Optical Waves in Crystals

Propagation and Control of Laser Radiation

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Preface

This book, intended as a text for a course in electro-optics for electrical-engineering or applied-physics students, has two primary objectives: to present a clear physical picture of the propagation of laser radiation in various optical media and to teach the reader how to analyze and design electro-optical devices.

Only through a study of the electromagnetic propagation can the characteristics of an optical device be appreciated and its limitation be understood. This characterization allows us to exploit the device as an element to manipulate the laser radiation. The emphasis is therefore on the fundamental principles. An effort is made to bridge the gap between theory and practice through the use of numerical examples based on real situations. Only classical electrodynamics is used in dealing with the coherent interaction of laser radiation with various optical media. The optical properties of these media are described by such material parameters as dielectric tensors, gyration tensors, electro-optic coefficients, photo-elastic constants, and nonlinear susceptibilities. A very wide range of topics is included, as may be seen from the Contents.

In writing this book we have assumed that the student has been introduced to Maxwell's equations in an intermediate course in electricity and magnetism. It is further expected that the student has some mathematical background in Fourier integrals, matrix algebra, and differential equations.

In the summer of 1982, the preliminary version of the manuscript was used as the text of a graduate course in modern optics which P. Yeh taught at the Taiwan University. He wishes to thank his colleagues and students at the University for countless helpful remarks and discussions. Special mention must be made of Professor K. P. Wang of the Taiwan University, who gave him the opportunity and encouragement to teach such a course. His thanks are also extended to Dr. Monte Khoshnevisan and Dr. Chun-Ching Shih for reading and commenting on the manuscript, and to Dr. Hidehiko Kuwamoto, Dr. Emilio Sovero, and Mark Ewbank for many helpful discussions.

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Contents

1

Electromagnetic Fields

The ideal laser emits coherent electromagnetic radiation which can be described by its electric and magnetic field vectors. The propagation of this radiation field is governed by Maxwell's equations. It is thus important to familiarize ourselves at the outset with some of the basic properties of electromagnetic fields.

In this introductory chapter we review and derive some basic relations involving classical electromagnetic fields. Starting with Maxwell's equations and the material equations, we obtain expressions for the energy density and the energy flow of an electromagnetic field. We also derive the Poynting theorem, the conservation laws, and the wave equations. We consider in some detail the propagation of monochromatic plane waves and some of their important properties. Finally, we discuss the concepts of phase velocity and group velocity of a wave packet propagating in a dispersive medium.

1.1. MAXWELL'S EQUATIONS AND BOUNDARY CONDITIONS

1.1.1. Maxwell's Equations

An electromagnetic field in space is described classically by two field vectors, **E** and **H**, called the electric vector and the magnetic vector, respectively. To include the effect of the field on matter, it is necessary to introduce a second set of vectors, **D** and **B**, called the electric displacement and the magnetic induction, respectively. These vectors are related by Maxwell's equations (in MKS units):

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{1.1-1}$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}, \tag{1.1-2}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{1.1-3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.1-4}$$

where **J** is the electric current density (amperes per square meter) and ρ is the electric charge density (coulombs per cubic meter).

These four equations are the basic laws of electricity and magnetism in their differential forms. Equation (1.1-1) is the differential form of Faraday's law of induction, which describes the creation of an induced electric field due to a time-varying magnetic flux. Equation (1.1-2) is the differential form of the generalized Ampère law, which describes the creation of an induced magnetic field due to charge flow. Equation (1.1-3) is the differential form of Coulomb's law, which describes the relation between the electric-field distribution and the charge distribution. Equation (1.1-4) may be regarded as a statement of the absence of free magnetic monopoles.

The Maxwell equations (1.1-1), (1.1-2), (1.1-3), and (1.1-4) fully describe the propagation of electromagnetic radiation in any medium.

The charge density ρ and the current density **J** may be regarded as the sources of the electromagnetic radiation. In many areas of optics one often deals with the propagation of electromagnetic radiation in regions far from the sources, where both ρ and **J** are zero. All the cases considered in this book fall within this category.

The Maxwell equations form a set of coupled partial differential equations involving the four basic quantities of the electromagnetic field, **E**, **H**, **D**, and **B**. To allow a unique determination of the field vectors from a given distribution of currents and charges, these equations must be supplemented by relations that describe the effect of the electromagnetic field on material media. These relations are known as constitutive equations (or material equations) and are given by

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \tag{1.1-5}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mathbf{H} + \mathbf{M}, \tag{1.1-6}$$

where the constitutive parameters ε and μ are tensors of rank 2 and are known as the dielectric tensor (or permittivity tensor) and permeability tensor, respectively; **P** and **M** are the electric and magnetic polarizations, respectively; and ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively. If the material medium is isotropic, these tensors reduce to scalars. In many cases the quantities ε and μ can be assumed to be independent of the field strengths. However, if the fields are sufficiently strong, such as those obtained, for example, by focusing a laser beam or applying a strong dc electric field to an electro-optic crystal, then the dependence of these quantities on **E** and **H** must be considered. These nonlinear optical effects will be considered in Chapter 7 and in Chapter 12.

1.1.2. Boundary Conditions

Maxwell's equations can be solved in regions of space where both ε and μ are continuous. In optics, one often deals with situations in which the physical properties (characterized by ε and μ) change abruptly across one or more smooth surfaces. The field vectors **E**, **H**, **D**, and **B** at a point on one side of a smooth surface between two media, 1 and 2, are related to the field vectors at the neighboring point on the opposite side of the interface by boundary conditions that can be derived directly from Maxwell's equations.

Consider now a very short cylinder drawn about a boundary surface, as in Fig. 1.1(a), such that the end faces of the cylinder are in region 1 and 2 and are parallel to the surface of discontinuity. The height of the cylinder is infinitesimal, such that the end faces are arbitrarily close to the boundary surface. An application of the Gauss divergence theorem

$$\int \nabla \cdot \mathbf{F}\, dV = \int \mathbf{F} \cdot d\mathbf{S} \tag{1.1-7}$$

to both sides of Eqs. (1.1-3) and (1.1-4) yields

$$\begin{aligned} \mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) &= 0, \\ \mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) &= \sigma, \end{aligned} \tag{1.1-8a}$$

where **n** is the unit normal to the surface directed from medium 1 into

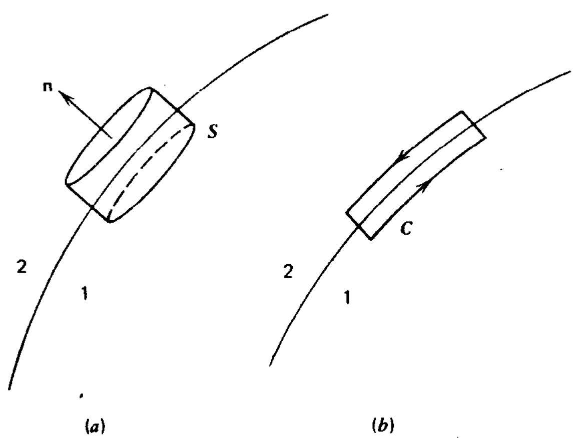

Figure 1.1. (a) A short cylinder about the interface between two media. S is the surface of this cylinder. (b) A narrow rectangle about the interface between two media. C is the boundary of this rectangle.

medium 2, σ is the surface charge density (coulombs per square meter), and the subscripts refer to values at the surface in the two media. The boundary conditions (1.1-8a) are often written as

$$B_{2n} = B_{1n},$$
$$D_{2n} - D_{1n} = \sigma, \qquad (1.1\text{-}8b)$$

where $B_{2n} = \mathbf{B}_2 \cdot \mathbf{n}$, $B_{1n} = \mathbf{B}_1 \cdot \mathbf{n}$, $D_{2n} = \mathbf{D}_2 \cdot \mathbf{n}$, and $D_{1n} = \mathbf{D}_1 \cdot \mathbf{n}$. In other words, the normal component of the magnetic induction **B** is always continuous, and the difference between the normal components of the electric displacement **D** is equal in magnitude to the surface charge density σ.

Next, consider a small, narrow, rectangular circuit loop around a section of the boundary surface, as in Fig. 1.1(b), such that the long sides of the rectangle are in regions 1 and 2 and parallel to the surface of discontinuity. The width of this rectangle is infinitesimal, so that the two long sides are arbitrarily close to the boundary. An application of the Stokes's theorem

$$\int \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int \mathbf{F} \cdot d\mathbf{l} \qquad (1.1\text{-}9)$$

to both sides of Eqs. (1.1-1) and (1.1-2) yields

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0,$$
$$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}, \qquad (1.1\text{-}10a)$$

where **K** is the surface current density (amperes per meter). Again, the boundary conditions for the electric and magnetic field vectors (1.1-10a) are often written as

$$\mathbf{E}_{2t} = \mathbf{E}_{1t},$$
$$\mathbf{H}_{2t} - \mathbf{H}_{1t} = \mathbf{K}, \qquad (1.1\text{-}10b)$$

where the subscript t means the tangential component of the field vector. (Note: The components of these field vectors tangential to the boundary surface are still vectors in the plane tangent to the surface.) In other words, the tangential component of the electric field vector **E** is always continuous at the boundary surface, and the difference between the tangential components of the magnetic field vector **H** is equal to the surface current density **K**.

In many areas of optics, one often deals with situations in which the surface charge density σ and the surface current density $\mathbf{K}$ both vanish. It follows that, in such a case, the tangential components of $\mathbf{E}$ and $\mathbf{H}$ and the normal components of $\mathbf{D}$ and $\mathbf{B}$ are continuous across the interface separating media 1 and 2. These boundary conditions are important in solving many wave propagation problems in optics, such as in guided-wave optics and wave propagation in layered media.

1.2. POYNTING'S THEOREM AND CONSERVATION LAWS

Conservation of energy for electromagnetic fields requires that the time rate of change of electromagnetic energy contained within a certain volume, plus the time rate of energy flowing out through the boundary surfaces of the volume, be equal to the negative of the total work done by the fields on the sources within the volume. For a point charge q, the rate of work done by an external electromagnetic field is $q\mathbf{v} \cdot \mathbf{E}$, where $\mathbf{v}$ is the velocity of the charge. The magnetic field does no work on the point charge, since the magnetic force is always perpendicular to the velocity. In the case of a distributed charge and current, the rate of work done by the fields per unit volume is $\mathbf{J} \cdot \mathbf{E}$. A continuity equation which describes this balance of energy exists. We will now derive this equation starting from the Maxwell equations. By using Eq. (1.1-2), we can express the rate of work done per unit volume by the electromagnetic fields as

$$\mathbf{J} \cdot \mathbf{E} = \mathbf{E} \cdot (\nabla \times \mathbf{H}) - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}. \tag{1.2-1}$$

If we now employ the vector identity

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}) \tag{1.2-2}$$

and use Eq. (1.1-1), the right-hand side of (1.2-1) becomes

$$\mathbf{J} \cdot \mathbf{E} = -\nabla \cdot (\mathbf{E} \times \mathbf{H}) - \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}. \tag{1.2-3}$$

If we now further assume that the material medium involved is linear in its electromagnetic properties (i.e., ε and μ are independent of the field strengths), Eq. (1.2-3) can be written

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}, \tag{1.2-4}$$

where U and $\mathbf{S}$ are defined as

$$U = \tfrac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}), \tag{1.2-5}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{1.2-6}$$

The scalar U represents the energy density of the electromagnetic fields and has the dimensions of joules per cubic meter. The vector $\mathbf{S}$, representing the energy flow, is called the Poynting vector and has the dimensions of joules per square meter per second. It is consistent to view $|\mathbf{S}|$ as the power per unit area (watts per square meter) carried by the field in the direction of $\mathbf{S}$. The quantity $\nabla \cdot \mathbf{S}$ thus represents the net electromagnetic power flowing out of a unit volume. Equation (1.2-4) is known as the continuity equation or the conservation of energy (Poynting's theorem). The conservation laws for the linear momentum of the electromagnetic fields can be obtained in a similar way. This is left as a problem for the student (Problem 1.4).

1.3. COMPLEX-FUNCTION FORMALISM

In optics, we generally deal with steady-state sinusoidal time-varying fields, for example, laser radiation. It is convenient to represent each field vector as a complex function. As an example, consider some component of the field vectors:

$$a(t) = |A|\cos(\omega t + \alpha), \tag{1.3-1}$$

where ω is the angular frequency and α is the phase. If we define a complex amplitude of $a(t)$ by

$$A = |A|e^{i\alpha}, \tag{1.3-2}$$

Eq. (1.3-1) can be written as

$$a(t) = \mathrm{Re}[Ae^{i\omega t}]. \tag{1.3-3}$$

We will often represent $a(t)$ by

$$a(t) = Ae^{i\omega t} \tag{1.3-4}$$

instead of by Eq. (1.3-1) or (1.3-3). This, of course, is not strictly correct; when it happens, it is always understood that what is meant (e.g.) by Eq. (1.3-4) is the real part of $Ae^{i\omega t}$. In most situations, representation of field vectors with the complex form (1.3-4) poses no problems insofar as linear mathematical operations, such as differentiation, integration, and summation, are concerned. The exceptions are cases that involve the product (or powers) of field vectors, such as the energy density and Poynting vector. In these cases, one must use the real form of the physical quantities.

As an example, consider the product of two sinusoidal functions $a(t)$ and $b(t)$, where

$$\begin{aligned} a(t) &= |A|\cos(\omega t + \alpha) \\ &= \mathrm{Re}[Ae^{i\omega t}] \end{aligned} \tag{1.3-5}$$

and

$$\begin{aligned} b(t) &= |B|\cos(\omega t + \beta) \\ &= \mathrm{Re}[Be^{i\omega t}] \end{aligned} \tag{1.3-6}$$

with $A = |A|e^{i\alpha}$ and $B = |B|e^{i\beta}$. Using the real functions, we get

$$a(t)b(t) = \tfrac{1}{2}|AB|[\cos(2\omega t + \alpha + \beta) + \cos(\alpha - \beta)]. \tag{1.3-7}$$

But if we were to evaluate the product $a(t)b(t)$ with the complex form of the functions, we would get

$$a(t)b(t) = ABe^{i2\omega t} = |AB|e^{i(2\omega t + \alpha + \beta)}. \tag{1.3-8}$$

A comparison of the last result with Eq. (1.3-7) shows that the time-independent (dc) term $\frac{1}{2}|AB|\cos(\alpha - \beta)$ is missing, and thus the use of the complex form led to an error. It is generally true that the product of the real parts of two complex numbers is not equal to the real part of the product of these two complex numbers. In other words, if x and y are two arbitrary complex numbers, the following is generally true:

$$\mathrm{Re}[x]\mathrm{Re}[y] \neq \mathrm{Re}[xy]. \tag{1.3-9}$$

1.3.1. Time-Averaging of Sinusoidal Products

In optical fields, the field vectors are rapidly varying functions of time. For example, the period of a time-varying field with a wavelength of $\lambda = 1\ \mu$m (one micrometer) is $T = \lambda/c = 0.33 \times 10^{-14}$ s. One often considers the time-averaged values rather than the instantaneous values of many physical quantities such as the Poynting vector and the energy density. It is fre-