MECHANICAL ENGINEERING SERIES

A.A. Shabana

Theory of Vibration

Volume I: An Introduction

Springer-Verlag
世界图书出版公司

A.A. Shabana

Theory of Vibration

Volume I: An Introduction

With 196 Figures

Springer-Verlag
New York Berlin Heidelberg London
Paris Tokyo Hong Kong Barcelona
世界图书出版公司
北京·广州·上海·西安

A.A. Shabana
Department of Mechanical Engineering
University of Illinois at Chicago
P.O. Box 4348
Chicago, IL 60680
USA

Series Editor

Frederick F. Ling
Director, Columbia Engineering Productivity Center, and Professor, Department of Mechanical Engineering, Columbia University, New York, NY 10027-6699; and Distinguished William Howard Hart Professor Emeritus, Department of Mechanical Engineering, Aeronautical Engineering and Mechanics, Rensselaer Polytechnic Institute, Troy, NY 12180-3590, USA

Library of Congress Cataloging-in-Publication Data
Shabana, Ahmed A., 1951–
Theory of vibration / A.A. Shabana.
p. cm.—(Mechanical engineering series)
Includes bibliographical references.
Contents: v. 1. An introduction
ISBN 0-387-97276-5 (alk. paper)
1. Vibration. I. Title. II. Series.
QA865.S49 1990
531'.32—dc20 90-9582

Reprinted by World Publishing Corporation, Beijing, 1992
for distribution and sale in The People's Republic of China only
ISBN 7 – 5062 – 1411 – 3

ISBN 0-387-97276-5 Springer-Verlag New York Berlin Heidelberg
ISBN 3-540-97276-5 Springer-Verlag Berlin Heidelberg New York

Mechanical Engineering Seri

Frederick F. Ling
Series Editor

Series Preface

Mechanical engineering, an engineering discipline borne of the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions, among others. The Mechanical Engineering Series is a new series, featuring graduate texts and research monographs, intended to address the need for information in contemporary areas of mechanical engineering.

The series is conceived as a comprehensive one which will cover a broad range of concentrations important to mechanical engineering graduate education and research. We are fortunate to have a distinguished roster of consulting editors on the advisory board, each an expert in one of the areas of concentration. The names of the consulting editors are listed on the first page of the volume. The areas of concentration are: applied mechanics; biomechanics; computational mechanics; dynamic systems and control; energetics; mechanics of materials; processing; thermal science; and tribology.

Professor Marshek, the consulting editor for dynamic systems and control, and I are pleased to present the fourth volume of the series: *Theory of Vibration, Volume I: An Introduction* by Professor Shabana. We note that this is the first of two volumes. The second will deal with *discrete and continuous systems.*

Frederick F. Ling

Preface

The aim of this book is to impart a sound understanding, both physical and mathematical, of the fundamentals of the theory of vibration and its applications. The book presents in a simple and systematic manner techniques that can be easily applied to the analysis of vibration of mechanical and structural systems. In most of the existing textbooks in this field, vibration problems are solved using an intuitive approach, with the assumption that the students have a sufficiently strong background in dynamics and mathematics. Furthermore, many of the techniques presented are tailored to deal with specific applications, without explanation of the more general concepts which can be applied to a larger class of problems. For example, the methods of developing the equations of motion of oscillatory rigid bodies (pendulums) use the special case of noncentroidal rotation when one point on the rigid body is fixed. As the result of using this approach, many students fail to develop the equations of motion of a pendulum with a moving base. Another example is the use of conservation of energy, at the beginning chapters of most existing vibration books, to solve special problems. This often leads to problems when students try to use these special techniques to solve other applications. It seems more appropriate to present the more general theorems of conservation of energy after covering the Lagrangian dynamics. Those theorems can then be simplified and applied to simple problems. In so doing, the students become aware of the assumptions made, and misconceptions regarding the conservation of energy can be avoided. From my experience, I have found that the use of shortcuts for solving vibration problems is often misleading. In this book an attempt has been made to provide the rational development of the methods of vibration analysis from their foundations and to develop the techniques in clearly understandable stages. I have found that adding a chapter which briefly discusses the solution of the vibration equations (Chapter 2) is helpful in this regard. This book, which is based on class notes which I have used for several years, is in many ways different from existing textbooks. Basic dynamic concepts are used to develop the equations of the oscillatory motion, the assumptions used to linearize the dynamic equations are clearly stated, and the relationship between the coefficients of the differential equations and the stability of mechanical systems is discussed more thoroughly.

The first volume of this book is intended as an introductory semester course on the theory of vibration. Since this volume is written for a first course in vibrations, new concepts have been presented in simple terms and the solution procedures have been explained in detail. The material covered in the volume comprises the following chapters.

In Chapter 1 basic definitions related to the theory of vibration are presented. The elements of the vibration models, such as inertia, elastic, and damping forces, are discussed in Section 2 of this chapter. Section 3 is devoted to the use of Newton's second law and D'Alembert's principle for formulating the equations of motion of simple vibratory systems. In Section 4 the dynamic equations that describe the translational and rotational displacements of rigid bodies are presented. It is also shown that these equations can be nonlinear because of the finite rotation of the rigid bodies. The linearization of the resulting differential equations of motion is the subject of Section 5. In Section 6 methods for obtaining simple finite number of degrees of freedom models for mechanical and structural systems are discussed.

Chapter 2 describes methods for solving both homogeneous and nonhomogeneous differential equations. The effect of the coefficients in the differential equations on the stability of the vibratory systems is also examined. Even though students may have seen differential equations in other courses, I have found that presenting Chapter 2 after discussing the formulation of the equations of motion in Chapter 1 is helpful.

Chapter 3 is devoted to the free vibrations of single degree of freedom systems. Both cases of undamped and damped free vibration are considered. The stability of undamped and damped linear systems is examined. The cases of viscous, structural, Coulomb, and negative damping are discussed, and examples for oscillatory systems are presented.

Chapter 4 is concerned with the forced vibration of single degree of freedom systems. Both cases of undamped and damped forced vibration are considered, and the phenomena of resonance and beating are explained. The forced vibrations, as the result of rotating unbalance and base excitation, are discussed in Sections 5 and 6. The theoretical background required for understanding the function of vibration measuring instruments is presented in Section 7 of this chapter. Methods for the experimental evaluation of the damping coefficients are covered in Section 8.

In the analysis presented in Chapter 4 the forcing function is assumed to be harmonic. Chapter 5 provides an introduction to the vibration analysis of single degree of freedom systems subject to nonharmonic forcing functions. Periodic functions expressed in terms of Fourier series expansion are first presented. The response of the single degree of freedom system to a unit impulse is defined in Section 6. The impulse response is then used in Section 7 to obtain the response of the single degree of freedom system to an arbitrary forcing function. In Section 8 computer methods for the vibration analysis of nonlinear systems are discussed.

In Chapter 6 the linear theory of vibration of two degree of freedom systems is presented. The equations of motion are presented in a matrix form. The case of damped and undamped free and forced vibration, as well as the theory of the vibration absorber of undamped and damped systems, are discussed.

I would like to thank many of my teachers and colleagues who contributed, directly or indirectly, to this book. I wish to acknowledge gratefully the many helpful comments and suggestions offered by my students. I would also like to thank Mr. D.C. Chen, Dr. W.H. Gau, and Mr. J.J. Jiang for their help in reviewing the manuscript and producing some of the figures. Thanks are due also to Ms. Denise Burt for the excellent job in typing the manuscript. The editorial and production staff of Springer-Verlag deserve special thanks for their cooperation and thorough professional work in producing this book. Finally, I thank my family for their patience and encouragement during the period of preparation of this book.

Chicago, Illinois — Ahmed A. Shabana

Contents

1
Introduction

The process of change of physical quantities such as displacements, velocities, accelerations, and forces may be grouped into two categories; oscillatory and nonoscillatory. The oscillatory process is characterized by alternate increases or decreases of a physical quantity. A nonoscillatory process does not have this feature. The study of oscillatory motion has a long history, extending back to more than four centuries ago. Such a study of oscillatory motion may be said to have started in 1584 with the work of Galileo (1564–1642) who examined the oscillations of a simple pendulum. Galileo was the first to discover the relationship between the frequency of the simple pendulum and its length. At the age of 26, Galileo discovered the law of falling bodies and wrote the first treatise on modern dynamics. In 1636 he disclosed the idea of the pendulum clock which was later constructed by Huygens in 1656.

An important step in the study of oscillatory motion is the formulation of the dynamic equations. Based on Galileo's work, Sir Isaac Newton (1642–1727) formulated the laws of motion in which the relationship between force, mass, and momentum is established. At the age of 45, he published his *Principle Mathematica* which is considered the most significant contribution to the field of mechanics. In particular, Newton's second law of motion has been a basic tool for formulating the dynamic equations of motion of vibratory systems. Later, the French mathematician Jean le Rond D'Alembert (1717–1783) expressed Newton's second law in a useful form, known as D'Alembert's principle, in which the inertia forces are treated in the same way as the applied forces. Based on D'Alembert's principle, Joseph Louis Lagrange (1736–1813) developed his well-known equations; Lagrange's equations, which were presented in his *Mechanique*. Unlike Newton's second law which uses vector quantities, Lagrange's equations can be used to formulate the differential equations of dynamic systems using scalar energy expressions. The Lagrangian approach, as compared to the Newtonian approach, lends itself easily to formulating the vibration equations of multidegree of freedom systems.

Another significant contribution to the theory of vibration was made by Robert Hooke (1635–1703) who was the first to announce, in 1676, the relationship between the stress and strain in elastic bodies. Hooke's law for deformable bodies states that the stress at any point on a deformable body is proportional to the strain at that point. In 1678, Hooke explained his law as "The power of any springy body is in the same proportion with extension." Based on Hooke's law of elasticity, Leonhard Euler (1707–1783) in 1744 and Daniel Bernoulli (1700–1782) in 1751 derived the differential equation that governs the vibration of beams. They also obtained the solution in the case of small deformation. Their work is known as Euler–Bernoulli beam theory. Daniel Bernoulli also examined the vibration of a system of *n* point masses and showed that such a system has *n* independent modes of vibration. He formulated the principle of superposition which states that the displacement of a vibrating system is given by a superposition of its modes of vibrations.

The modern theory of mechanical vibration was organized and developed by Baron William Strutt, Lord Rayleigh (1842–1919), who published his book in 1877 on the theory of sound. He also developed a method known as Rayleigh's method for finding the fundamental natural frequency of vibration using the principle of conservation of energy. Rayleigh made a correction to the technical beam theory (1894) by considering the effect of the rotary inertia of the cross section of the beam. The resulting equations are found to be more accurate in representing the propagation of elastic waves in beams. Later, in 1921, Stephen Timoshenko (1878–1972) presented an improved theory for the vibrations of beams. His theory has become known as Timoshenko beam theory. Among the contributors to the theory of vibrations is Jean Baptiste Fourier (1768–1830) who developed the well-known Fourier series which can be used to express periodic functions in terms of harmonic functions. Fourier series are widely used in the vibration analysis of discrete and continuous systems.

1.1 BASIC DEFINITIONS

Vibration theory is concerned with the oscillatory motion of physical systems. The motion may be harmonic, periodic, or a general motion in which the amplitude varies with time. The importance of vibration to our comfort and needs is so great that it would be pointless to try to list all the examples which come to mind. Vibration of turbine blades, chatter vibration of machine tools, electrical oscillations, sound waves, vibrations of engines, torsional vibrations of crankshafts, and vibrations of automobiles on their suspensions can all be regarded as coming within the scope of vibration theory. We shall, however, be concerned in this book with the vibrations of mechanical and structural systems.

Vibrations are encountered in many mechanical and structural applications, for example, mechanisms and machines, buildings, bridges, vehicles, and aircraft. Some of these systems are shown in Fig. 1. In many of these systems excessive vibrations produce high stress levels, which in turn may cause mechanical failure. Vibration can be classified as *free* or *forced* vibration. In free vibration, there are no external forces that act on the system. Forced

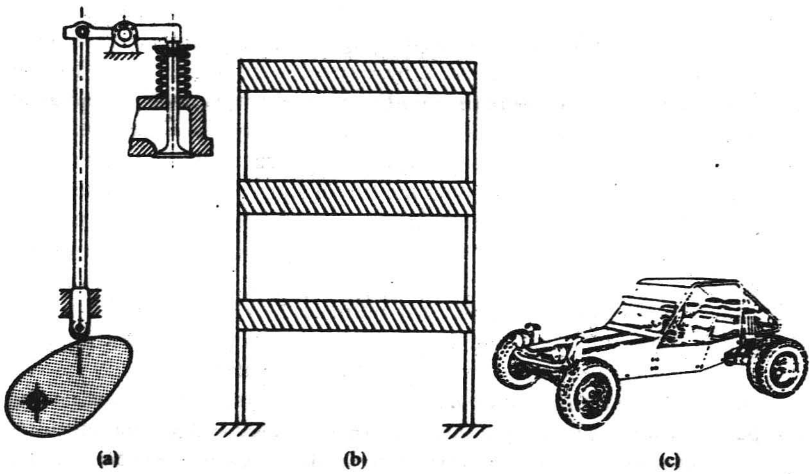

FIG. 1.1. Physical systems: (a) mechanism systems; (b) multistory buildings; (c) vehicle systems.

vibrations, on the other hand, are the result of external excitations. In both cases of free and forced vibration the system must be capable of producing restoring forces which tend to maintain the oscillatory motion. These restoring forces can be produced by discrete elements such as the linear and torsional springs shown, respectively, in Fig. 2(a) and (b) or by continuous structural elements such as beams and plates (Fig. 2(c), (d)).

These discrete and continuous elastic elements are commonly used in many systems, such as the suspensions and frames of vehicles, the landing gears, fuselage, and wings of aircraft, bridges, and buildings. Clearly, the restoring forces produced by the elastic elements are proportional to the deflection or the elastic deformation of these elements. If the vibration is small, it is customary to assume that the force–deflection relationship is linear, that is, the force is equal to the deflection multiplied by a proportionality constant. In this case the *linear theory of vibration* can be applied. If the assumptions

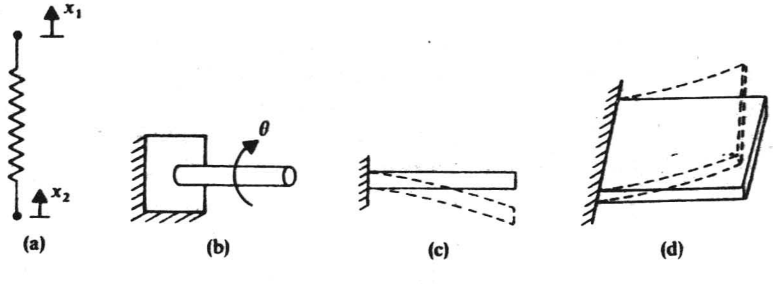

FIG. 1.2. Elastic elements.

of the linear theory of vibration are not valid, for example, if the displacement-force relationship cannot be described using linear equations, the ***nonlinear theory of vibration*** must be applied. Linear systems are usually easier to deal with since in many cases, where the number of equations is small, closed-form solutions can be obtained. The solution of nonlinear system equations, however, requires the use of approximation and numerical methods. Closed-form solutions are usually difficult to obtain even for simple nonlinear systems. In many applications linearization techniques are used, in order to obtain a linear system of differential equations whose solution can be obtained in a closed form.

The level of vibration is significantly influenced by the amount of energy dissipation. Energy is dissipated as a result of *dry friction* between surfaces, *viscous damping*, and/or *structural damping* of the material. The dry friction between surfaces is also called *Coulomb damping*. In many applications, energy dissipated as the result of damping can be evaluated using damping forcing functions that are velocity-dependent. In this book we also classify our vibratory systems according to the presence of damping. If the system has a damping element, it is called a *damped system*. Otherwise, it is called an *undamped system*.

Mechanical systems can also be classified according to the number of degrees of freedom which is defined as the minimum number of coordinates required to define the system configuration. In textbooks on the theory of vibration, mechanical and structural systems are often classified as *single degree of freedom systems*, *two degree of freedom systems*, *multi-degree of freedom systems*, or *continuous systems* which have an infinite number of degrees of freedom. The vibration of systems which have a finite number of degrees of freedom is governed by second-order ordinary differential equations. On the other hand, the vibration of continuous systems which have infinite degrees of freedom is governed by partial differential equations, which depend on time as well as on the spatial coordinates. Finite degree of freedom models, however, can be obtained for continuous systems by using approximation techniques such as the *Rayleigh–Ritz method* and the *finite element method*.

1.2 ELEMENTS OF THE VIBRATION MODELS

Vibrations are the result of the combined effects of the *inertia* and *elastic* forces. Inertia of moving parts can be expressed in terms of the masses, moments of inertia, and the time derivatives of the displacements. Elastic restoring forces, however, can be expressed in terms of the displacements and stiffness of the elastic members. While damping has a significant effect, vibration may occur without damping. Damping, however, remains as a basic element in the vibration analysis.

Inertia Inertia is the property of an object that causes it to resist any effort to change its motion. For a particle, the inertia force is defined as the product of the mass of the particle and the acceleration, that is,

$$\mathbf{F}_i = m\ddot{\mathbf{r}}$$

where $\mathbf{F}_i$ is the vector of the inertia forces, m is the mass of the particle, and $\ddot{\mathbf{r}}$ is the acceleration vector defined in an inertial frame of reference. Rigid bodies, on the other hand, have inertia forces and moments. For the planar motion of a rigid body, the inertia forces and moments are given by

$$\mathbf{F}_i = m\ddot{\mathbf{r}}$$
$$M_i = I\ddot{\theta}$$

where $\mathbf{F}_i$ is the inertia forces, m is the total mass of the rigid body, $\ddot{\mathbf{r}}$ is the acceleration vector of the center of mass of the body, M_i is the inertia moment, I is the mass moment of inertia of the rigid body about its center of mass, and $\ddot{\theta}$ is the angular acceleration. The units for the inertia forces and moments are, respectively, the units of forces and moments.

Elastic Forces Components with distributed elasticity are used in mechanical and structural systems to provide flexibility, and to store or absorb energy. These elastic members produce restoring forces which depend on the stiffness of the member as well as the displacements. Consider, for example, the spring connecting the two masses shown in Fig. 3(a). If the displacement of the first mass is x_1 and the displacement of the second mass is x_2, and if we assume for the moment that x_1 is greater than x_2, the total deflection in the spring is given by

$$\Delta x = x_1 - x_2$$

where Δx is the total deflection of the spring due to the displacements of the two masses. Using Taylor's series, the spring force after the displacement Δx can be written as

$$F_s(x_0 + \Delta x) = F_s(x_0) + \left.\frac{\partial F_s}{\partial x}\right|_{x=x_0} \Delta x + \frac{1}{2!}\left.\frac{\partial^2 F_s}{\partial x^2}\right|_{x=x_0} (\Delta x)^2 + \cdots \quad (1.1)$$

where F_s is the spring force and x_0 may be defined as the pretension or

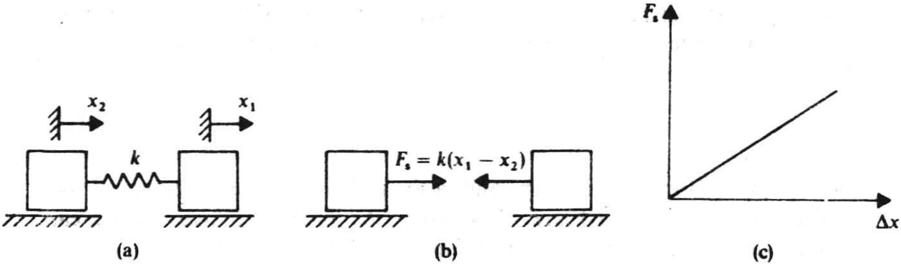

FIG. 1.3. Linear spring force.

precompression in the spring before the displacement Δx. If there is no pretension or compression in the spring, the spring force $F_s(x_0)$ is identically zero.

As a result of the displacement Δx, the spring force $F_s(x_0 + \Delta x)$ can be written as

$$F_s(x_0 + \Delta x) = F_s(x_0) + \Delta F_s \tag{1.2}$$

where ΔF_s is the change in the spring force as a result of the displacement Δx. By using Eq. 1, ΔF_s of Eq. 2 can be written as

$$\Delta F_s = \left.\frac{\partial F_s}{\partial x}\right|_{x=x_0} \Delta x + \frac{1}{2}\left.\frac{\partial^2 F_s}{\partial x^2}\right|_{x=x_0} (\Delta x)^2 + \cdots \tag{1.3}$$

If the displacement Δx is assumed to be small, higher-order terms in Δx can be neglected and the spring force ΔF_s can be linearized. In this case, Eq. 3 yields

$$\Delta F_s = \left.\frac{\partial F_s}{\partial x}\right|_{x=x_0} \Delta x \tag{1.4}$$

This equation can be written in a simpler form as

$$\begin{aligned} \Delta F_s &= k\Delta x \\ &= k(x_1 - x_2) \end{aligned} \tag{1.5}$$

where k is a proportionality constant called the *spring constant*, the *spring coefficient*, or the *stiffness coefficient*. The spring constant k is defined as

$$k = \left.\frac{\partial F_s}{\partial x}\right|_{x=x_0} \tag{1.6}$$

The effect of the spring force F_s on the two masses is shown in Fig. 3(b), and the linear relationship between the force and the displacement of the spring is shown in Fig. 3(c). Springs are commonly used in many mechanical systems, as shown in Fig. 4.

Continuous elastic elements such as rods, beams, and shafts produce restoring elastic forces. Figure 5 shows some of these elastic elements which behave like springs. In Fig. 5(a) the rod produces a restoring elastic force that resists the longitudinal displacement in the system. If the mass of the rod is negligible compared to the mass m, one can write, from strength of materials, the following relationship

$$F = \frac{EA}{l} u \tag{1.7}$$

where F is the force acting at the end of the rod, u is the displacement of the end point, and l, A, and E are, respectively, the length, cross-sectional area, and modulus of elasticity of the rod. Equation 7 can be written as

$$F = ku \tag{1.8}$$

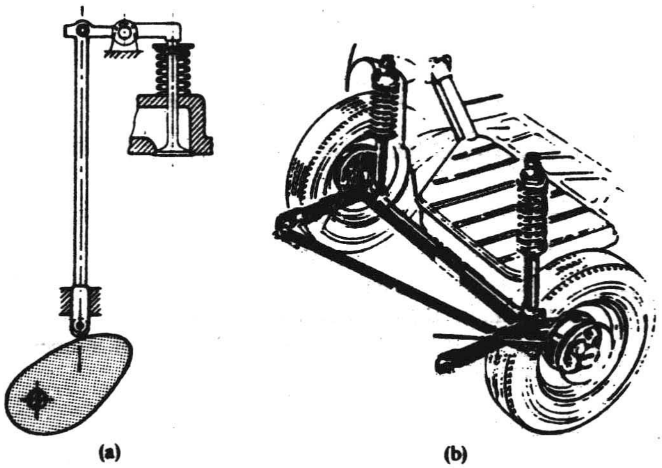

FIG. 1.4. Use of springs in mechanical systems: (a) cam mechanisms; (b) vehicle suspensions.

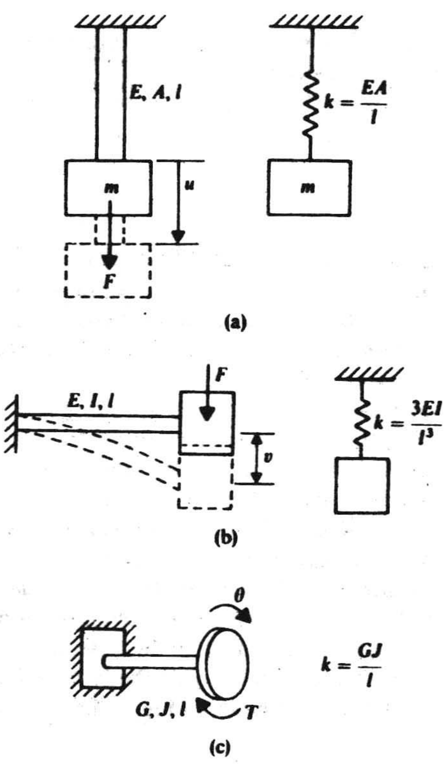

FIG. 1.5. Continuous elastic systems: (a) longitudinal vibration of rods; (b) transverse vibration of cantilever beams; (c) torsional system.