Passive and Active Microwave Circuits

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Preface

This book was written primarily as an introductory text in microwave engineering for final undergraduate or first-year graduate students in electrical engineering. It is also likely to be of value to engineers in industry. Since it is component rather than system oriented, it incorporates examples of the most important devices used in microwave engineering. An effort has been made throughout to maintain the length and standard of each chapter as uniform as possible. Except for the chapter on scattering matrices, which is required reading, each chapter is fairly self-contained. No background other than an introduction to transmission line theory and waveguide fields is necessary for understanding the text. Although the material included in the text is too large to be covered in one semester, it will allow each professor to construct a mix of topics without too much restriction and will also allow some latitude for varying curriculum from year to year. It is hoped that the student will find the text sufficiently interesting to follow up the material not covered in the classroom.

The three classes of devices treated in this text are passive components, nonreciprocal ferrite devices, and semiconductor circuits. The classic electronic tubes such as the klystron, magnetron, and traveling wave tube have been omitted since their descriptions are readily available in many standard textbooks. The book starts with scattering and immittance matrices, the former being essential reading. It continues with passive networks such as directional couplers, impedance and mode transducers, phase and attenuation networks, resonators, and filters in Chapters 3–7, respectively. The classic nonreciprocal ferrite devices, gyrator circuits, and circulators are described next in Chapters 8–10. The semiconductor class of devices is studied in Chapters 11–15, which deal with variable-resistance and -capacitance devices, negative-resistance bulk devices, nonlinear resistive mixer circuits, and field-effect transistor circuits.

At microwave frequencies the measured quantities of both passive and active circuits are most often the scattering parameters of the device. These parameters describe transmission and reflection at the different ports of the device. The scattering coefficients are therefore used as far as possible to characterize the behavior of the devices dealt with in the text. For the symmetric passive devices, the entries of the scattering matrix are formed by constructing the eigenvalue problem used in the classic book by Montgomery, Dicke, and Purcell *Principles of Microwave Circuits* (McGraw-Hill, New York, 1948). This approach is also utilized to formulate the nonreciprocal ferrite devices such as the circulator and gyrator circuits. Although the eigenvalue problem is not extended to the class of semiconductor devices, the scattering variables are still the measured quantities there, no more so than in the case of the microwave transistor amplifier.

Since this is essentially a teaching rather than a research text, no effort has been made to acknowledge individual contributions specifically, but it goes without saying that this work is but a reflection of many individual efforts over the past 50 years.

Wholehearted thanks are due to Sheila Murray, Moira Tullis, and Helen Vaughan of the Department of Electrical Engineering, Heriot-Watt University, for their good will and cheer, without which this task would not have taken root.

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CHAPTER ONE

The Scattering Matrix

The scattering matrix dealt with in this chapter is admirably suited for the description of a large class of passive microwave components and is used as much as possible throughout this text. In many cases it leads to a complete understanding of the microwave device while avoiding the need to construct a formal electromagnetic boundary-value problem for the structure.

The entries of the scattering matrix of an m-port junction are a set of quantities that relate incident and reflected waves at the ports of the junction. It describes the performance of a network under any specified terminating conditions. The coefficients along the main diagonal of the scattering matrix are reflection coefficients, whereas those along the off-diagonal are transmission coefficients. A scattering matrix exists for every linear, passive, and time-invariant network. It is possible to deduce important general properties of junctions containing a number of ports by invoking such properties of the junction, as symmetry reciprocity, and power conservation.

Since the entries of the $\bar{S}$, $\bar{Z}$, or $\bar{Y}$ matrices of a symmetrical network are linear combinations of the circuit eigenvalues, their direct evaluation or measurement provides an alternative formulation of network parameters. The m eigenvalues of a symmetrical m-port junction are 1-port reflection coefficients or immittances at any port of the junction corresponding to the m eigenvectors of the device. These eigenvectors are the m possible ways that it is possible to excite the junction and are determined by its symmetry only. The 1-port circuits formed in this way are known as the eigennetworks of the network. In the case of symmetrical 2-port networks the eigenvalues may be obtained from measurement or by calculation by applying in-phase or out-of-phase eigenvectors at the ports of the network.

The scattering parameters of symmetrical 2-port networks can be readily obtained from their equivalent circuits by forming their eigennetworks. These eigennetworks are obtained by bisecting the 2-port network and opencircuiting and shortcircuiting the exposed terminals. The reflection coefficients of these two 1-port eigennetworks are just the two eigenvalues of the scattering matrix. Since the scattering coefficients are the sum and difference of the two eigenvalues, this approach immediately yields the entries of the scattering matrix. A simple microwave test set that allows these two eigenvalues to be measured is also described in this chapter.

1.1 *THE SCATTERING MATRIX*

The scattering matrix of a general m-port junction is defined by

$$\bar{b} = \bar{S}\bar{a} \tag{1.1}$$

where $\bar{S}$ is a square matrix that for a 2-port network has the form

$$\bar{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1.2}$$

The elements along the main diagonal are reflection coefficients, whereas those along the off-diagonal are transmission ones.

The vectors $\bar{a}$ and $\bar{b}$ are column matrices given by

$$\bar{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{1.3}$$

$$\bar{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{1.4}$$

Thus the relation between the incoming and outgoing waves for a 2-port

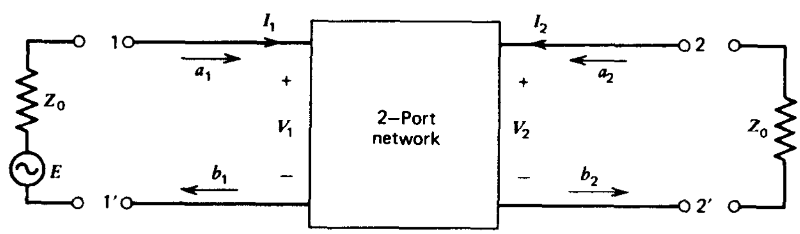


Figure 1.1. Schematic diagram indicating definition of incoming and outgoing waves for a 2-port network.

network becomes

$$b_1 = a_1 S_{11} + a_2 S_{12} \tag{1.5}$$

$$b_2 = a_1 S_{21} + a_2 S_{22} \tag{1.6}$$

This relation is given schematically in Figure 1.1.

The scattering parameters of the 2-port can be expressed in terms of the incident and reflected waves as

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \tag{1.7}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \tag{1.8}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \tag{1.9}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \tag{1.10}$$

Figure 1.2*a* and *b* illustrates one way that the scattering parameters may be obtained experimentally.

It is assumed that a_i and b_i are normalized so that $\frac{1}{2}a_i a_i^*$ is the available power at port *i* and $\frac{1}{2}b_i b_i^*$ is the emergent power at the same port. For a 2-port network the *a*'s and *b*'s are defined by

$$a_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{R_0}} + \sqrt{R_0}\, I_1\right) \tag{1.11}$$

$$b_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{R_0}} - \sqrt{R_0}\, I_1\right) \tag{1.12}$$

$$a_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{R_0}} + \sqrt{R_0}\, I_2\right) \tag{1.13}$$

$$b_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{R_0}} - \sqrt{R_0}\, I_2\right) \tag{1.14}$$

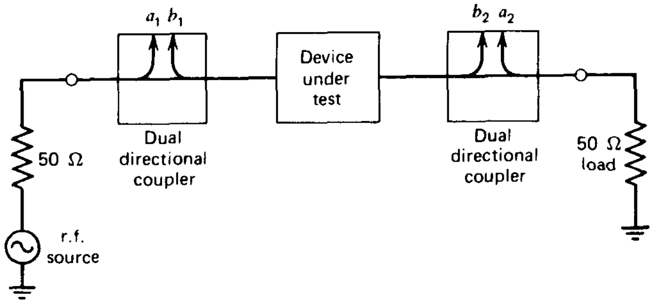


Figure 1.2a. Microwave test set for evaluating S_{11} *and* S_{21}.

To show that $\frac{1}{2}a_i a_i^*$ is the available power at port 1, it is only necessary to form the voltage V_1 in terms of the generator voltage E_1 and internal impedance R_0

$$V_1 = E_1 - R_0 I_1 \tag{1.15}$$

Substituting this value of V_1 into the definition of a_1 gives

$$a_1 = \frac{1}{2}\frac{E_1}{\sqrt{R_0}} \tag{1.16}$$

the result is

$$\tfrac{1}{2}a_1 a_1^* = \frac{E_1^2}{8R_0} \tag{1.17}$$

This is just the available power of a generator of e.m.f. E_1 and internal impedance R_0.

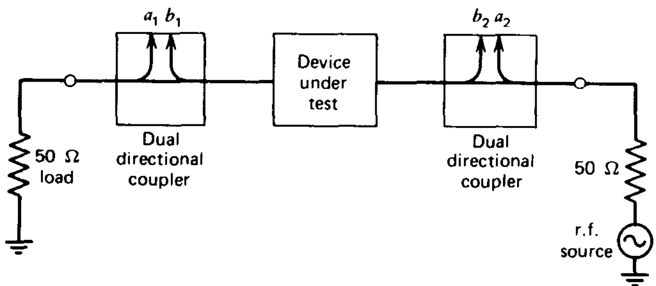


Figure 1.2b. Microwave test set for evaluating S_{22} *and* S_{12}.

To show that $\frac{1}{2} B_i b_i^*$ is the emergent power at port 2, it is necessary to combine (1.13) and (1.14) with $a_2=0$. This gives

$$b_2=\frac{V_2}{\sqrt{R_0}} \tag{1.18}$$

Thus the power in the load is

$$\tfrac{1}{2}b_2b_2^*=\frac{V_2^2}{2R_0} \tag{1.19}$$

The significance of the transmission parameters may now be inferred by forming S_{21} as defined by Eq. 1.8

$$S_{21}=\left.\frac{b_2}{a_1}\right|_{a_2=0}=2\frac{V_2}{E_1} \tag{1.20}$$

so that S_{21} is the voltage transfer ratio of the network.

The meaning of the reflection parameters may also be obtained by using the definition for S_{11} given by Eq. 1.7

$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0}=\frac{\left(V_1/\sqrt{R_0}\right)-I_1\sqrt{R_0}}{\left(V_1/\sqrt{R_0}\right)+I_1\sqrt{R_0}} \tag{1.21}$$

Thus

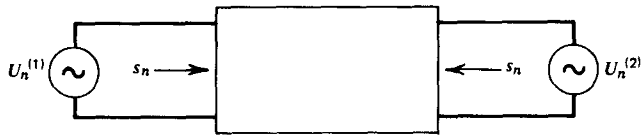
$$S_{11}=\frac{R_1-R_0}{R_1+R_0} \tag{1.22}$$

This is just the familiar reflection coefficient of a 1-port network. Dual relations to those above apply to S_{12} and S_{22}.

1.2 *THE SCATTERING MATRIX EIGENVALUES*

The relation between the scattering matrix and its eigenvalues can be obtained from the eigenvalue equation of the square matrix $\bar{S}$ shown schematically in Figure 1.3

$$\bar{S}\,\bar{U}_n=s_n\bar{U}_n \tag{1.23}$$



$s_n\bar{U}_n = \bar{S}\,\bar{U}_n \quad (\bar{b} = \bar{S}\bar{a})$

Figure 1.3. Schematic diagram illustrating eigenvalue equation.

where $\bar{U}_n$ is an eigenvector and s_n is an eigenvalue. Through comparison with Eq. 1.1 it can be seen that $\bar{U}_n$ represents a possible excitation in the junction with the fields at the terminal planes proportional to the elements of the eigenvector, and s_n represents a reflection coefficient measured at any terminal plane. Equation 1.23 has a nonvanishing value for $\bar{U}_n$ provided

$$\det|\bar{S} - s_n\bar{I}| = 0 \tag{1.24}$$

where $\bar{I}$ is a unit vector.

Equation 1.24 is known as the characteristic equation. The determinant given by the last equation is a polynomial of degree m. Its m roots are the m eigenvalues, of $\bar{S}$, some of which may be equal (degenerate). For a lossless junction, they lie in the complex plane with unit amplitude. These eigenvalues can be obtained once the entries of the scattering matrix are stated.

The characteristic equation for a 2-port network is

$$\begin{vmatrix} S_{11} - s_n & S_{21} \\ S_{21} & S_{11} - s_n \end{vmatrix} = 0 \tag{1.25}$$

provided the junction is both reciprocal and symmetrical. Expanding this determinant gives

$$(S_{11} - s_n)^2 - S_{21}^2 = 0 \tag{1.26}$$

The two roots of the characteristic equation are

$$s_1 = S_{11} + S_{21} \tag{1.27}$$

$$s_2 = S_{11} - S_{21} \tag{1.28}$$

Thus the eigenvalues are linear combinations of the entries of the scattering matrix.

The scattering coefficients may also be written in terms of the eigenvalues as

$$S_{11} = \frac{s_1 + s_2}{2} \tag{1.29}$$

$$S_{21} = \frac{s_1 - s_2}{2} \tag{1.30}$$

This suggests that if either set of variables is known the other may be formed. The boundary conditions of junctions may therefore be established in terms of either set of variables. If we assume that the junction is matched, the relation between the two eigenvalues can be obtained from Eq. 1.29 by

$$s_1 = -s_2 \tag{1.31}$$

which leads to

$$S_{11} = 0 \tag{1.32}$$

$$|S_{21}| = 1 \tag{1.33}$$

These two entries satisfy the unitary condition to be introduced later in this chapter.

1.3 *EIGENVECTORS*

A junction eigenvector is a unique set of incident waves determined by the symmetry of the network for which the reflection coefficient at any port is the corresponding eigenvalue of the scattering matrix. Since the eigenvectors are completely determined by the junction symmetry, a symmetrical perturbation of the junction alters the phase angles of the eigenvalues but leaves the eigenvectors unchanged. For the 2-port network illustrated in Figures 1.4*a* and 1.4*b* the two eigenvectors may be obtained by forming the eigenvalue equation given by Eq. 1.23 one at a time.

For the eigenvalue s_1, the eigenvalue equation becomes

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix} \begin{bmatrix} U_1^{(1)} \\ U_1^{(2)} \end{bmatrix} = (S_{11} + S_{21}) \begin{bmatrix} U_1^{(1)} \\ U_{(1)}^{(2)} \end{bmatrix} \tag{1.34}$$

Expanding this equation gives

$$S_{11} U_1^{(1)} + S_{21} U_1^{(2)} = (S_{11} + S_{21}) U_1^{(1)} \tag{1.35}$$

$$S_{21} U_1^{(1)} + S_{11} U_1^{(2)} = (S_{11} + S_{21}) U_1^{(2)} \tag{1.36}$$

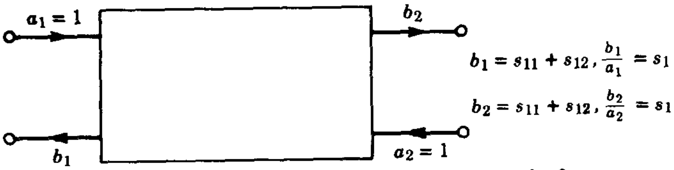


Figure 1.4a. Schematic diagram for in-phase eigensolution for 2-port network.

These two equations are satisfied provided

$$U_1^{(1)} = U_1^{(2)} = \frac{1}{\sqrt{2}} \tag{1.37}$$

This eigenvector corresponds to equal amplitude in-phase waves at ports 1 and 2 of the network in the manner illustrated in Figure 1.4*a*.

For the eigenvector s_2, the eigenvalue equation is

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix} \begin{bmatrix} U_2^{(1)} \\ U_2^{(2)} \end{bmatrix} = (S_{11} - S_{21}) \begin{bmatrix} U_2^{(1)} \\ U_2^{(2)} \end{bmatrix} \tag{1.38}$$

Expanding this equation gives

$$S_{11} U_2^{(1)} + S_{21} U_2^{(2)} = (S_{11} - S_{21}) U_2^{(1)} \tag{1.39}$$

$$S_{21} U_2^{(1)} + S_{11} U_2^{(2)} = (S_{11} - S_{21}) U_2^{(2)} \tag{1.40}$$

The two equations are consistent provided

$$U_2^{(1)} = -U_2^{(2)} = \frac{1}{\sqrt{2}} \tag{1.41}$$

This solution is shown schematically in Figure 1.4*b*.

These two excitations either produce an opencircuit or a shortcircuit at the plane of symmetry of the network. The equivalent circuits or eigen-

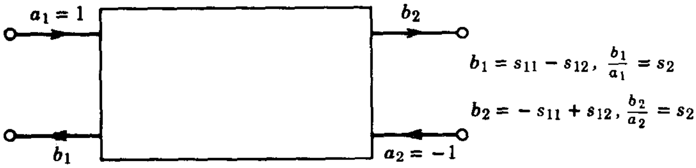


Figure 1.4b. Schematic diagram for out-of-phase eigensolution for 2-port network.

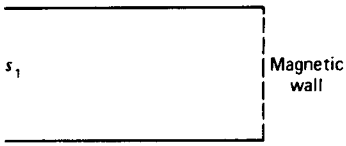


Figure 1.5a. One-port eigennetwork for in-phase eigensolution.

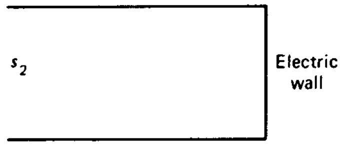


Figure 1.5b. One-port eigennetwork for out-of-phase eigensolution.

networks are therefore the 1-port opencircuited or shortcircuited transmission lines depicted in Figures 1.5*a* and 1.5*b*.

1.4 *DIAGONALIZATION OF SCATTERING MATRIX*

If the eigenvalues are known, it is possible to form the coefficients of the matrix $\bar{S}$. The relation between the two is obtained by diagonalizing $\bar{S}$. This can be done by a matrix $\bar{U}$ having for its columns the eigenvectors of $\bar{S}$

$$\bar{S}=\bar{U}\bar{\lambda}\bar{U}^{-1} \tag{1.42}$$

where $\bar{\lambda}$ is a diagonal matrix with the eigenvalues of $\bar{S}$ along its main diagonal and $\bar{U}^{-1}$ is the inverse of $\bar{U}$. If the eigenvectors of $\bar{S}$ are those obtained earlier

$$\bar{U}^{-1}=(\bar{U}^{*})^{T} \tag{1.43}$$

where $(\bar{U}^{*})^{T}$ is the transpose of the complex conjugate of $\bar{U}$. The relation between the eigenvalues and the coefficients of the scattering matrix is obtained by multiplying out Eq. 1.42.

The diagonalization procedure will now be developed for a 2-port junction. This gives the relation between the eigenvalues and the scattering coefficients of the scattering matrix. The matrix $\bar{U}$ that has the eigenvectors of $\bar{S}$ as its columns is

$$\bar{U}=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{1.44}$$

The diagonal matrix $\bar{\lambda}$ is

$$\bar{\lambda}=\begin{bmatrix}s_1 & 0\\ 0 & s_2\end{bmatrix} \tag{1.45}$$

Diagonalizing the matrix $\bar{S}$ gives

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Thus

$$S_{11} = \frac{s_1 + s_2}{2} \tag{1.46}$$

$$S_{21} = \frac{s_1 - s_2}{2} \tag{1.47}$$

which is the result obtained earlier.

1.5 *SCATTERING PARAMETERS OF 2-PORT NETWORKS*

The scattering parameters of symmetrical networks may be readily obtained from their equivalent circuits by forming their eigennetworks. These are obtained by bisecting the network and opencircuiting and shortcircuiting the exposed terminals, which is described in Chapter 2. This approach will be illustrated now in the case of a uniform section of transmission line and also for shunt and series immittances loading a transmission line.

For a uniform transmission line of electrical length θ in Figure 1.6*a*, the two eigennetworks are opencircuited and shortcircuited lines of electrical lengths $\theta/2$. The two reflection eigenvalues are therefore

$$s_1 = \frac{Z_{o/c} - Z_0}{Z_{o/c} + Z_0} \tag{1.48}$$

$$s_2 = \frac{Z_{s/c} - Z_0}{Z_{s/c} + Z_0} \tag{1.49}$$

It is observed that s_1 is associated with the opencircuited eigennetwork,

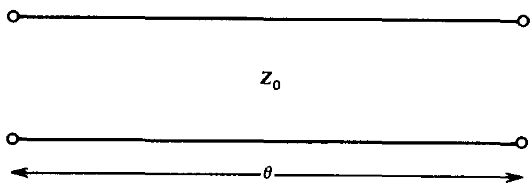


Figure 1.6a. Section of uniform line.

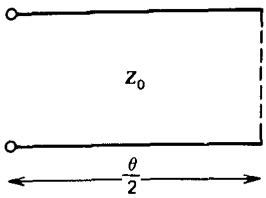


Figure 1.6b. In-phase eigennetwork for uniform section of line.

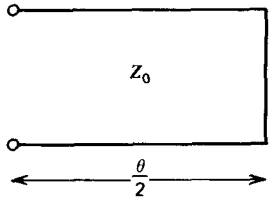


Figure 1.6c. Out-of-phase eigennetwork for uniform section of line.

since its eigenvector corresponds to in-phase waves at the two ports of the network, whereas s_2 corresponds to the shortcircuited eigennetwork, since it is related to the eigenvector having out-of-phase waves at the ports. The opencircuited and shortcircuited impedances for the eigennetworks in Figures 1.6*b* and 1.6*c* are

$$Z_{o/c} = Z_0 \coth \frac{\theta}{2} \tag{1.50}$$

$$Z_{s/c} = Z_0 \tanh \frac{\theta}{2} \tag{1.51}$$

and Z_0 is the characteristic impedance of the line.

Substituting Eqs. 1.50 and 1.51 into Eqs. 1.48 and 1.49 gives

$$s_1 = \exp(-\theta) \tag{1.52}$$

$$s_2 = -\exp(-\theta) \tag{1.53}$$

Thus

$$S_{11} = S_{22} = \frac{s_1 + s_2}{2} = 0 \tag{1.54}$$

$$S_{21} = S_{12} = \frac{s_1 - s_2}{2} = \exp(-\theta) \tag{1.55}$$

The scattering matrix for the shunt admittance loading a transmission line of characteristic admittance Y_0 in Figure 1.7*a* proceeds once more by constructing the two eigennetworks. The eigennetworks shown in Figures 1.7*b* and 1.7*c* are obtained by bisecting the network and opencircuiting and shortcircuiting the exposed terminals. The reflection eigenvalues for

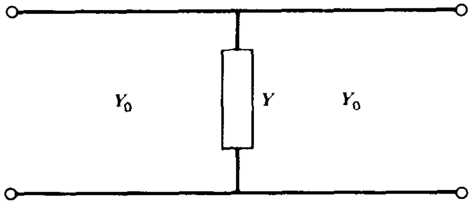


Figure 1.7a. Section of uniform line loaded with shunt network.

these two networks are

$$s_1 = \frac{Y_0 - Y_{o/c}}{Y_0 + Y_{o/c}} \tag{1.56}$$

$$s_2 = \frac{Y_0 - Y_{s/c}}{Y_0 + Y_{s/c}} \tag{1.57}$$

where

$$Y_{o/c} = \frac{Y}{2} \tag{1.58}$$

$$Y_{s/c} = \infty \tag{1.59}$$

and Y_0 is the characteristic admittance of the transmission line. Combining Eqs. 1.56 to 1.59 yields

$$S_{11} = S_{22} = \frac{s_1 + s_2}{2} = \frac{-Y}{Y + 2Y_0} \tag{1.60}$$

$$S_{21} = S_{12} = \frac{s_1 - s_2}{2} = \frac{2Y_0}{Y + 2Y_0} \tag{1.61}$$

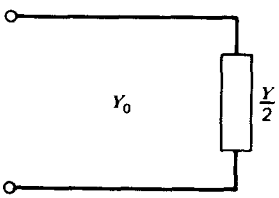


Figure 1.7b. In-phase eigennetwork for uniform line loaded with shunt network.

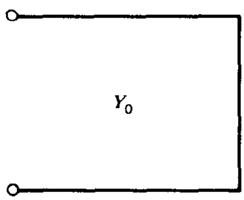


Figure 1.7c. Out-of-phase eigennetwork for uniform line loaded with shunt network.