or from center of twist to extreme fiber
<i>E</i>	modulus of elasticity in tension or compression
<i>e</i>	eccentricity
Eq.	equation
<i>F</i>	force, or body force, Fahrenheit
<i>f</i>	normal force per unit of length
ft	foot or feet
ft-lb	foot-pound
<i>G</i>	modulus of rigidity (modulus of elasticity in shear)
<i>g</i>	acceleration of gravity
<i>h</i>	height, depth of beam
hp	horsepower
<i>I</i>	moment of inertia or second moment of area
in.	inch
in.-lb	inch-pound
<i>J</i>	polar moment of inertia of area
J	joule (N·m)
<i>K</i>	stress concentration factor
<i>k</i>	modulus of spring, factor of safety
kg	kilogram
kip	kilopound (1000 lb)
ksi	kilopounds (or kips) per square inch
<i>L</i>	length
lb	pound

*The symbols and abbreviations used in this book conform essentially with those approved by the International Standards Organization.

M	mega
M	bending moment
m	meter
max	maximum
min	minimum
N	normal force
N	newton
$N \cdot m$	newton · meter
n	ratio of elastic moduli
P	concentrated load, force
p	fluid pressure
Pa	pascal (N/m^2)
psi	pounds per square inch
Q	first moment of area
q	shearing force per unit of length
R	radius, modulus of rupture, resultant force
r	radius of gyration
rad	radian
rpm	revolutions per minute
rps	revolutions per second
S	surface force, resultant stress, stress vector, section modulus ($S = I/c$)
sec	second
T	torque, temperature, tension
t	thickness, tangential deviation
U	strain energy
u	strain energy per unit volume
u, v, w	components of displacement
V	shearing force
W	weight
W	watt ($N \cdot m/s$)
W_k	work
w	load per unit of length
x, y, z	cartesian coordinates
y	deflection
α (alpha)	coefficient of thermal expansion
γ (gamma)	shearing strain, specific weight
δ (delta)	total deformation
ϵ (epsilon)	normal strain

θ (theta)	total angle of twist in radians, slope of deflected beam
μ (mu)	micro (10^{-6})
ν (nu)	Poisson's ratio
ρ (rho)	radius, radius of curvature
σ (sigma)	normal stress
τ (tau)	shearing stress
ω (omega)	angular velocity
$^{\circ}$	degree
C12 \times 30	a 12-in. rolled channel section with a weight of 30 lb/ft
C305 \times 45	a 305-mm rolled channel section with a mass of 45 kg/m
L6 \times 4 \times 1/2	a rolled angle section with 6-in. and 4-in. legs 1/2 in. thick
L102 \times 89 \times 13	a rolled angle section with 102-mm and 89-mm legs 13 mm thick
S10 \times 35	a 10-in. American standard rolled section (I-beam) with a weight of 35 lb/ft
S127 \times 22	a 127-mm American standard rolled section (I-beam) with a mass of 22 kg/m
W16 \times 100	a 16-in. wide-flange rolled section with a weight of 100 lb/ft
W914 \times 342	a 914-mm wide-flange rolled section with a mass of 342 kg/m
WT8 \times 50	an 8-in. structural T-section (cut from a W-section) with a weight of 50 lb/ft
WT457 \times 171	a 457-mm structural T-section (cut from a W-section) with a mass of 171 kg/m

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CHAPTER 1

INTRODUCTION TO STRESS, STRAIN, AND THEIR RELATIONSHIPS

1-1

INTRODUCTION

The primary objective of a course in mechanics of materials is the development of relationships between the loads applied to a nonrigid body and the internal forces and deformations induced in the body. Ever since the time of Galileo Galilei (1564–1642), scientists and engineers have studied the problem of the load-carrying capacity of structural members and machine components, and have developed mathematical and experimental methods of analysis for determining the internal forces and the deformations induced by the applied loads. The experiences and observations of these scientists and engineers of the last three centuries are the heritage of the engineer of today. The fundamental knowledge gained over the last three centuries, together with the theories and analysis techniques developed, permit the modern engineer to design, with complete competence and assurance, structures and machines of unprecedented size and complexity.

The subject matter of this book forms the basis for the solution of three general types of problems:

1. Given a certain function to perform (the transporting of traffic over a river by means of a bridge, conveying scientific instruments to

Mars in a space vehicle, the conversion of water power into electric power), of what materials should the machine or structure be constructed, and what should the sizes and proportions of the various elements be? This is the designer's task, and obviously there is no single solution to any given problem.

2. Given the completed design, is it adequate? That is, does it perform the function economically and without excessive deformation? This is the checker's problem.
3. Given a completed structure or machine, what is its actual load-carrying capacity? The structure may have been designed for some purpose other than the one for which it is now to be used. Is it adequate for the proposed use? For example, a building may have been designed as an office building, but is later found to be desirable for use as a warehouse. In such a case, what maximum loading may the floor safely support? This is the rating problem.

Since the complete scope of these problems is obviously too comprehensive for mastery in a single course, this book is restricted to a study of individual members and very simple structures or machines. The design courses that follow will consider the entire structure or machine, and will provide essential background for the complete analysis of the three problems.

The principles and methods used to meet the objective stated at the beginning of this chapter depend to a great extent on prerequisite courses in mathematics and mechanics, supplemented by additional concepts from the theory of elasticity and the properties of engineering materials. The equations of equilibrium from *statics* are used extensively, with one major change in the free-body diagrams; namely, most free bodies are isolated by *cutting through* a member instead of removing a pin or some other connection. The forces transmitted by the cut sections are *internal forces*. The intensities of these internal forces (force per unit area) are called *stresses*.

It will frequently be found that the equations of equilibrium (or motion) are not sufficient to determine all the unknown loads or reactions acting on a body. In such cases it is necessary to consider the geometry (the change in size or shape) of the body after the loads are applied. The *deformation* per unit length in any direction or dimension is called *strain*. In some instances, the specified maximum deformation and not the specified maximum stress will govern the maximum load that a member may carry.

Some knowledge of the physical and mechanical properties of materials is required in order to create a design, to properly evaluate a given design, or even to write the correct relation between an applied load and the resulting deformation of a loaded member. Essential in-

formation will be introduced as required, and more complete information can be obtained from textbooks and handbooks on properties of materials.

1-2

LOAD CLASSIFICATION

Certain terms are commonly used to describe applied loads; their definitions are given here so that the terminology will be clearly understood.

Loads may be classified with respect to time:

1. A *static load* is a gradually applied load for which equilibrium is reached in a relatively short time.
2. A *sustained load* is a load that is constant over a long period of time, such as the weight of a structure (called *dead load*). This type of load is treated in the same manner as a static load; however, for some materials and conditions of temperature and stress, the resistance to failure may be different under short-time loading and under sustained loading.
3. An *impact load* is a rapidly applied load (an energy load). Vibration normally results from an impact load, and equilibrium is not established until the vibration is eliminated, usually by natural damping forces.
4. A *repeated load* is a load that is applied and removed many thousands of times. The helical springs that close the valves on automobile engines are subjected to repeated loading.

Loads may also be classified with respect to the area over which the load is applied:

1. A *concentrated load* is a load or force applied at a point. Any load applied to a relatively small area compared with the size of the loaded member is assumed to be a concentrated load; for example, a truck wheel load on the longitudinal members of a bridge.
2. A *distributed load* is a load distributed along a length or over an area. The distribution may be uniform or nonuniform. The weight of a concrete bridge floor of uniform thickness is an example of a uniformly distributed load.

Loads may be classified with respect to the location and method of application: