Microwave Transmission Line Filters

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Preface

This book concerns microwave filters constructed of transmission-lines or coupled transmission lines, a field in which considerable advances have been made during the last decade, specifically in exact filter design methods. Much of this material has not been available in textbook form, presenting the novice with the task of collecting and evaluating material of which he has as yet little knowledge or experience. It is the purpose of this book, therefore, to provide a unified treatment of the most relevant material, both old and new. Students at graduate level (the text is based on part of a one semester course in microwave networks at Stellenbosch University) or practicing engineers interested in entering the field should therefore find the book of value.

One obvious omission is that of filter approximation theory. However, there are a large number of excellent texts on filters and modern network theory to which the reader can refer; only in a number of special cases where the treatment for microwave filters differs from that for conventional (lumped element) filters will approximation theory be considered. Simultaneously it is assumed that the reader is familiar with such basic concepts as maximally flat and equiripple prototypes, scaling, and so forth.

The microwave filter principles are introduced as first principles so that the reader need not be equipped with any knowledge in the field other than basic transmission-line theory and high-frequency methods in general. Extensive use is made of a series of equivalent circuits or network models of coupled line circuits and circuit transforms which are developed in the first chapter. The treatment is purposely complete so that the principles can be applied to material outside the contents of this book. In addition, references and suggestions for further reading are included at the end of each chapter.

Chapters 2 and 3 are devoted to the study of bandstop filters using lumped element prototypes: Butterworth and Chebyschev in Chapter 2 and elliptic function prototypes in Chapter 3. Similarly, bandpass filters using lumped element prototypes are treated in Chapters 4 and 5.

Nonredundant filter synthesis in both the S- and Z-planes is described in Chapter 6, a necessary amount of approximation theory being included. The design procedures described in Chapters 2 to 6 are applied to the design of multiplexers in Chapter 7 while Chapter 8 describes linear phase filters. It should be noted that most of Chapter 7 can be covered should the reader prefer to omit Chapter 6.

Simple computer analysis methods are described in Appendix A while Appendix B is a convenient summary of design formulae used in Chapters 1 to 5.

The author remains indebted to Mr. J.H. Cloete for his constructive criticism and the use of his design example and advice in Chapter 8, and to Mrs. M.M. Kruger for an exceptional effort in typing the manuscript.

J.A.G. Malherbe
Stellenbosch
1 November 1977

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Chapter 1
TEM
Network Elements

The study of transmission-line filters introduces a number of new and physically different principles to the field of conventional lumped element network theory. These concepts are so basic to the theory and practice of TEM (Transverse Electro-Magnetic) filters that it is necessary to first consider them in detail. The transform proposed by Richards[1-1] in 1948 forms the basis of the TEM network theory, and although specific filter designs will only be studied in Chapter 2, an example is introduced in the first paragraph to at least partially motivate the long excursion into TEM network elements, of which the basis are sections of transmission-line, single or coupled.

1.1 RICHARDS' TRANSFORM

At frequencies in the kilohertz or lower Megahertz region, the change in phase due to the finite propagation velocity of an electrical signal is usually so small as to be considered negligible for most physical structures. At these frequencies the networks are termed lumped element (LE) networks. At high (radio) frequencies, however, the distributed nature of a physical structure is taken into consideration, leading to the well known transmission-line theory. The driving-point impedance of a transmission-line of characteristic impedance Z_0 and length ℓ is given by

$$Z_s = Z_0 \frac{Z_L \cosh \gamma\ell + Z_0 \sinh \gamma\ell}{Z_0 \cosh \gamma\ell + Z_L \sinh \gamma\ell}$$

where Z_0 and γ are given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

and R, L, G and C are the distributed parameters resistance, inductance, conductance and capacitance per meter of the line. For electrically short linelengths at very high frequencies, $R \ll \omega L$, $G \ll \omega C$ so that, for $\alpha = 0$,

$$Z_0 \simeq \sqrt{\frac{L}{C}} = \frac{1}{vC}, \beta = \omega\sqrt{LC}, v = \frac{1}{\sqrt{LC}} .$$

Consider a capacitor made up of two long strips of foil, separated by a dielectric. The input admittance of such a line will be given by

$$Y = jvC \tan \beta\ell.$$

At low frequencies, $\beta\ell$ is small so that $\tan \beta\ell \simeq \beta\ell$, and

$$Y = jvC\beta\ell = jvC \frac{\omega}{v} \ell = j\omega C\ell.$$

that is, the structure behaves as a capacitor of capacitance $C\ell$. At high frequencies the input admittance is given by

$$Y_{oc} = \frac{j}{Z_0} \tan \beta\ell$$
$$= \frac{j}{Z_0} \tan\left(\frac{\pi}{2} \frac{\omega}{\omega_0}\right)$$

where ω_0 is defined as the frequency at which $\ell = \lambda_0/4$. The transform proposed by Richards states that, if

$$S = j\Omega = j \tan\left(\frac{\pi}{2} \frac{\omega}{\omega_0}\right), \tag{1-1}$$

then

$$Y_{oc} = j\Omega/Z_0 = j\Omega C' ,$$

that is, the open-circuited transmission-line behaves as a capacitor in the S-plane. For the case of a short circuited line,

$$Z_{sc} = jZ_0 \tan\left(\frac{\pi}{2}\frac{\omega}{\omega_0}\right)$$

$$= j\Omega Z_0 = j\Omega L' .$$

Under Richards' transform, open circuited transmission-lines map into capacitors, while short circuited lines become inductors. This is shown graphically in Figure 1.1.

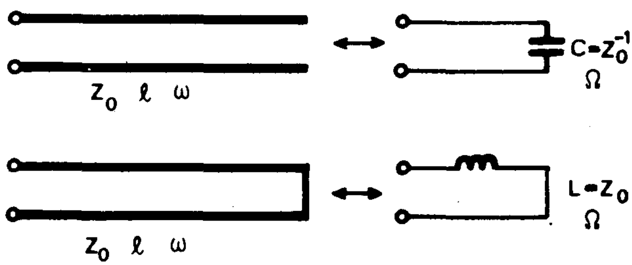


Figure 1.1. Richards' transform.

In order to indicate the application and to illustrate the shortcomings of Richards' transform, it is necessary to consider the performance of a simple lumped element filter, as shown in Figure 1.2 (a) for a 3rd order Butterworth lowpass prototype, with the frequency response as shown in Figure 1.2 (b). Applying Richards' transform, the network of Figure 1.2 (c) results, after replacing capacitors by open circuit lines, and inductors by short circuit lines.

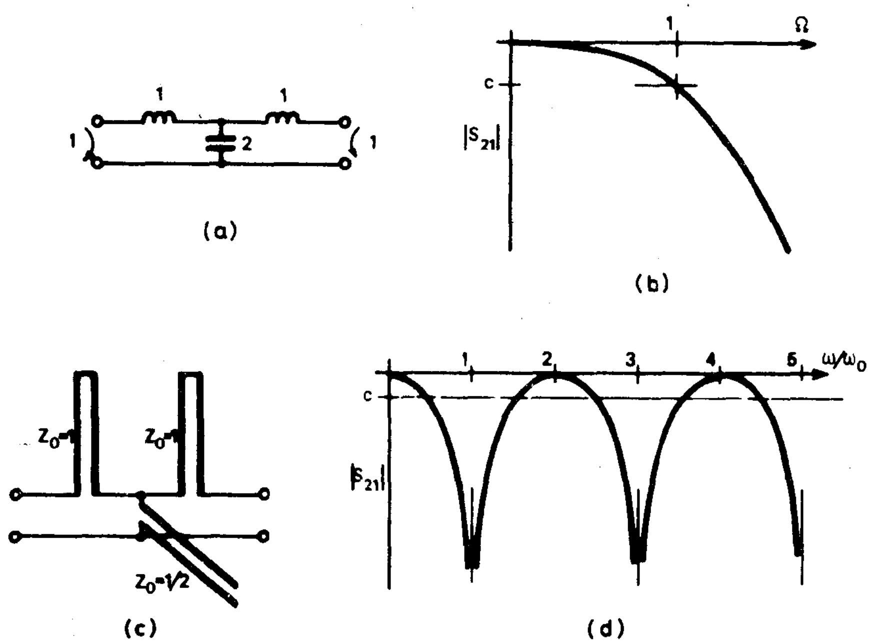


Figure 1.2. Application of Richards' transform to filter synthesis.
(a): *lumped element prototype with frequency response shown in* (b);
(c): *transmission-line filter with harmonic response,* (d).

Now the frequency variable Ω varies as $\tan\left(\frac{\pi}{2}\,\frac{\omega}{\omega_0}\right)$, so that for $\omega = n\omega_0$, n odd, a singularity in Ω results. Therefore, as ω increases from zero to ω_0, Ω changes through all values from zero to infinity, and must consequently cause the full response (all the way to infinity) of Figure 1.2 (b) to be compressed between zero and ω_0. As ω exceeds ω_0 for the first time, the sign of Ω changes, and Ω decreases from negative infinity to zero as ω approaches $2\omega_0$. This causes the mirror image of the first response to be plotted between ω_0 and $2\omega_0$. This process is repeated between $2\omega_0$ and $4\omega_0$, and so on, resulting in the odd harmonic response shown in Figure 1.2 (d). It should also be noted that the lowpass response has been mapped into a bandstop response in exactly the same way a highpass response maps into a bandpass response. If the initial prototype had had a Chebyschev rather than Butterworth response, the ripples occurring in the band $0 \leqslant \omega \leqslant 1$ would also have appeared in the frequency bands $0 \leqslant \omega \leqslant \omega_0/2$,

$1.5\,\omega_0 \leqslant \omega \leqslant 2\omega_0,\ 2\omega_0 \leqslant \omega \leqslant 2.5\,\omega_0$ . . . . . In both cases the response would also be Butterworth or Chebyschev in the sense of being maximally flat or equal ripple.

The purpose of the transform has been shown; it remains to be stated that, as such, it has extremely limited application because the filter of Figure 1.2 (c) cannot be physically constructed, due to the series connected stubs. (The parallel, open circuit stub would present no problem.) At this stage, an element without equivalent under the inverse of Richards' transform is introduced: the *Unit Element* (U), which consists simply of a length $\lambda_0/4$ of transmission-line Z_0. The unit element is readily described in matrix form as

$$[Z] = \frac{Z_0}{S}\begin{bmatrix} 1 & \sqrt{1-S^2} \\ \sqrt{1-S^2} & 1 \end{bmatrix} \tag{1-2a}$$

$$[Y] = \frac{1}{SZ_0}\begin{bmatrix} 1 & -\sqrt{1-S^2} \\ -\sqrt{1-S^2} & 1 \end{bmatrix} \tag{1-2b}$$

$$\begin{bmatrix} AB \\ CD \end{bmatrix} = \frac{1}{\sqrt{1-S^2}}\begin{bmatrix} 1 & Z_0 S \\ \dfrac{S}{Z_0} & 1 \end{bmatrix} \tag{1-2c}$$

The basic TEM network elements can be summarized as in Figure 1.3 below.

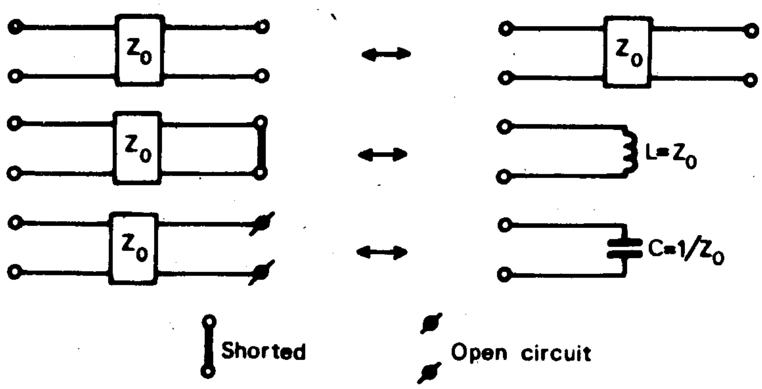


Figure 1.3. TEM Network Elements.

Consider again the transmission-line filter of Figure 1.2 (c). Extending the connection lines on both sides of the filter will leave the amplitude response of the network invariant (although the phase response does change) providing the extending line is of the same characteristic impedance as the generator or load into which the filter operates. It is acceptable therefore to introduce any number of unit elements to each port of the filter, without changing its performance. In that case, if one unit element is added to each port, it would be possible to construct the series lines as series internal coaxial stubs, making the network realizable. Such a form of construction is shown below in Figure 1.4.

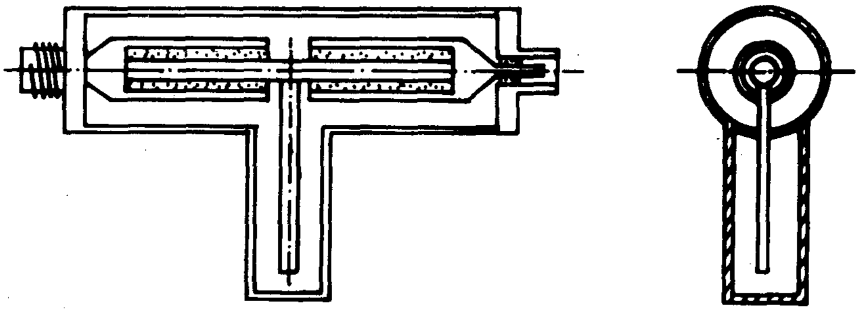


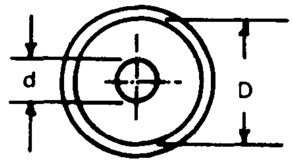
Figure 1.4. Coaxial construction of 3rd order Butterworth bandstop filter.

At this point the range of realizable filter structures is very limited; prototypes of different order are freely available but the limitations of physical construction prohibit their use. For this reason, transforms will be developed to introduce unit elements deeper into more complex networks, and exact synthesis methods for the optimal use of such elements investigated. The range of useful networks will also be vastly extended by the use of coupled lines.

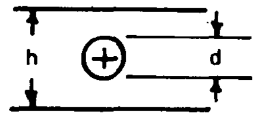
1.2 CHARACTERISTIC IMPEDANCE

Characteristic impedance is viewed as being of prime importance in the design of transmission-line filters. Some of the more important types of transmission-line are mentioned here without going into derivations of the equations. For further reference the reader should consult the bibliography at the end of this chapter. The most common forms of TEM line cross section using round cylindrical center conductors are shown in Figure 1.5.

$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln D/d$



$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log \frac{4h}{\pi d}$, if $d/h < 0.75$



$Z_0 \approx (138 \log_{10} q + 6.48 - 2.34A - 0.48B - 0.12C)\epsilon^{-1/2}$

where $q = D/d$

$A = (q^4 + 0.405)/(q^4 - 0.405)$

$B = (q^8 + 0.163)/(q^8 - 0.163)$

$C = (q^{12} + 0.067)/(q^{12} - 0.067)$

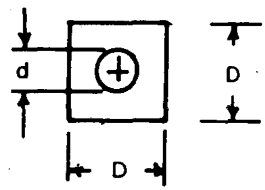


Figure 1.5. Characteristic Impedance.

Lines with rectangular or square center conductor cross sections are generally classified as striplines. Figure 1.6 shows a typical cross section, the capacitance being made up out of the parallel-plate components c_{p1} and c_{p2} and the fringe-fields c_{f1} and c_{f2}. The total capacitance is given by

$$c/\epsilon = c_{p1}/\epsilon + c_{p2}/\epsilon + 2c_{f1}/\epsilon + 2c_{f2}/\epsilon \; ,$$

and

$$Z_0 = \frac{1}{vC} = \frac{\eta}{\sqrt{\epsilon_r}} \; \frac{1}{c/\epsilon} \; .$$

Cohn[1-2] gives the fringe-field capacitance as

$$c_f/\epsilon = \frac{1}{\pi} \left[2\xi \; \ln(\xi + 1) - (\xi - 1) \ln(\xi^2 - 1) \right] ,$$

$$\xi = \begin{cases} \dfrac{1}{1 - t/(b-s)} \ldots (c_{f1}) \\ \\ \dfrac{1}{1 - t/(b+s)} \ldots (c_{f2}) \end{cases} \qquad (1\text{-}3)$$

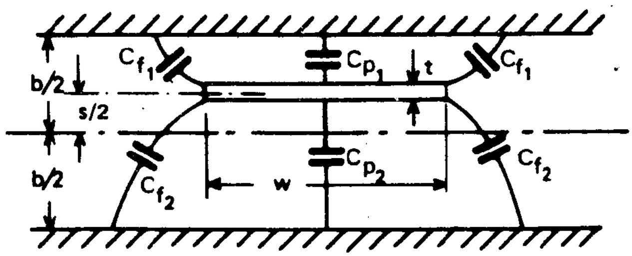


Figure 1.6. Stripline cross section.

The parallel-plate capacitance values are given by

$$c_{p1}/\epsilon = \frac{2w/(b-s)}{1 - t/(b-s)}$$

$$c_{p2}/\epsilon = \frac{2w/(b+s)}{1 - t/(b+s)} .$$

For symmetrically spaced stripline, the equation for Z_0 simplifies to

$$Z_0 = \frac{\eta}{4\sqrt{\epsilon_r}} \left(\frac{w/b}{1 - t/b} + c_f/\epsilon \right)^{-1} . \qquad (1\text{-}4)$$

In cases where the normalized width, $w/(b-t) \leqslant 0.35$, the linewidth must be compensated because of the loss of fringe field capacitance. According to Getsinger[1.3] the new width must be

$$w'/b = [0.07\,(1 - t/b) + w/b] \div 1.2. \qquad (1\text{-}5)$$