The search for GRAVITY WAVES

P.C.W. DAVIES

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PREFACE

The search for gravity waves is an exciting and somewhat bizarre episode in the development of science and technology in the past twenty years. Few physicists seriously doubt that waves in the gravitational field, analogous to waves in the electromagnetic field, really exist. However, calculations indicate that although the emission of gravitational radiation by distant astronomical objects could have a vastly greater impact on their evolution and structure than their electromagnetic counterpart, only something like 10^{-76} of the energy released is likely to register its presence in laboratory detectors on Earth. This is because, notwithstanding the colossal quantities of energy sure to be contained in the larger gravity wave outbursts in space, the interaction of gravitational radiation with matter is minute.

The extreme weakness of the sought-for effects demands a technology of dazzling capabilities. The detection of just one quantum of vibration in a tonne of metal is being planned. Movements of only 10^{-21} m in a highly refrigerated metre-long bar must be measured. Great strides towards achieving these extraordinary accuracies have been made, and the development of gravity wave detectors is proceeding apace. A pivotal event in this programme occurred in the early 1970s when Professor Joseph Weber of the University of Maryland claimed to have discovered gravity waves in the first detector. Although subsequent work has not confirmed these results, the establishment of a new branch of astronomy, using gravity wave detectors as 'gravity telescopes', is on the horizon. With such a facility we could 'see' into the dense hearts of quasars and neutron stars, probe to the very edges of black holes and maybe eventually listen to the rumble of the primordial big bang itself.

An added impetus to this fascinating development came with the discovery of the so-called binary pulsar in 1974. This object displays unmistakable signs of emitting gravity waves.

It is therefore timely to give an account for non-specialists of the subject of gravity waves and their detection. No advanced knowledge of physics or astronomy is needed to understand this book. The subjects of gravity and Einstein's theory of relativity are explained from basics; mathematics is kept to a minimum, employing only elementary high school algebra and calculus. In many cases I have used words in equations rather than resort to a proliferation of formal symbols. The level of exposition corresponds roughly to that of *Scientific American* or *New Scientist*.

The treatment is not intended to constitute a textbook, but rather a survey of a scientific adventure story that promises to bring a rich harvest of rewards in the coming decades.

I am grateful to N. D. Birrell, S. A. Huggett, M. J. Rees, D. C. Robinson and W. G. Unruh for helpful information and comments. I should also like to thank S. Mitton, G. A. Papini and J. Weber for supplying photographs.†

Note on units and nomenclature. In this book I have used the internationally accepted system of units (SI units), except occasionally in the discussion of astronomical distances, where the parsec ($= 3.09 \times 10^{16}$ m $= 3.26$ light years), abbreviated pc, or astronomical unit (average Earth-orbit radius $= 1.50 \times 10^{11}$ m), abbreviated AU, are more appropriate. Where 'billion' is used, the USA value of 10^9 (one thousand million) is intended.

Many of the numerical estimates and some of the formulae quoted are only accurate to within an order of magnitude, in which case the equality sign is replaced by $\sim$. In other cases it has proved convenient for exposition to round off numerical factors, and in these cases the approximate equality sign $\approx$ is used.

Newcastle-upon-Tyne P. C. W. Davies

†Since this book went to press, Professor Papini's experiment has been discontinued.

CONTENTS

1 Electromagnetic waves

Most people are familiar with electric and magnetic forces from daily life or elementary laboratory experiments. What can be hard to understand is how these forces can be translated into wave motion which can leave the laboratory and travel off into space as an apparently independent entity.

Sound waves and water waves do not seem so remarkable because we can detect a tangible medium which vibrates when the wave passes. An electromagnetic wave, on the other hand, is not a disturbance in any substance, and can travel unimpeded through a perfect vacuum. Heat and light radiation from the Sun – perhaps the two most familiar electromagnetic waves – reach the Earth across 150 million kilometres of empty space.

What are electromagnetic waves? In the following sections it will be explained how electric and magnetic forces can operate on each other to produce wavelike disturbances in free space. Some of the properties of these disturbances will also be described for later comparison with gravity waves.

1.1 Forces and fields

Electricity manifests itself as a force which acts on electrically charged bodies. The simplest examples of such a force are the so-called electrostatic effects, such as occur when combing one's hair or when a stroked rubber balloon adheres to the ceiling.

Electric charges come in two types, called positive and negative. Like charges repel each other, but unlike charges attract. Both forces diminish rapidly with distance between the charges. The existence of two varieties of electricity, and hence both attractive and repulsive forces, is in contrast to gravity, which always attracts.

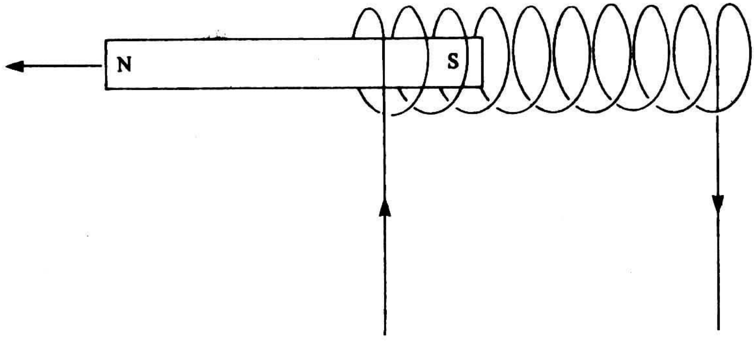
It is now known that all electricity is fixed to subatomic particles in definite multiples of a fundamental quantity – an 'atom' of electricity. Not all types of subatomic particles carry electric charge, but of those that do, the lightest is the electron and this carries one unit of charge. All normal atoms contain electrons, and often they can be fairly easily detached. In some substances, such as metals, free electrons are prolific. It is usually the rearrangement of electrons that causes a macroscopic body to become electrically charged.

A normal atom has equal quantities of both positive and negative electricity, so it is electrically neutral. The number of 'units' of electricity carried by all the atom's electrons, which by convention is taken to be negative, is exactly balanced by an equal and opposite positive charge located on the atomic nucleus. If a body contains a surfeit of electrons it will become negatively charged, while a deficit of electrons implies some unbalanced positive charges on the nuclei, and hence the body will be overall positively charged.

Electrons are fairly mobile particles, and travel easily through most metals. Under the action of electric forces, electrons repel each other, so there is a natural tendency for a local aggregation of them to disperse themselves. For example, if a crowd of electrons is located at the end of a metal wire, the electrons will flow along the wire to escape their mutual repulsion. In this way electric currents occur.

In the early nineteenth century it was discovered that electric currents produce magnetic forces and that, conversely, if a magnet is waved about near a wire, then a current can be induced to flow (see Fig. 1.1). This interplay between electric and magnetic forces is

Fig. 1.1. Electromagnetic induction. If the magnet is suddenly withdrawn from the interior of the coil, the change in magnetic field threading the wire causes an electric field which drives a current round the coil.



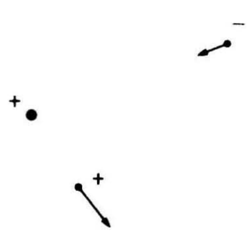
central to an understanding of electromagnetic waves. Today it is believed that all magnetism is caused by electric currents; i.e., there are no 'atoms of magneticity' that play the role for magnetism that electrons play for electricity. In the case of the Earth's magnetism, for example, there are electric currents deep in the planet's interior. The magnetism of an ordinary bar magnet is due to circulating currents at the molecular level.

One convenient way of describing the forces which act between electric charges and currents is in terms of the *field* concept, first introduced by Michael Faraday. Rather than say that two charges attract (or repel) across empty space, one says that every electric charge produces an electric field around itself, the strength of which diminishes with distance. The force that is experienced by a nearby charge is then attributed to the interaction between the latter charge and the field. Of course, this charge is also the source of its own field, which will react back on the former charge as well. The strength of the force is proportional to the strength of the field at that point. These ideas are depicted in Fig. 1.2.

The advantage of the field concept is that the interaction between separated charges is reduced from a non-local action-at-a-distance, to a local charge–field interaction. Magnetic fields may be introduced similarly.

It is possible to give meaning to the *shape* of the field as a means of representing the pattern of force around a particular configuration of charges of magnets. Fig. 1.3 shows the force field around a single point electric charge. The radially symmetric lines

Fig. 1.2. Electric field. The fixed positive charge is surrounded by an invisible electric field. The test charges sense the field in their vicinity and are driven towards (−) or away from (+) the central charge.



are so-called 'lines of force' and they represent the direction of force acting on a positive charge placed at that point. When the nature of the electromagnetic field is being probed using charged particles, we have in mind a so-called test charge which, although it responds to the presence of the field, does not itself react back electrically on the field. Thus the test charge does not disturb the system being investigated. In practice this situation can be well approximated by using a test particle with a very small charge. Similarly, when we come on to the subject of the gravitational field, test masses, whose reaction on the gravitational field may be neglected, will also be discussed. As shown, the effect is a repulsion directly away from the central charge, as indicated by the outwardly directed arrows. If the central charge were negative the arrows would point inwards, indicating a radial attraction.

A measure of the force is provided by the relative density of lines. Near the centre, where the force is strong due to the close proximity of the charge, the lines are crowded, but they fan out with distance, indicating a weakening of the field and its corresponding force on a test charge. As lines of force cannot end, except on other charges, the total number is constant. If one considers concentric spheres about the central charge, then each sphere is threaded by the same number of lines. As the surface area of the spheres increases like $(\text{radius})^2$, the density of lines diminishes with distance like $1/(\text{radius})^2$, or an inverse square law.

Fig. 1.3. Lines of force. The radial pattern diverging from the positive charge maps the shape of the electric field. As the lines fan out, their density falls off inversely as the square of distance from the charge (in three dimensions), indicating an inverse square law of force.

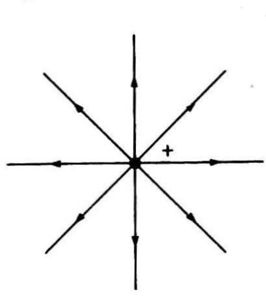


Fig. 1.4 shows the field lines around a more complicated charge system, and the analogous lines of magnetic force around a bar magnet. Magnetic field lines represent the force acting on a point test north pole.

To the physicists of the nineteenth century, electric and magnetic fields assumed an almost tangible status, and were envisaged as a sort of invisible fluid medium. The mysterious, ephemeral medium was given the name aether, and was supposed to fill all of space. Electric and magnetic fields were then identified with stresses in the aether medium.

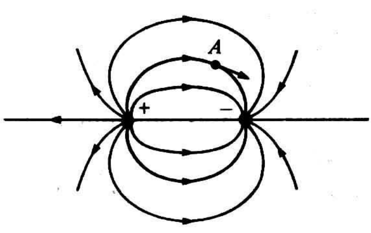
It is clear that the aether cannot be a medium of a familiar sort, for electrically neutral material bodies may pass through it without encountering any resistance. For example, the Earth orbits the Sun in apparently drag-free conditions.

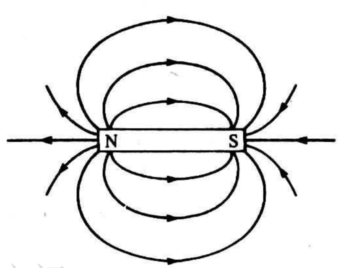
1.2 Electromagnetism

As a simple conceptual aid, electric and magnetic fields are invaluable, but at first sight they appear to be redundant physically. Nothing as yet separates the behaviour of the field from the electric and magnetic sources to which it is tied. We could either describe the forces as due to electric charges and currents acting at a distance, or regard them as sources of fields and look to the fields for an explanation of electric and magnetic force. It is merely a matter of linguistic convenience.

All this changed with the revolutionary work of James Clerk

Fig. 1.4. Dipole fields. (*a*) The conjunction of equal and opposite (+, −) charges is called a dipole and produces a static field with a complicated structure. The test charge *A* moves obliquely to both + and −. (*b*) A magnetic dipole (bar magnet) formed from the conjunction of two magnetic poles (N, S; north, south) has a similar magnetic field shape.





Maxwell in the early 1860s. Maxwell's achievement provides a beautiful example of how mathematical symmetry and elegance can be employed to improve our understanding of nature.

The fact that magnetic fields are produced by, and can act on, electric currents indicates a deep connection between electricity and magnetism. There are, however, two ways in which the relationship between them seems lopsided. The first is the absence of magnetic charge – magnetic fields seem only to be produced by electric currents. This asymmetry has long been a puzzle to physicists, and some believe that magnetic charges do exist on hitherto undiscovered subatomic particles, but there is no experimental evidence to support this conjecture. The second lopsidedness between electricity and magnetism is that, while a changing magnetic field will induce an electric field that can make an electric current flow (see Fig. 1.1), the reciprocal effect was not known in the mid nineteenth century. Maxwell puzzled over this because the equations which connect the strength of a field to the behaviour of its sources are inconsistent unless a changing *electric* field can induce a *magnetic* field, as well as vice versa.

To achieve mathematical consistency, Maxwell introduced a new term into the field equations, representing the missing effect. Thus, one of the asymmetries in the theory was removed. More importantly, Maxwell's bold step transformed the nature of the fields. If a changing electric field can induce a magnetic field, then as this latter field builds up, it will in turn induce an electric field. But the build-up of the new electric field goes on to produce its own changing magnetic field, and so on. The exciting possibility arises that changing electric and magnetic fields can sustain each other in a sort of perpetual motion. Moreover, because each field acts as a source of the other, the fields can even exist and move in regions of space where there are no electric charges or currents to act as sources. The fields therefore acquire an independence that was quite unsuspected before Maxwell. No longer tied to charges and currents, the fields are freed to assume a separate mechanical existence. They have been elevated from the status of a linguistic convenience, to that of a real, independent, physical system.

Because the self-sustaining electric and magnetic disturbances always require both electric and magnetic fields together – each to feed off the other – we are really dealing with a single, unified, *electromagnetic* field, of which electric and magnetic fields individually are merely components. Maxwell investigated the behaviour of these self-sustaining electromagnetic motions, and soon discovered that one particularly simple solution to his equations exists. The pattern of field motion can assume a very familiar form – that of a wave. The equations indicated that the speed of the wave depends on the electric and magnetic properties of the medium in which it propagates. In free space, the speed works out at around $3\times10^8\,\mathrm{m\,s^{-1}}$, which is the speed of light. Maxwell concluded that light is an electromagnetic wave, and thereby achieved a brilliant synthesis of the science of optics with that of electromagnetic theory.

1.3 Electromagnetic waves

What is an electromagnetic wave? In the nineteenth century it was fashionable to envisage the wave disturbance as a vibration of the mysterious aether, rather as a sound wave is a vibration of the air. As we shall see, this picture is both unnecessary and, if taken too literally, incorrect.

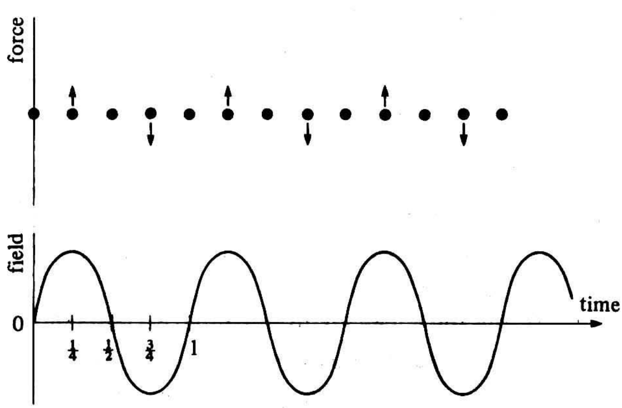
An electromagnetic wave is primarily an undulation of electric and magnetic force. If we regard it as a moving, changing field, we can map how the field changes by stationing electric charges in its path to measure the local field strength. Because all *periodic* undulations of a linear system can, by the theorem of Jean Fourier, be built up from a superposition of pure *harmonic*, or sinusoidal, waves, we need only consider a wave motion of the form $\sin(\omega t+\phi)$ where ω is the angular frequency of the oscillations, t is the time and ϕ is a constant phase angle.

Fig. 1.5 shows the effect on a positive electric test charge caused by the passage of an electromagnetic wave travelling perpendicular to the page. As the wave passes, the magnitude of the electric field **E** rises and falls in the sinusoidal fashion shown. This field exerts a force on the test charge and wiggles it up and down periodically. At time zero, there is no electric field and no force. Then the field starts to build up strength, peaking at $\frac{1}{4}$ cycle, driving the charge upwards with its greatest force. The field then starts to decline and the upward force falls away until at $\frac{1}{2}$ cycle it vanishes. Thereafter the direction of the field and the force is reversed as we move down into the 'trough' of the wave. The particle is forced downwards. Once again the field reaches maximum intensity at $\frac{3}{4}$ cycle, and then declines, until after one cycle it has dwindled to zero again and is ready to repeat the next cycle.

It is worth noting that the *motion* of the test charge is $\frac{1}{2}$ cycle out of phase with the driving force; i.e., when the force is directed upwards, the particle is moving downwards, but decelerating. It reaches the bottom of its trajectory when the upward force is a maximum. The arrows in Fig. 1.5 depict the *force*, not the motion.

The words 'upwards' and 'downwards' have been used purely schematically, for the electromagnetic wave need have no relation to up or down. Indeed, the undulating electric field can point in any direction perpendicular to the direction of propagation of the wave (see Fig. 1.6). Waves of this variety are called *transverse*. This can be expressed in terms of vectors (directed quantities denoted by arrows). In Fig. 1.6 the vector **k** marks the propagation direction of the wave, while the unit vector **e** represents the electric force direction. As the wave is transverse, **k** and **e** are perpendicular.

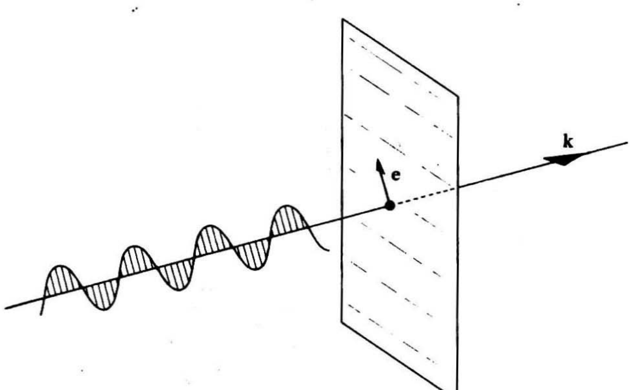
Fig. 1.5. Electromagnetic wave. The electric field undulates sinusoidally as shown below. The positive test charge (above) experiences the changing forces indicated. The absence of an arrow implies instantaneously zero force.



The direction of **e** is known as the *polarization* vector. Light from an ordinary source will contain electromagnetic waves of many different polarization directions all mixed together, but if it is passed through a polarizer, only waves vibrating in one particular direction will be passed, the others being filtered out.

A mathematical analysis shows that the magnetic field **B** oscillates with the same frequency as the electric field, but in a perpendicular direction (see Fig. 1.7). It is interesting to consider the effect of both the electric and magnetic forces on the test charge. When the charge starts to move under the action of the electric field, it constitutes a tiny electric current, which has *magnetic* action (this is the principle of the electric motor, where a current-carrying coil is forced to rotate in the field of a magnet). The magnetic force acting on a current is perpendicular to the current and to the applied magnetic field, so here it is directed along the line of propagation (i.e., along **k**). The main effect of the wave is to wiggle the test charge perpendicular to **k**, but as a small secondary effect it will also drive the charge along somewhat. If the charged particle is subject to some damping forces, it will oscillate slightly out of phase (i.e., it will lag behind the driving field), and the average effect

Fig. 1.6. Transverse wave. The wave travels forward in the direction of vector **k**, but the electric (and magnetic) fields are transverse to this. The electric field undulates along the direction of **e**, perpendicular to **k**. The test charge (blob) is driven back and forth along direction **e** by the oscillating electric field.



of the action along **k** will be the exertion of a pressure. In other words, the charged particle *recoils*, and we see that the electromagnetic wave must carry momentum, some of which is imparted to the charge.

That electromagnetic waves carry momentum and can exert a force when they strike matter is beautifully illustrated by the tails of comets, which consist of gas that is literally blown out of the cometary head by the pressure of sunlight. Ambitious proposals have been made to 'sail' a spacecraft to distant planets using this light pressure as a driving force.

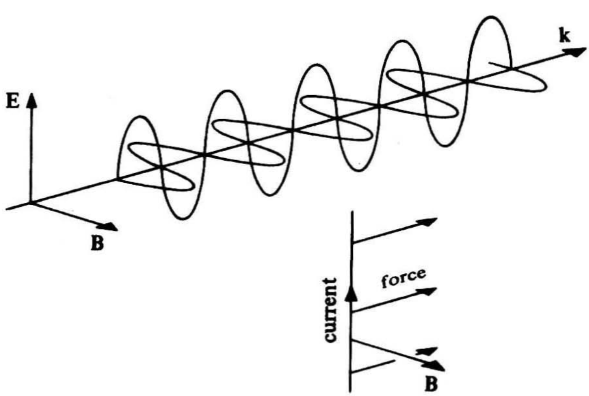
Clearly, if the charged particle is set in motion it acquires energy, which tells us that the wave carries energy as well as momentum. Maxwell's theory shows that the energy density is simply, in Gaussian units,

$$\frac{1}{8\pi}(\mathbf{E}^2+\mathbf{B}^2)$$

where **E** and **B** are the electric and magnetic field strengths respectively, while the momentum is

$$\frac{c}{4\pi}(\mathbf{E}\times\mathbf{B}),$$

Fig. 1.7. The electric (**E**) and magnetic (**B**) fields oscillate in phase, perpendicular to each other and to the propagation direction **k**. When a test charge moves along **E**, it makes a current which the magnetic field **B** forces in the direction **k**.



the vector product of the two, multiplied by *c*, the speed of the wave.

It was mentioned above that any periodic waveform can be built up from sine waves of various frequencies. This is only true if all the constituent waves have the same direction of polarization; we cannot build a wave which vibrates, say, east–west, from waves which vibrate north–south. To take into account this additional requirement of polarization it is necessary to build up a general wave by using sinusoidal waves belonging to two independent sets, each with polarization perpendicular to the other. Any intermediate polarization direction can then be built up by vector addition (see Fig. 1.8).

As remarked, it is possible to create waves that are polarized in one particular direction. Another possibility is to combine together two perpendicularly polarized waves in a different way, as shown in Fig. 1.9. Here the waves are set $\frac{1}{4}$ cycle ($\pi/2$ radians) out of phase, so that when one wave reaches peak field strength the other is zero, and vice versa. This means that if the waves are of equal strength, and the time dependence of one wave is described by $\mathbf{e}_1 \sin \omega t$, the other is described by $\mathbf{e}_2 \sin(\omega t + \pi/2) = \mathbf{e}_2 \cos \omega t$. As $\mathbf{e}_1$ and $\mathbf{e}_2$ are perpendicular, the *strength* of the resultant superposed wave is, from the rule of vector addition,

$$\sqrt{(\mathbf{e}_1{}^2 \sin^2 \omega t + \mathbf{e}_2{}^2 \cos^2 \omega t)} = 1$$

Fig. 1.8. Superposition of polarized waves. Adding two unequal strength waves with perpendicular polarizations produces a wave with intermediate polarization vector **e**. The magnetic fields are not shown.