

Mechanics of

Aircraft *structures*

A large, stylized graphic of an aircraft wing and a jet engine, rendered in a reddish-orange hue, dominates the center of the cover. The wing is shown from a perspective view, and the engine is depicted as a large, circular component. The background is a dark, textured surface with a grid of thin, light-colored lines. A bright, circular light source, possibly the sun, is visible in the upper right quadrant, casting a glow over the scene.

C. T. SUN

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PREFACE

This book is intended for junior or senior level aeronautical engineering students with a background in the first course of mechanics of solids. The contents can be covered in a semester at a normal pace.

The selection and presentation of materials in the course of writing this book were greatly influenced by the following developments. First, commercial finite element codes have been used extensively for structural analyses in recent years. As a result, many simplified ad hoc techniques that were important in the past have lost their useful roles in structural analyses. This development leads to the shift of emphasis from the problem-solving drill to better understanding of mechanics, developing the student's ability in formulating the problem, and judging the correctness of numerical results. Second, fracture mechanics has become the most important tool in the study of aircraft structure damage tolerance and durability in the past thirty years. It seems highly desirable for undergraduate students to get some exposure to this important subject, which has traditionally been regarded as a subject for graduate students. Third, advanced composite materials have gained wide acceptance for use in aircraft structures. This new class of materials is substantially different from traditional metallic materials. An introduction to the characteristic properties of these new materials seems imperative even for undergraduate students.

In response to the advent of the finite element method, consistent elasticity approach is employed. Multidimensional stresses, strains, and stress-strain relations are emphasized. Displacement, rather than strain or stress, is used in deriving the governing equations for torsion and bending problems. This

approach will help the student understand the relation between simplified structural theories and 3-D elasticity equations.

The concept of fracture mechanics is brought in via the original Griffith's concept of strain energy release rate. Taking advantage of its global nature and its relation to the change of the total strain energies stored in the structure before and after crack extension, the strain energy release rate can be calculated for simple structures without difficulty for junior and senior level students.

The coverage of composite materials consists of a brief discussion of their mechanical properties in Chapter 1, the stress-strain relations for anisotropic solids in Chapter 2, and a chapter (Chapter 8) on analysis of symmetric laminates of composite materials. This should be enough to give the student a background to deal correctly with composites and to avoid regarding a composite as an aluminum alloy with the Young's modulus taken equal to the longitudinal modulus of the composite. Such a brief introduction to composite materials and laminates is by no means sufficient to be used as a substitute for a course (or courses) dedicated to composites.

A classical treatment of elastic buckling is presented in Chapter 7. Besides buckling of slender bars, the postbuckling concept and buckling of structures composed of thin sheets are also briefly covered without invoking an advanced background in solid mechanics. Postbuckling strengths of bars or panels are often utilized in aircraft structures. Exposure, even very brief, to this concept seems justified, especially in view of the mathematics employed, which should be quite manageable for student readers of this book.

The author expresses his appreciation to Mrs. Marilyn Engel for typing the manuscript and to James Chou and R. Sergio Hasebe for making the drawings.

C. T. SUN

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CHARACTERISTICS OF AIRCRAFT STRUCTURES AND MATERIALS

1.1 INTRODUCTION

The main difference between aircraft structures and materials and civil engineering structures and materials lies in their weight. The main driving force in aircraft structural design and aerospace material development is to reduce weight. In general, materials with high stiffness, high strength, and light weight are most suitable for aircraft applications.

Aircraft structures must be designed to ensure that every part of the material is used to its full capability. This requirement leads to the use of shell-like structures (monocoque constructions) and stiffened shell structures (semi-monocoque constructions). The geometrical details of aircraft structures are much more complicated than those of civil engineering structures. They usually require the assemblage of thousands of parts. Technologies for joining the parts are especially important for aircraft construction.

The size and shape of an aircraft structural component are usually determined based on nonstructural considerations. For instance, the airfoil is chosen according to aerodynamic lift and drag characteristics. Then the solutions for structural problems in terms of global configurations are limited. Often, the solutions resort to the use of special materials developed for applications in aerospace vehicles.

Because of their high stiffness/weight and strength/weight ratios, aluminum and titanium alloys have been the dominant aircraft structural materials for many decades. However, the recent advent of advanced fiber-reinforced composites has changed the outlook. Composites may now achieve weight savings of 30–40 percent over aluminum or titanium counterparts.

1.2 BASIC STRUCTURAL ELEMENTS IN AIRCRAFT STRUCTURE

Major components of aircraft structures are assemblages of a number of basic structural elements, each of which is designed to take a specific type of load, such as axial, bending, or torsional load. Collectively, these elements can efficiently provide the capability for sustaining loads on an airplane.

1.2.1 Axial Member

Axial members are used to carry extensional or compressive loads applied in the direction of the axial direction of the member. The resulting stress is uniaxial:

$$\sigma = E\varepsilon \quad (1.1)$$

where E and ε are the Young's modulus and normal strain, respectively, in the loading direction. The total axial force F provided by the member is

$$F = A\sigma = EA\varepsilon \quad (1.2)$$

where A is the cross-sectional area of the member. The quantity EA is termed the axial stiffness of the member, which depends on the modulus of the material and the cross-sectional area of the member. It is obvious that the axial stiffness of axial members cannot be increased (or decreased) by changing the shape of the cross-section. In other words, a circular rod and a channel (see Figs. 1.1a and 1.1b) can carry the same axial load as long as they have the same cross-sectional area.

Axial members are usually slender and are susceptible to buckling failure when subjected to compression. Buckling strength can be increased by increasing the bending stiffness and by shortening the length of the buckle mode. For buckling, the channel section is better since it has higher bending

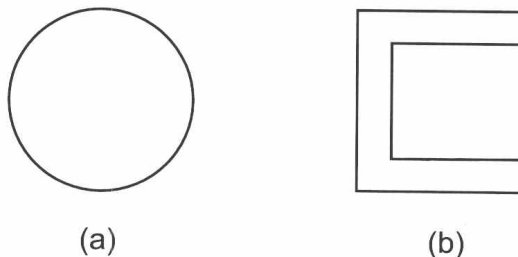


Figure 1.1 (a) Circular rod. (b) Channel.

stiffness than the circular section. However, because of the slenderness of most axial members used in aircraft (such as stringers), the bending stiffness of these members is usually very small and is not sufficient to achieve the necessary buckling strength. In practice, the buckling strength of axial members is enhanced by providing lateral supports along the length of the member with more rigid ribs (in wings) and frames (in fuselage).

1.2.2 Shear Panel

A shear panel is a thin sheet of material used to carry in-plane shear load. Consider a shear panel under uniform shear stress τ as shown in Fig. 1.2. The total shear force in the x -direction provided by the panel is given by

$$V_x = \tau \cdot t \cdot a = G\gamma ta \quad (1.3)$$

where G is the shear modulus, and γ is the shear strain. Thus, for a flat panel, the shear force V_x is proportional to its thickness and the lateral dimension a .

For a curved panel under a state of constant shear stress τ (see Fig 1.3), the resulting shear force of the shear stress on the thin-walled section may be decomposed into a horizontal component V_x and a vertical component V_y as

$$V_x = \tau \cdot t \cdot a = Gt a \gamma \quad (1.4)$$

$$V_y = \tau \cdot t \cdot b = Gt b \gamma \quad (1.5)$$

If there are no other design constraints, the most efficient shear panel for resisting shear loads V_x and V_y is to choose a and b so that

$$\frac{a}{b} = \frac{V_x}{V_y}$$

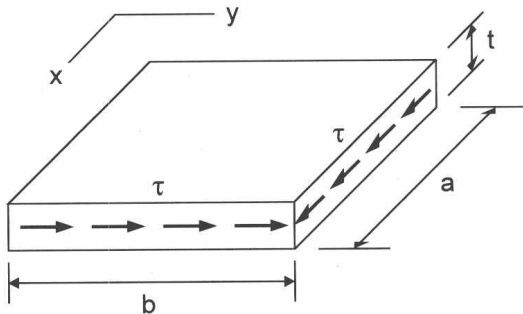


Figure 1.2 A shear panel under uniform shear stress.

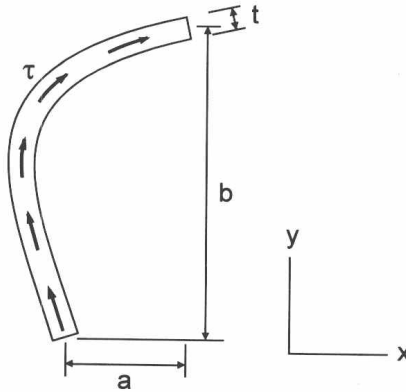


Figure 1.3 A curved panel under a state of constant shear stress.

Since the contour shape of the panel is irrelevant, it is obvious that a flat panel is most cost effective in resisting shear forces.

1.2.3 Bending Member (Beam)

A structural member that can carry bending moments is called a beam. A beam can also act as an axial member carrying longitudinal tension and compression. According to simple beam theory, bending moment M is related to beam deflection w as

$$M = -EI \frac{d^2 w}{dx^2} \quad (1.6)$$

where EI is the bending stiffness of the beam. The area moment of inertia I depends on the geometry of the cross-section.

Except for pure moment loading, a beam is designed to carry both bending moments and transverse shear forces as the latter usually produce the former. For a beam of a large span/depth ratio, the bending stress is usually more critical than the transverse shear stress. This is illustrated by the example of a cantilever beam shown in Fig. 1.4.

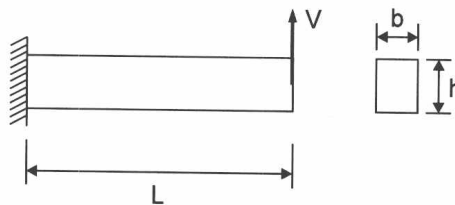


Figure 1.4 A cantilever beam.

It is easy to see that the maximum bending moment and bending stress occur at the fixed root of the cantilever beam. We have

$$\sigma_{\max} = \frac{M_{\max}(h/2)}{I} = \frac{VL(h/2)}{bh^3/12} = \frac{6VL}{bh^2} \quad (1.7)$$

The transverse shear stress distribution is parabolic over the beam depth with maximum value occurring at the neutral plane, i.e.,

$$\tau_{\max} = \frac{3}{2} \frac{V}{bh} \quad (1.8)$$

From the ratio

$$\frac{\sigma_{\max}}{\tau_{\max}} = \frac{4L}{h} \quad (1.9)$$

it is evident that bending stress plays a more dominant role than transverse shear stress if the span-to-depth ratio is large (as in wing structure). For such beams, attention is focused on optimizing the cross-section to increase bending stiffness.

In the elastic range, bending stress distribution over depth is linear with maximum values at the farthest positions from the neutral axis. The material near the neutral axis is underutilized. Thus, the beam with a rectangular cross-section is not an efficient bending member.

In order to utilize the material to its full capacity, material in a beam must be located as far as possible from the neutral axis. An example is the wide flange beam shown in Fig. 1.5a. Although the bending stress distribution is still linear over the depth, the bending line force (bending stress times the width) distribution is concentrated at the two flanges as shown in Fig. 1.5b because $b \gg t_w$. For simplicity, the small contribution of the vertical web to bending can be neglected.

The transverse shear stress distribution in the wide flange beam is shown in Fig. 1.5c. The vertical web is seen to carry essentially all the transverse shear load; its variation over the web is small and can be practically assumed to be constant. For all practical purposes, the wide flange beam can be regarded as two axial members (flanges) connected by a flat shear panel.

1.2.4 Torsion Member

Torque is an important form of load to aircraft structures. In a structural member, torque is formed by shear stresses acting in the plane of the cross-section. Consider a hollow cylinder subjected to a torque T as shown in Fig.