ELECTROMAGNETIC WAVES

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'At a time when many undergraduate textbooks illustrate inflation in poundage, both in weight and cost, an interesting countertrend is established by the introduction from Routledge of a series of small carefully written paperbacks devoted to key areas of physics. The enterprising authors are E.R. Dobbs (*Electricity and Magnetism*), B.P. Cowan (*Classical Mechanics*), R.E. Turner (*Relativity Physics*) and Paul Davies (*Quantum Mechanics*). The student is offered an account of a key area of physics summarised within an attractive small paperback, and the lecturer is given the opportunity to develop a lecture treatment around this core.' – Daphne Jackson and David Hurd, *New Scientist*

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Preface

Electromagnetism began in the nineteenth century when Faraday showed electricity and magnetism were not distinct, separate phenomena, but interacted when there were time-varying electric or magnetic fields. In *Electricity and Magnetism* I have shown from first principles how Faraday's experiments led finally to Maxwell's four equations, which with the electromagnetic-force law summarise the whole of classical electromagnetism. This book therefore begins with Maxwell's equations and then uses them to study the propagation and generation of electromagnetic waves.

Physics is a subject in which the more advanced the treatment of a topic, the deeper the understanding of common occurrences that is revealed. In studying the solutions of Maxwell's equations you will find answers to such questions as: What is an electromagnetic wave? Why does a radio wave travel through space at the speed of light? How is a radio wave generated? Why does light pass through a straight tunnel when a radio wave does not? How does light travel down a curved glass fibre?

It is a remarkable fact that the classical laws of electromagnetism are fully consistent with Einstein's special theory of relativity and this is discussed in Chapter 2. The following four chapters provide solutions of Maxwell's equations for the propagation of electromagnetic waves in free space, in dielectrics, across interfaces and in conductors respectively. In Chapter 7 the generation of radio waves from dipoles and of microwaves from other antennas is explained, while the final chapter shows how these waves can be transmitted down waveguides and coaxial lines. In conclusion, the use of resonant and re-entrant cavities leads to a discussion of the classical theory of cavity radiation and its usefulness as a limiting case of the quantum theory of radiation.

The spectrum of electromagnetic radiation covers an enormous range of frequencies, from the very low frequencies (VLF) used to communicate with submerged submarines to the enormous frequencies (10^{24} Hz) associated with some cosmic rays from outer space. The complete spectrum is illustrated (opposite p. 1), where it is characterised by both the classical, wave properties of frequency (ν) and wavelength (λ) and the quantised, photon properties of energy ($h\nu$) and temperature ($h\nu/k_B$). Classical electromagnetism provides a theory of the wave properties of radiation over a wide frequency range, including for example the diffraction of X-rays by crystals, but for interactions of radiation with matter classical theory only applies in the long wavelength, low frequency, low energy ($h\nu \ll k_B$) limit. The generation of electromagnetic radiation is similarly the classical process of acceleration of electrons in producing a radio wave, where the wavelength is macroscopic, but quantum processes are involved in the production of X-rays by electronic transitions in atoms, or gamma rays by nucleonic transitions in nuclei, where the wavelengths are microscopic. The production of light by laser action is an interesting example of the combination of the classical process of reflection with the quantum process of stimulated emission. In this text the limits of classical electromagnetism are explained and the usefulness of the wave and particle properties of radiation are discussed, so that the reader is provided with an understanding of the applicability and limitations of classical theory.

SI units are used throughout and are listed for each electromagnetic quantity in Appendix 1. Since Gaussian units are still in use in some research papers on electromagnetism, Appendix 2 lists Maxwell's equations in these units and states the conversion from the Gaussian to the SI systems. The physical constants used in the text are listed in Appendix 3 with their approximate values and units. Vector calculus was introduced in *Electricity and Magnetism* and is used here from the beginning. In Appendices 4 and 5 there are summaries of the most useful relations in vector calculus and special relativity. Finally each chapter, except the first, has a set of associated exercises in Appendix 6, with answers in Appendix 7.

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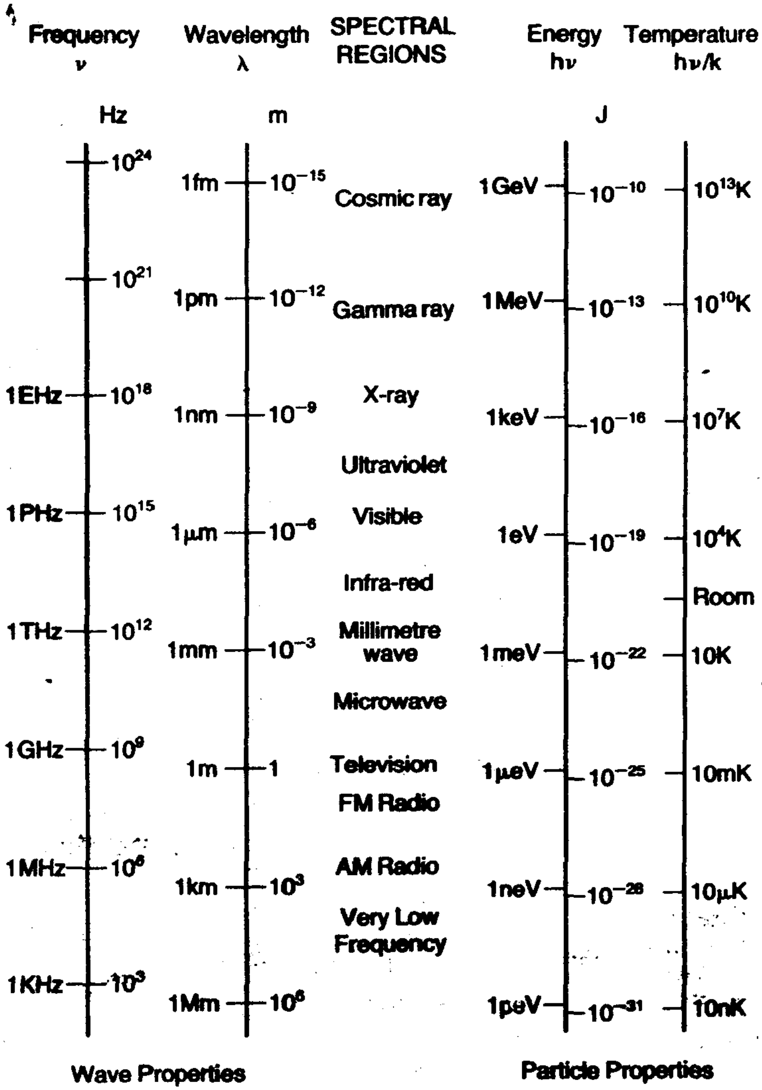
It is a pleasure to thank colleagues in the Universities of London and Sussex for their helpful comments and criticisms and my wife for her constant support. I am indebted especially to Mrs Sheila Pearson for her rapid production of an accurate typescript at a particularly busy time, as we were planning our move from Regent's Park to Egham Hill in the restructuring of the University of London.



The Electromagnetic Spectrum

[1Hz ≡ 3.00 × 10^8m ≡ 4.14 × 10^{-15}eV ≡ 4.80 × 10^{-11} K].

Contents

Chapter 1
The electromagnetic field

Electromagnetic theory is a triumph of classical physics. It was completed in a set of differential equations by Maxwell between 1855 and 1865. These are Maxwell's equations for the electromagnetic field. In this chapter they are first given in the form derived from first principles in *Electricity and Magnetism* in this series and then reformulated for free space and for matter.

Maxwell's equations for the electric field **E** and magnetic field **B** of any electromagnetic field at any frequency are:

$$\operatorname{div} \mathbf{E} = \rho/\epsilon_0 \qquad [1.1]$$

$$\operatorname{div} \mathbf{B} = 0 \qquad [1.2]$$

$$\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad [1.3]$$

$$\operatorname{curl} \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \qquad [1.4]$$

where ρ is the total electric charge density, $\mathbf{j}$ is the total electric current density, ϵ_0 is the electric constant and μ_0 is the magnetic constant (defined in Appendix 3). The electric and magnetic fields in Maxwell's equations refer to a classical 'point', which is conceived as an infinitesimal volume of a macroscopic field, but containing a very large number of atoms. In matter therefore the fields **E** and **B**, and the densities ρ and $\mathbf{j}$, are averages over large numbers of microscopic particles (electrons, protons, neutrons). The equations are not limited to linear, isotropic media, but apply to non-linear, anisotropic and non-homogeneous media.

In completely empty, or free, space there can be no electric charges and no electric currents, so that Maxwell's equations become:

$$\text{div } \mathbf{E} = 0 \qquad [1.5]$$

$$\text{div } \mathbf{B} = 0 \qquad [1.6]$$

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad [1.7]$$

$$\text{curl } \mathbf{B} = \mu_0 \, \varepsilon_0 \, \frac{\partial \mathbf{E}}{\partial t}. \qquad [1.8]$$

The surprising result of these equations, as Maxwell first showed in 1864, is that electric and magnetic fields do not merely exist in free space, but can propagate at the speed of light over galactic distances. So using satellites modern astronomy is able to explore the universe over the entire electromagnetic spectrum from cosmic rays to long-wavelength radio waves. We shall solve equations [1.5] to [1.8] for the electric and magnetic fields of electromagnetic waves in Chapter 3.

In the presence of matter, many physicists prefer to reformulate Maxwell's equations [1.1] to [1.4] in terms of the four fields **E**, **D**, **B** and **H**, where the electric displacement **D** and the magnetising field **H** are defined by:

$$\mathbf{D} = \epsilon_0 \, \mathbf{E} + \mathbf{P} \qquad [1.9]$$

and

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}. \qquad [1.10]$$

Here **P** is the electric polarisation in a dielectric medium and **M** is the magnetisation in magnetic matter. The result is that equations [1.1] and [1.4] are changed, but equations [1.2] and [1.3], which do not contain any sources, remain as before. We will now show explicitly how first equation [1.1] and then equation [1.4] can be rewritten in terms of **D** and **H** for use in dielectrics and magnetic matter.

When a dielectric medium is present the charge density ρ in

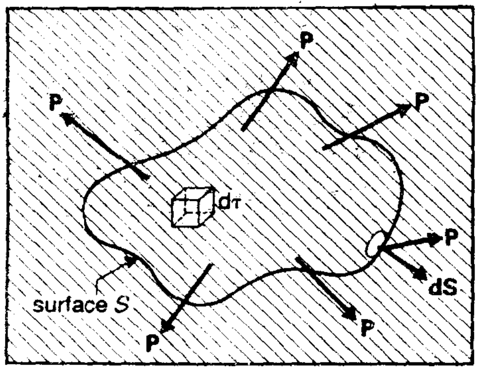


Fig. 1.1 Non-uniform polarisation of a dielectric

equation [1.1] is the sum of the density ρ_p of any polarisation charges and the density ρ_f of any free charges. Therefore equation [1.1] becomes:

$$\text{div}\, \epsilon_0 \mathbf{E} = \rho_p + \rho_f. \qquad [1.11]$$

For an arbitrary surface S inside a dielectric (Fig. 1.1) it is the normal components of the polarisation vectors **P** that produce a surface charge. A non-uniform polarisation at the surface S therefore produces a total displacement of charge q_p across S given by:

$$q_p = \int_S \mathbf{P}.\mathrm{d}\mathbf{S}.$$

Since a dielectric is electrically neutral this must be compensated by a charge density $-\rho_p$ such that:

$$\int_V -\rho_p \mathrm{d}\tau = -q_p.$$

Hence the flux of **P** is given by a type of Gauss's law for polarised dielectrics:

$$\int_S \mathbf{P}.\mathrm{d}\mathbf{S} = -\int_V \rho_p \mathrm{d}\tau. \qquad [1.12]$$

Applying Gauss's divergence theorem (Appendix 4) to this equation we have:

$$\int_V \operatorname{div} \mathbf{P} d\tau = -\int_V \rho_p d\tau$$

and so

$$\operatorname{div} \mathbf{P} = -\rho_p. \qquad [1.13]$$

Substituting for ρ_p in equation [1.11] gives:

$$\operatorname{div}(\epsilon_0 \mathbf{E} + \mathbf{P}) = \rho_f$$

or

$$\operatorname{div} \mathbf{D} = \rho_f. \qquad [1.14]$$

The fourth Maxwell equation, [1.4], includes a term $\partial \mathbf{E}/\partial t$ for electric fields that are varying with time. In the presence of such time-dependent fields the motion of the polarisation charges in a dielectric produces a polarisation current of density $\mathbf{j}_p$. Since charge is conserved, the outward flux of such a current density from a volume V must be equal to the rate of decrease of the polarisation charges within it:

$$\int_S \mathbf{j}_p . d\mathbf{S} = -\frac{\partial}{\partial t} \int_V \rho_p d\tau. \qquad [1.15]$$

From equation [1.12] this becomes

$$\int_S \mathbf{j}_p . d\mathbf{S} = \frac{\partial}{\partial t} \int_S \mathbf{P} . d\mathbf{S}$$

and, since the time derivative can be taken either before or after the integration,

$$\mathbf{j}_p = \frac{\partial \mathbf{P}}{\partial t}. \qquad [1.16]$$

Applying equation [1.4] to a polarisable and magnetisable medium we must put

$$\mathbf{j} = \mathbf{j}_f + \mathbf{j}_p + \mathbf{j}_m \qquad [1.17]$$

where the total electric current density $\mathbf{j}$ is the sum of the conduction current density $\mathbf{j}_f$ due to the free charges ρ_f, the polarisation current density $\mathbf{j}_p$ due to the polarisation charges ρ_p, and the magnetisation current density $\mathbf{j}_m$ associated with magnetised matter. This arises from the atomic currents inside the matter which are equivalent to small magnetic dipoles.

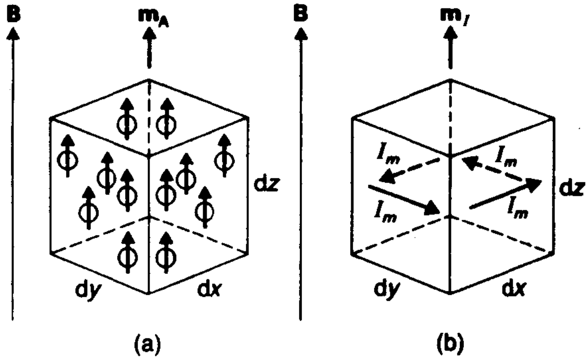


Fig. 1.2 (a) An elementary volume of uniformly magnetised matter is equivalent to (b) a surface magnetisation current I_m

Magnetisation of matter by applied magnetic fields is a similar phenomenon to the polarisation of matter by applied electric fields. In Fig. 1.2(a) an elementary cube $\mathrm{d}x\mathrm{d}y\mathrm{d}z$ of a paramagnetic has been magnetised in the uniform applied field $\mathbf{B}$ and the aligned magnetic dipoles add to a magnetic moment $\mathbf{m}_A$. This can be exactly equivalent to a single current loop, shown in Fig. 1.2(b), where a current I_m around the volume element produces a magnetic moment:

$$m_I = I_m \, \mathrm{d}x\mathrm{d}y = m_A.$$

By definition the magnetisation $\mathbf{M}$ of the elementary volume is the magnetic moment $\mathbf{m}_A$ per unit volume, so that its magnitude is:

$$M = \frac{m_A}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{I_m}{\mathrm{d}z} = i_m$$

where i_m is the surface current density or surface current per unit length normal to the current. The uniform magnetisation M of a block can thus be replaced by an equivalent surface current density i_m which acts in the direction given by:

$$\mathbf{M} \times \hat{\mathbf{n}} = \mathbf{i}_m \qquad [1.18]$$

where $\hat{\mathbf{n}}$ is the outward normal of the surface of the block containing the current. In this case the volume current density $\mathbf{j}_m$ is zero and there is only a surface current density $\mathbf{i}_m$.

For a non-uniformly magnetised material, however, there is also an equivalent current density $\mathbf{j}_m$, by analogy with Ampère's law:

$$\oint_C \frac{\mathbf{B}}{\mu_0} . \mathbf{ds} = \int_S \mathbf{j} . \mathbf{dS} \qquad [1.19]$$

namely

$$\oint_C \mathbf{M} . \mathbf{ds} = \int_S \mathbf{j}_m . \mathbf{dS}. \qquad [1.20]$$

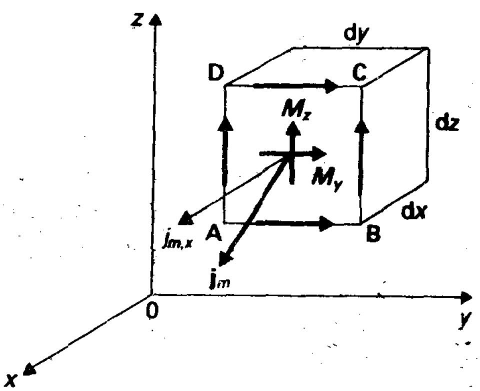


Fig. 1.3 An elementary volume of non-uniform magnetisation is equivalent to a volume current density j_m