FEEDBACK

FRED D. WALDHAUER

Preface

Feedback has been one of the more fascinating concepts of technology for centuries, from sixteenth century furnaces that controlled their own temperature to contemporary theories of social interaction. The intuitive understanding of feedback systems at the most elementary level has been made more difficult than necessary by an "endless chain of dependencies" that seems to arise whenever we attempt to analyze a feedback system.

In a system of three interacting things, for example, when thing *A* affects thing *B* and thing *B*, in turn, affects thing *C*, we feel ourselves to be on firm ground in our understanding of the system, even if we do not know all the details of the interactions between *A* and *B* or between *B* and *C*. We have a mental picture that follows a cause-and-effect path sequentially from *A* to *B* to *C*.

If *C* turns around and affects *A*, however, our mental picture of the interactions is no longer so clear. By introducing *feedback* from *C* to *A*, we establish an endless chain of dependencies. The mathematics of the process is well established, but the *schema*, or mental picture, is more complex than it need be. At this point we become involved with the mathematical analysis of the whole process to make sure that we have accounted for everything. We run the risk of getting bogged down in mathematical detail and losing sight of what we are trying to accomplish.

This book adopts a basic change in outlook that greatly simplifies feedback analysis and design. It allows us to retain a clear mental picture of the interactions. In the view developed in this book, we assume that the system output at C is known. (It must, after all, satisfy some design specification, for example.) Then one part of A is known—the part that comes directly from C through a feedback path. But if the output at C is known, we can infer the input at B from the characteristics of the connecting path from B to C. If we know the input at B, we can similarly infer the contribution to A implied by the input at B (through the characteristics of the connecting path from A to B).

Finally, we *add* the two contributions to A to find the total input to the system. We can thus find the loss of the system—the input divided by the output. No endless chain of dependencies arises, and our mental picture is one of sequential reasoning through the two paths from output to input, in this case from C to A.

To express the distinction between the new theory and the old, we use the term "anticausal analysis" to describe the direction of analysis from output to input.

By applying this change in point of view to many practical areas of circuit analysis and design, we show (1) how it can be used in studying feedback systems and (2) how it is applied to the problems of circuit design. One of the chief benefits of the new approach is that we obtain a traceable path from the initial, rough design approximations to the final, exact analysis and design.

Most of the examples in this book come from electrical circuits, where I have had most of my experience. Examples from audio frequency design to designs of microwave integrated circuits are employed; a uniform approach is adopted over the whole range.

Knuth has said that "the enjoyment of the tools one works with is, of course, an essential ingredient of successful work."* An object of this book is to provide the reader with an enjoyable set of tools for designing feedback systems. I hope that it will also kindle interest in circuit theory and design.

For readers who would like to apply the methods developed here directly to obtain individual designs of their own, or to check the designs given in the book, 31 programs are given in three appendices. They are written for the Hewlett-Packard HP 41C or 41CV calculator and cover most aspects of the material in the book.

Among these programs is one that synthesizes feedback systems for a prescribed performance. Another converts the HP 41C calculator into a "two-port network calculator." Included are the four basic functions of addition, subtraction, multiplication and the matrix inverse, as well as lead interchange operations (e.g., conversion from common emitter to common base or common collector), all available at the touch of a button. This "calculator within a calculator" is itself programmable, and means are provided for converting numerical results into network properties, including loss, input and output impedances, and sensitivities as functions of frequency.

These programs were originally written as an aid to the author to assure himself that the approaches taken could be expressed algorithmically. I believe

*Quoted by J. A. Ball in his preface to *Algorithms for RPN Calculators*, Wiley, New York, 1978.

that they have turned out to be more generally useful as teaching tools in themselves. To avail oneself of this feature, a calculator must be acquired (with printer and card reader). Alternatively, the programs of interest can be rewritten for the computer in the reader's own operating system and language.

This book is intended for upper-division undergraduate students of electrical engineering and for professionals who have an interest in designing feedback systems and circuits. It arose from notes written for an in-hours two semester course that I taught at Bell Laboratories. After finishing the book, the reader should be able to design feedback systems in a very direct way, with confidence in the sensitivities of the important design specifications to the devices and components used. I hope that the reader will also be motivated to do original work in this area.

The book is intended for either individual or classroom study at several levels of reader involvement. A good overview of the subject can be obtained by reading the book and following the mathematical developments. To become adept at applying the methods in actual circuit design, the reader should complete the homework problems. Further study is facilitated by the fully documented calculator programs in the appendices.

ORGANIZATION OF THE BOOK

The subject matter is separated into three hierarchical levels: (1) the system level, (2) the circuit level, and (3) the device level. In the interests of clarity in both thought and programs, interaction between hierarchical levels has been restricted to adjacent levels to the fullest extent possible. The book is divided into two parts. Part 1 concerns the relationship between system and circuit levels, and Part 2 concerns the interactions between circuit and device levels. In Part 2 the system considerations of Part 1 are also included.

Separation into hierarchical levels is helpful in breaking the design process into manageable pieces, particularly in the design of monolithic integrated circuits. It is also invaluable in rational programming of the design on the calculator or computer.

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Part 1

Systems and Circuits

Chapter 1

Feedback Amplifiers: An Alternate Foundation

Modern feedback theory may be said to have begun on the Lackawanna Ferry between Hoboken, New Jersey and Manhattan on the morning of August 2, 1927. Harold Black was a passenger on his way to work at Bell Laboratories, where he had been working for some six years on the problem of reducing distortion in amplifiers to be used in repeaters for telephone transmission. On a blank space in his copy of *The New York Times*, he drew the diagram and wrote the equation shown in Fig. 1.1.[1,2] The diagram has become a commonplace in fields far removed from telephone transmission, appearing in books and journals on control theory, system theory, biology, cybernetics, sociology, and economics. The diagram and the equation represent the canonical view of feedback.

Pinpointing the beginning of feedback theory at this event is arbitrary, perhaps, since Maxwell had analyzed what we recognize as a feedback system —the flyball governor—some 60 years earlier.[3] This analysis was based on inventions that preceded it over several centuries, including furnace regulators of Cornelius Drebble from the sixteenth century, windmill regulators of Mead and others, and steam engines of James Watt in the eighteenth and nineteenth centuries.[4]

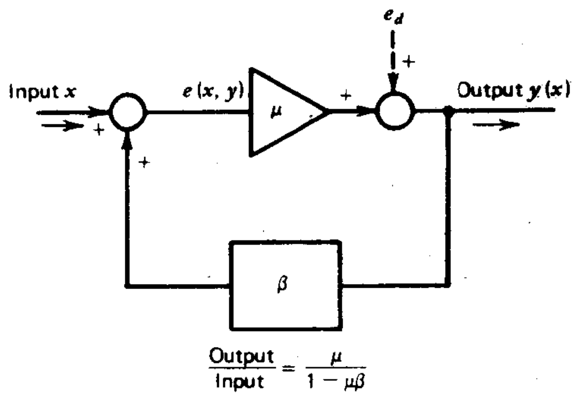

Figure 1.1. Black's feedback diagram and equation.

Nevertheless, Black's diagram and equations were central because they established a *language* with which to talk about feedback systems. This language was later picked up and used in many other disciplines, after Nyquist and Bode had contributed their mathematical insights to problems of amplifier design in the face of inherent instability.[5,6] In this and the following chapters, we investigate an alternative interpretation of the set of facts represented by Black's diagram and equation. We begin by briefly reviewing feedback under the aspect of the canonical theory introduced by Black.

1.1 CANONICAL FEEDBACK DIAGRAM AND EQUATION

Black was seeking a way of reducing distortion in electronic amplifiers to be used as repeaters for telephone transmission, where even small amounts of distortion would build up to unacceptable levels in many tandem repeaters. To see how the circuit represented by the system diagram in Fig. 1.1 does this, we now develop Black's equation from the diagram. A source signal is applied to the input of an electronic amplifier or *active path* that amplifies it by a factor μ and presents it to the output. A fraction β of the output signal is fed back to the input of the amplifier through a *feedback path* and is of polarity appropriate to reduce the active path input signal. The reason that this reduces distortion in the amplifier is that the undistorted portion of the output signal almost cancels the signal from the source, but the distorted component is not canceled. Its presence at the active-path input tends to cancel the distortion in the active path: it may be regarded as a corrective predistortion applied to the input of the amplifier.

To make these ideas quantitative, we derive Black's equation from the diagram in Fig. 1.1. The output y is related to the input x by solution of the following simultaneous equations:

$$y = \mu e \tag{1.1-1}$$

$$e = x + \beta y \tag{1.1-2}$$

Substituting (1.1-2) in (1.1-1), we obtain Black's equation

$$y=\frac{\mu}{1-\mu\beta}x \tag{1.1-3}$$

This has been called the *fundamental formula of control theory*.[7] The quantity $\mu\beta$ is the *loop gain*, and $1-\mu\beta$ is the *return difference*, so called because if the loop is broken (e. g. at e), and 1 V is applied at the right side of the break, the signal returned to the input is $\mu\beta$, and the difference between this signal and the originating 1 V is $1-\mu\beta$.

A note on signs is in order. For the closed-loop gain to be stable, it is necessary (but not sufficient) for the sign of either μ or β to be negative. We take up the question of stability in later chapters.

The benefits of feedback are considerable. To see the effect of feedback on distortion, we add a distortion generator e_d in series with the output of the active path, as shown in Fig. 1.1. This generator represents a distortion signal generated in the amplifier. Thus eq. (1.1-1) becomes

$$y=\mu e+e_d \tag{1.1-4}$$

Solving this simultaneously with eq. (1.1-2), we have

$$y=\frac{\mu}{1-\mu\beta}x+\frac{1}{1-\mu\beta}e_d \tag{1.1-5}$$

The distortion is reduced by the factor $1-\mu\beta$. For a substantial reduction in distortion, therefore, the magnitude of $1-\mu\beta$ must be large: factors of 30th –100 are common. The beneficial effects of feedback are seen to come from the denominator of the gain expression $1-\mu\beta$, the return difference.

Another benefit of feedback important to Black's repeaters is the stabilization of gain. An accumulation of gain deviations in many tandem repeaters could lead to overload for an increase in gain and to reduction in signal : noise ratio for a reduction in gain. Bode defined the term "sensitivity" to describe the ratio of the per unit variation in closed-loop gain $K=y/x$ to a small per unit variation in μ:

$$S_\mu^K=\frac{dK/K}{d\mu/\mu}=\frac{d\ln K}{d\ln\mu} \tag{1.1-6}$$

From this equation we can find the sensitivity of the closed-loop gain K to the active-path gain μ:

$$\begin{aligned} S_\mu^K &= \frac{dK}{d\mu}\cdot\frac{\mu}{K}=\frac{1-\mu\beta+\mu\beta}{(1-\mu\beta)^2}\cdot\frac{\mu(1-\mu\beta)}{\mu} \\ &= \frac{1}{1-\mu\beta} \end{aligned} \tag{1.1-7}$$

This equation says that a 1% variation in μ will cause a $1/(1-\mu\beta)$ percent variation of closed-loop gain. For the canonical diagram in Fig. 1.1, the sensitivity is simply the reciprocal of the return difference. The sensitivity can be found for any parameter in an amplifier. The sensitivity of K to β for the fundamental equation is

$$S_{\beta}^{K}=\frac{\mu\beta}{1-\mu\beta} \tag{1.1-8}$$

If $\mu=-1000$ and $\beta=0.1$, for example, the sensitivity to variations in μ is $1/101$, and sensitivity to β is $100/101$. The basic assumption is that the value of β is well controlled (e.g., a ratio of resistors), so that high sensitivity to β is tolerable whereas the value of μ is much less well controlled.

The effect of feedback on noise and other unwanted disturbances is most easily calculated by referring the noise to the input of the amplifier; this is common practice for characterizing and specifying noise. Noise originating internally in the active path is represented in Fig. 1.2. The active path has been

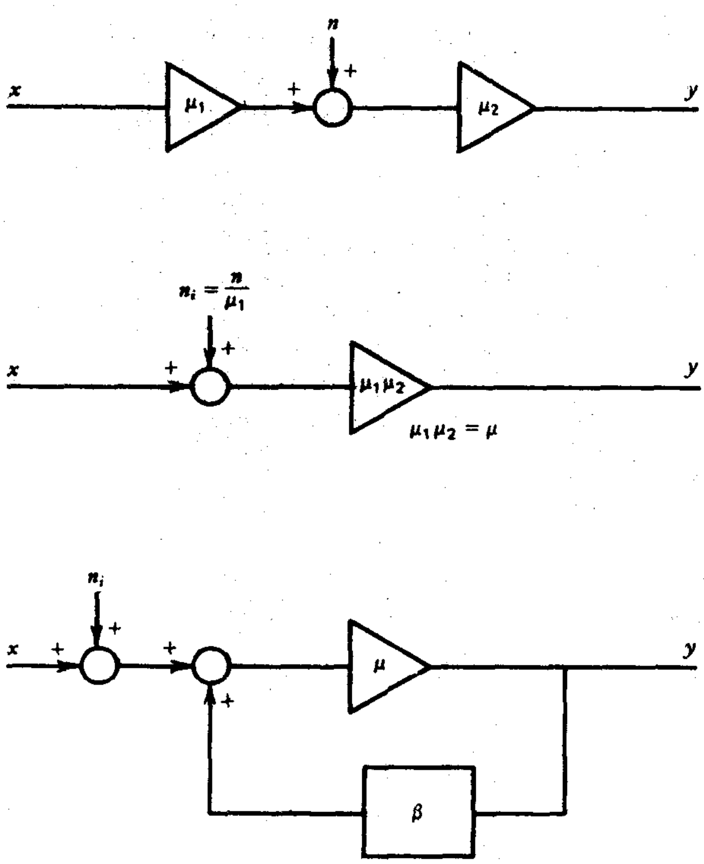

Figure 1.2. By representing all noise sources in the amplifier by an equivalent noise source at the input, noise may be removed from the feedback loop.

split into two (noiseless) portions μ_1 and μ_2, and a noise source is added between them. This is equivalent to the second diagram, in which the noise source has been divided by μ_1 and moved to the input of the active path. Any other noise sources in the amplifier may be similarly treated, so that the equivalent input noise source n_i will serve to represent them all. To compare the noise performance of the amplifier with and without feedback, we can write

$$y=\mu\beta y+\mu x+\mu n_i \tag{1.1-9}$$

$$y=\frac{\mu x+\mu n_i}{1-\mu\beta} \tag{1.1-10}$$

Feedback reduces the gain and the noise by the same amount at the output. Thus the signal/noise ratio, *keeping the input signal constant*, is unchanged by the feedback. If noise n_o is injected at the *output*, we may find $n_i=n_o/\mu$ and use it in eq. (1.1-10) to find that the noise is reduced by the factor $1-\mu\beta$ and that the signal/noise ratio is improved by this factor.

For the fundamental feedback diagram, the benefits of feedback can be summarized to include reduction of distortion and active-path gain variation by a factor of $1-\mu\beta$, and an improvement in signal/noise ratio (*for given output signal level*) of the same factor. At the input, the signal must also rise by $1-\mu\beta$ to maintain the given output, so that the improvement in signal/noise ratio comes from the increased input signal.

We have defined two concepts for the canonical diagram that require further discussion: (1) return difference, the denominator of eq. (1.1-3); and (2) sensitivity, in eq. (1.1-6).

Bode made these two concepts precise for general circuits, not just for the canonical diagram due to Black. He chose return difference as the primary concept because, as he said, it "most nearly agrees with the usual conception of feedback."[6] In the following section we introduce an alternate formulation of the problem in which return difference disappears but in which sensitivity retains the general meaning given to it by Bode.

The benefits of feedback are not attainable without some cost. First, the gain is reduced by the factor $1-\mu\beta$ so that additional active-path gain must be provided. A more important and fundamental limitation arises because of *bandwidth limitations* in the active path and signal *propagation delay* around the feedback loop. These effects can cause unsatisfactory dynamic behavior such as ringing and overshoot of the output signal, and even instability. Much of what follows in this and later chapters is concerned with these fundamental limitations and the optimization of performance in their presence. The means by which this is done in this book is considerably simplified by taking an approach that is quite different from the one taken above. We begin the study of the new approach in the following section. In Section 1.3 we discuss bandwidth limitations in the active path. Propagation delay is studied later in this chapter and in Chapter 5.

1.2 AN ALTERNATE FOUNDATION FOR FEEDBACK THEORY

Harold Black wrote his equation on his copy of *The New York Times* as the circuit *gain* of the feedback amplifier. He could as easily have expressed his result as circuit *loss*, merely the reciprocal formulation of eq. (1.1-3):

$$x=\left(\frac{1}{\mu}-\beta\right)y \tag{1.2-1}$$

in which the input x and the active-path input e are expressed in terms of the output y. The negative sign of β arises because feedback signals were defined as adding to e in the previous section; now they are seen to subtract from $e(y)=(1/\mu)y$. Fig. 1.3 contrasts the summation of signals under the conventional and reciprocal formulations. The quantity $(1/\mu)-\beta$ is the *loss ratio*. Loss was used to express the characteristics of transmission lines, to which his repeaters were to be applied, so that the concept of loss as the reciprocal of gain would not have been strange. The loss of a repeater amplifier would have had to be a number less than one. If Black had expressed his result in this way the development of feedback theory might well have taken a different direction. This book builds feedback theory from this alternate point of view. It is shown later that the description of feedback can thereby be simplified substantially.[8]

One conceptual problem with the reciprocal equation concerns the common-sense notion of causality. When we write an equation that says that the input x depends on the output y, we express a mathematical relationship: x is *functionally dependent* on y. We know, on the other hand, that x *causes* y. Most equations that we write in engineering and science are expressed in cause-and-effect form, in which the effect is expressed as functionally dependent on the cause. No doubt this is why Black wrote his equation in the way he did. The output y depends on the input x: thus it seems "natural" to write the equation

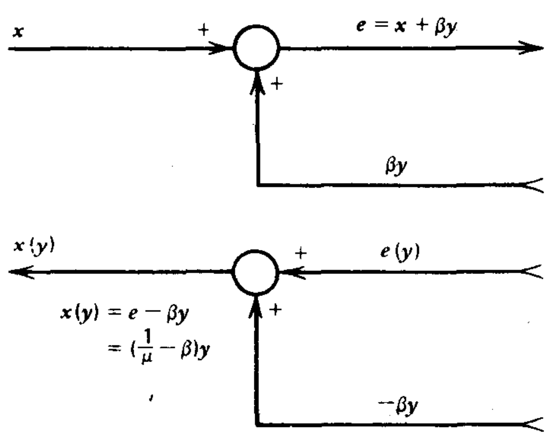

Figure 1.3. Canonical feedback diagram under the reciprocal formulation.

as $y=f(x)$. In this way the mathematical description follows the causal description. When the mathematical description proceeds in an "anticausal" direction, it seems unnatural.

Consider Black's situation when he developed feedback, however. He knew the output he wanted. It was to be an undistorted signal. After many years of effort, he finally found how to modify the input signal to obtain the desired output. In this sense the reciprocal equation can be read as the answer to the question regarding what input signal is needed to give the required output signal. The reciprocal equation can be considered conceptually as a "designer's equation." Although it takes some getting used to, the reciprocal formulation is as intuitively satisfying as the canonical one.

How should we interpret eq. (1.2-1)? The loss ratio of the equation is simply the sum of two components—the loss of the active path and the β loss. The loss of the active path is the reciprocal of μ and is the loss ratio when the β path is set to zero, that is, when the feedback is removed. Likewise, β is the loss ratio when $1/\mu$ is set to zero, that is when the loss of the active path vanishes, or when the gain goes to infinity. The equation contains no denominator; thus the return difference as defined previously is unity.

Let us repeat the gain stability and distortion calculations of the previous section for the reciprocal formulation. Although there is no return ratio or return difference, the physical quantities representing the performance of the amplifier must remain the same. Denoting the ratios $x/y=1/K=L$, the loss ratio, we rewrite eq. (1.2-1) as

$$L=\frac{1}{\mu}-\beta \tag{1.2-2}$$

Applying the sensitivity definition of eq. (1.1-6) to this equation, we find that $dL/d(1/\mu)=1$, so that

$$S^{L}_{1/\mu}=1\cdot\frac{1/\mu}{L}=\frac{1/\mu}{(1/\mu)-\beta}$$
$$=\frac{1}{1-\mu\beta} \tag{1.2-3}$$

Therefore, the sensitivity of L to $1/\mu$ is the same as that of K to μ, as we would expect since the loss equation is merely a different description of the same physical situation.

There is one important difference, however. The sensitivity under the reciprocal formulation *is also applicable for large changes in the parameter* $1/\mu$ since for a change $\Delta(1/\mu)$, we have

$$L=\frac{1}{\mu}-\beta$$

$$\Delta L=\Delta\frac{1}{\mu}$$