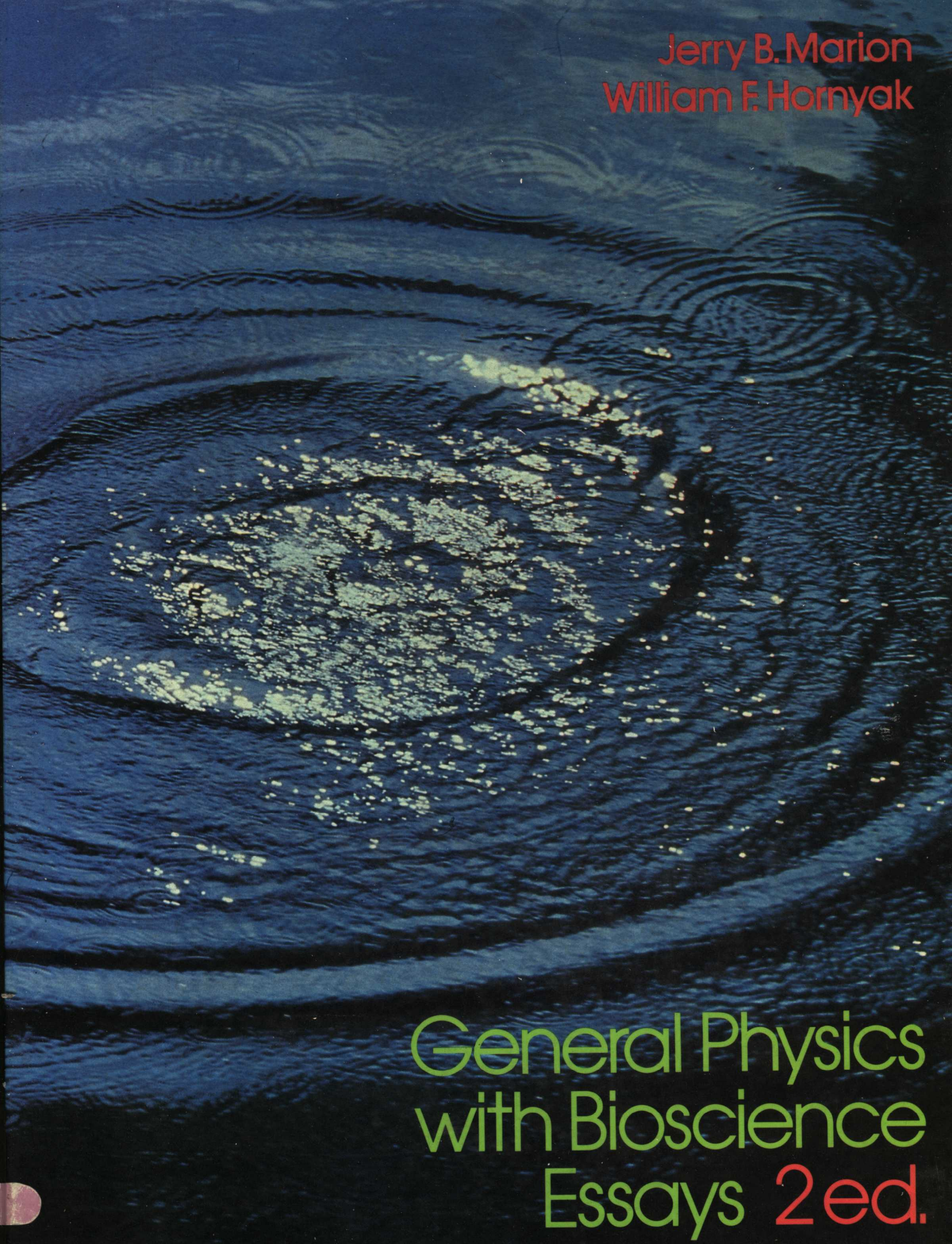

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General Physics with Bioscience Essays 2 ed.

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SECOND EDITION

General Physics with Bioscience Essays

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Preface

The wide acceptance of the first edition of General Physics with Bioscience Essays has also provided a valuable source of suggestions for improvements by its many users. In addition, several detailed reviews have been solicited resulting in further comments. In response to these suggestions, this new edition has been prepared.

The present edition continues as a basic text for a general introductory course in physics for students whose main interests and careers lie in other areas. The mathematical ability necessary to master the material presented here is not great—high school algebra is used extensively and simple trigonometry is used where necessary—the methods of calculus are not used at all. The Appendix summarizes all of the mathematical techniques required to understand the discussions and to solve the problems.

For most of the college and university students who take an introductory physics course at the level of this book, it will be the only formal course in physics that they will take. Consequently, we have again presented a comprehensive and balanced overview of the subject. Some sections have been partially rewritten, some new sections added, and the motion of rigid bodies expanded into a new chapter. There is a strong emphasis on classical physics, with discussions of all the important topics, but there is also a generous amount of material on modern physics. The present edition has brought this material up-to-date, adding the most recent findings in physics and astronomy. Brief discussions are now included of such topics as: black holes, pulsars, neutron stars, the big bang theory, nucleogenesis, quarks and gluons, color forces, and the recent progress in a unified field theory. (If insufficient time is available to cover all the topics here, Chapter 17, Relativity, Chapter 20, The Structure of Matter, and the section in Chapter 21 on elementary particles can be omitted.)

To understand and appreciate the various concepts and applications of physics, considerable drill in problem solving is required. Each chapter contains a collection of worked examples covering all of the important points. Moreover, at the end of each chapter there is a list of questions to test the student's comprehension of the concepts and a generous number of problems to test his or her problem-solving abilities. In the present edition the problems have also been identified by the relevant section. The more difficult problems are indicated by an asterisk, and the answers to the odd-numbered problems are at the back of the book. Altogether, with many new additions, there are now approximately 1200 questions and problems in the 21 chapters of this edition.

For the student who wishes assistance in a planned program of study, a new Study Guide is available to accompany this text.

Many of the students who take an introductory physics course are looking toward careers in medicine, dentistry, nursing, medical technology, microbiology, chemistry, or a variety of other professions in or related to the life sciences. These students, in particular, sometimes wonder how the subject of physics plays a role in the behavior of living things. The attempt to provide a partial answer to this curiosity has led to the development of the unique aspect of both the first edition and of its continuation in the present edition, that is, the inclusion of a number of essays on bioscience topics that emphasize the importance of physical principles in the operation of living systems. These essays—generally, two or three pages in length and identified by a colored border on the pages—will be found in every chapter (except the chapter on relativity theory). Each essay is related directly to the material in the chapter of which it is a part. But the essays are supplementary to and separate from the material in the chapters themselves. That is, the essays are optional and can be completely skipped without a loss in the flow of physics ideas. However, by omitting the essays, some of the most interesting physics in the book will be lost!

One of the ideas in preparing this collection of bioscience essays was to provide, for every main physics topic, some *quantitative* life-science application. Therefore, every essay contains some numerical discussion of the topic and some numerical problems so the student can see how calculations are carried out in a different area of science.

These essays do not represent a complete survey of the many ways that physics touches upon living systems. Nor do they represent even the most profound of the topics in the biological sciences that involve physics concepts. Instead, the selection of material has been made to illustrate the impact of physics in a variety of areas, with applications that have an interest or appeal to most students and which do not require mathematics beyond the level of the main text. A final consideration was to limit the amount of material to that which could be incorporated into a typical one year physics course without overbalancing the content of the course. (The essays here amount to about 12 percent of the text.) Consequently, some of the topics that might have been included were omitted in favor of others in different areas or because of space or mathematics limitations.

We would like to thank the several persons who read the new revised text with diligence and care and who offered so many suggestions that have resulted in a significant improvement in the text: Professors Fred Becchetti (University of Michigan), Charles E. Brient (Ohio University), Irvin G. Clator (University of North Carolina), Susanta K. Ghorai (Alabama State University), Eastman N. Hatch (Utah State University), Thomas R. Manney (Kansas State University), James L. Monroe (Pennsylvania State University), and Richard M. Prior (University of Arkansas). Finally, we thank Cheryl Connor for her usual excellent work in preparing the manuscript.

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Contents

1 The Physical View of the World Around Us

The science of physics is a growing, changing body of knowledge about the way in which Nature behaves. The physicist seeks to describe this behavior with the simplest possible models. In this chapter we examine the tools that the physicist uses in making these descriptions—mathematics and standards of measurement. Without an organized system for reporting the results of experiments (measurement standards) and without a precise language for formulating theories (mathematics), an accurate description of natural phenomena would not be possible. Therefore, we begin with a discussion of the methods we use for describing things.

Throughout this book we are concerned with the behavior of *matter*. Accordingly, in this chapter we also take a first look at the properties and structure of matter, the forms in which it occurs, and its ultimate composition. As we proceed with our discussions, we will build on these ideas and examine the workings of Nature in the large scale of the macroscopic world as well as in the small scale of the microscopic domain.

1.1 Physics as an Experimental Science

The Final Test Is in the Laboratory

The scientist seeks to learn the "truth" about Nature. In physics we can never learn "absolute truth" because physics is basically an experimental science; experiments are never perfect and, therefore, our knowledge of Nature must always be imperfect. We can only state at a certain moment in time the extent and the precision of our knowledge of Nature, with the full realization that both the extent and the precision will increase in the future. Our understanding of the physical world has as its foundation experimental measurements and observations; on these are based our theories that organize our facts and deepen our understanding of Nature.

Physics is not an armchair activity. The ancient Greek philosophers debated the nature of the physical world, but they would not test their conclusions, they would not experiment. Real progress was made only centuries later, when it was finally realized that the key to scientific knowledge lay in observation and experiment, combined with logic and reason. Of course, the formulation of ideas in physics involves a certain amount of just plain *thinking,* but when the final analysis is made, the crucial questions can only be answered by experiments.

The Philosophy of Discovery

The mere accumulation of facts does not constitute good science. Certainly, facts are a necessary ingredient in any science, but facts alone are of limited value. In order to utilize our facts fully, we must understand the relationships among them; we must systematize our information and discover how one event produces or influences another event. In doing this, we follow the *scientific method:* the coupling of observation, reason, and experiment.

The scientific method is not a formal procedure or a detailed map for the exploration of the unknown. In science we must always be alert to a new idea and prepared to take advantage of an unexpected opportunity. Progress in science occurs only as the result of the symbiotic relationship that exists between observational information and the formulation of ideas that correlate the facts and allow us to appreciate the interrelationships among the facts. The scientific method is actually not a "method" at all; instead, it is an attitude or philosophy concerning the way in which we approach the real physical world and attempt to gain an understanding of the way Nature works.

Johannes Kepler (1571–1630), the greatest of the early astronomers, followed the scientific method when he analyzed an incredible number of observations of the positions of planets in the sky. From these facts he was able to deduce the correct description of planetary motion: the planets move in elliptical orbits around the Sun.

Kepler's procedure—amassing facts and trying various hypotheses until he found one that accounted for all the information—is not the only way to utilize the scientific method. When Erwin Schrödinger was working on the problems associated with the new experiments in atomic physics in the 1920s, he set out to find a description of atomic events that could be formulated in a mathematically beautiful way. Schrödinger deviated from the "normal" procedure of the scientific method. Instead of closely following the experimental facts and attempting to relate them, he sought only to find an aesthetically pleasing mathematical description of the general trend of the results. This pursuit of mathematical beauty led Schrödinger to develop modern quantum theory. In the realm of atoms, where quantum theory applies, Nature does indeed operate in a beautiful way. At essentially the same time, Werner Heisenberg followed the more conventional approach and formulated an alternative version of quantum theory, which is equivalent to the theory constructed by Schrödinger in every respect that can be experimentally tested.

The Language of Physics

One of the significant steps forward in our understanding of the behavior of Nature was the realization that it is the Earth that moves around the Sun, and not the Sun that moves around the Earth. The simple statement that "the Earth moves around the Sun" represents a new dimension in physical thinking. As important as this idea is, nevertheless, it is incomplete. We cannot say that we really understand a physical phenomenon until we have reduced the description to a statement involving *numbers.* Physics is a precise science and its natural language is mathematics. Only when Johannes Kepler gave a mathematical description of planetary motion and Isaac Newton derived the same results on the basis of his theory of universal gravitation could it be said that a proper analysis of the motion of the Earth and the planets had finally been made.

Although the most sophisticated mathematics can be used in developing physical theories, in this book we restrict the use of mathematics to algebra, geometry, and trigonometry. The reader will find a mathematical review in the appendices at the end of this book.

1.2 The Fundamental Units of Measure

Units and Standards

In our subsequent discussions we will encounter a variety of physical quantities—for example, length, time, mass, force, momentum, energy, and so forth. These quantities not only have magnitudes but they have *dimensions* and *units* as well. It makes no sense to state that a certain length is 12—we must also specify the units in which the magnitude has this value. Whether the length is 12 centimeters or 12 miles makes a considerable difference!

It is also necessary to have *standards* for the units of physical measure. If we state that the size of a building lot is 30 paces by 60 paces, we have only a crude idea of the area. But if we state that the size is 20 meters by 40 meters, we know the area precisely because the *meter* is a well-defined and standard unit of length.

In this book we will use the metric system of physical measure. The particular metric units and abbreviations used will be those that have been recommended by the commission on "Le Système International d'Unités." We will refer to these as SI units.

The Standard of Length

Although an enormous number of units for the specification of length have been invented, only those of the British system and metric systems survive today. The unit of length in the British system is the *yard;* the derived units are the *foot* ($\frac{1}{3}$ yard), the *inch* ($\frac{1}{36}$ yard), and the *statute mile* (1760 yards). In the metric system the unit of length is the *meter,* which was originally conceived as 10^{-7} of the distance from the equator to the North Pole along a meridian passing through Paris. In order to provide a more practical standard, in 1889 the meter was officially defined as the distance between two parallel scribe marks on a specially constructed bar of platinum-iridium.

The meter-bar definition of the meter suffers from two disadvantages: not only is the precision inadequate for many scientific purposes, but comparisons of lengths with a bar that is kept in a standards laboratory are quite inconvenient. These difficulties have been overcome with the definition, by international agreement in 1961, of a *natural* unit of length based on an atomic radiation. Because all atoms of a given species are identical, their radiations are likewise identical. Therefore, an atomic definition of length is reproducible everywhere. We now accept as the standard of length the wavelength of a particular orange radiation emitted by krypton atoms. The standard was arrived at by carefully measuring the length of the standard meter bar in terms of the wavelength of krypton light. It was then decided that exactly 1,650,763.73 wavelengths would constitute 1 meter. This definition is then consistent with the previous definition in terms of the distance between the scribe marks on the meter bar, but is has the advantage of being approximately 100 times as precise. Now the standard can be reproduced in many laboratories throughout the world, not in standards laboratories alone.

In the early part of this book we will occasionally refer to the British system of units for length (with which you are probably more familiar). When we have completed the introductory material, we will forgo completely the use of British units.

Table 1.1 gives the conversion factors connecting some of the units of length in the metric and British systems. Notice that the inch, the foot, and the yard are now defined *exactly* in terms of centimeters. Thus, the krypton wavelength is the standard of length for both systems.

Table 1.1 Conversion Factors for Length

1 cm = 0.3937 in.	1 in. = 2.54 cm	exactly
1 m = 3.281 ft.	1 ft = 30.48 cm	exactly
1 km = 0.6214 mi	1 yd = 91.44 cm	exactly
= 3281 ft	1 mi = 5280 ft	
	= 1.609 km	

Table 1.2 shows in a schematic way the enormous range of distances we encounter in the Universe. Notice that there is a factor of 1000 between successive marks on the vertical scale. Between the smallest and the largest things about which we have any comprehension, the span is more than 40 factors of 10!

Table 1.3 gives the values of some of the distances that we will find useful in our discussions.

Table 1.2 The Range of Distances in the Universe

meters	
10^{27}	
	Edge of the visible Universe
10^{24}	
	Nearest galaxy
10^{21}	Radius of our Galaxy
10^{18}	
	Nearest star (Alpha Centauri)
10^{15}	1 light year
	Radius of Pluto's orbit
10^{12}	
	Earth–Sun
10^{9}	
	Earth–Moon
10^{6}	New York–Chicago
10^{3}	
	Height of Washington Monument
	Height of a man
1	
	1 cm
10^{-3}	
	Grain of salt
10^{-6}	
10^{-9}	Virus
	Radius of hydrogen atom
10^{-12}	
	Nuclear radius
10^{-15}	

Table 1.3 Some Useful Distances

To Alpha Centauri (nearest star)	4.04×10^{16} m
1 light year (L.Y.)	9.460×10^{15} m
1 astronomical unit (A.U.) (Earth–Sun distance)	1.496×10^{11} m
Radius of Sun	6.960×10^{8} m
Earth–Moon distance	3.844×10^{8} m
Radius of Earth	6.378×10^{6} m
Wavelength of yellow sodium light	5.89×10^{-7} m
1 Ångstrom (Å)	10^{-10} m
Radius of hydrogen atom	5.292×10^{-11} m
Radius of proton	1.2×10^{-15} m

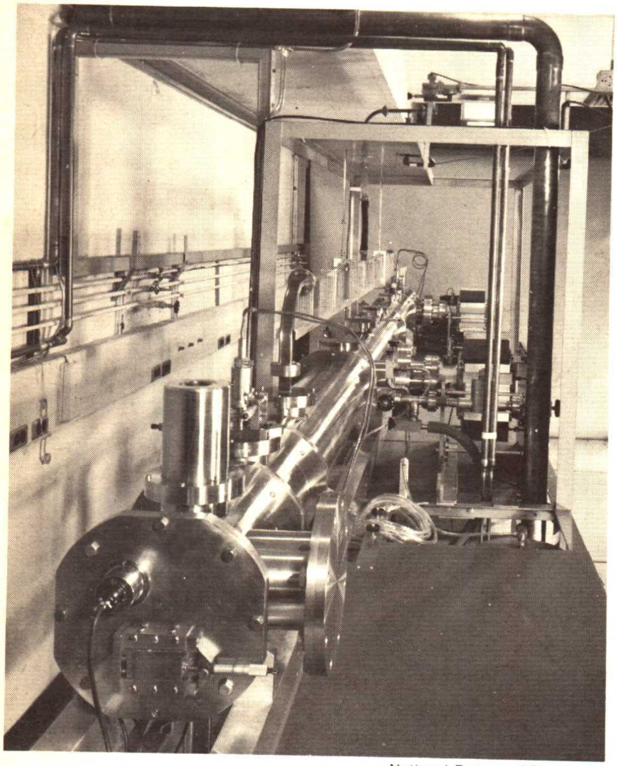
Table 1.4 Relative Precision of Various Types of Clocks

Type of Clock	Precision: 1 s in	Precision: 1 part in
Hour glass	1.5 min	10^2
Pendulum clock	3 h	10^4
Tuning fork	1 day	10^5
Quartz crystal oscillator	3 y	10^8
Ammonia resonator	30 y	10^9
Cesium resonator	3×10^4 y	10^{12}
Hydrogen maser	3×10^6 y	10^{14}

The Standard of Time

The unit of time in both the British and metric systems is the *second,* which, until recently, was defined as 1/86,400 of the mean solar day. The determination of time by observation of the rotation of the Earth is inadequate for high precision work because of minor but quite perceptible changes in the speed of the Earth's rotation.

In order to improve the precision of time measurements, in 1967 we adopted a *natural* unit for time just as we had done previously for a length standard. Our present-day *atomic clocks* (Fig. 1.1) depend on the characteristic vibrations of cesium atoms. The *second* is defined as the time required for 9,192,631,770 complete vibrations to occur in cesium. With this definition of the second, it is possible to compare time intervals to 1 part in 10^{12}, which corresponds to 1 second in 30,000 years. Current research with other atomic vibrations indicates that we will soon have a clock that will be precise to 1 part in 10^{14} or to 1 second in 3 million years!



National Bureau of Standards

Figure 1.1 A cesium clock constructed at the National Bureau of Standards (Boulder Laboratories). This clock can measure time intervals to a precision equivalent to 1 second in 30,000 years.

Time standards for practical working purposes are provided by radio station WWV, located in Fort Collins, Colorado, and operated by the National Bureau of Standards. WWV operates on frequencies of 2.5, 5, 10, 15, 20, and 25×10^6 cycles per second, which are controlled to 1 part in 10^{10} by comparison with a cesium clock. A beat is given every second and 10 times per hour the time is given by voice. Several other countries also maintain radio stations that broadcast time signals.

Table 1.5 shows the range of time intervals that we encounter in the Universe. Notice that the span from the shortest to the longest time interval is greater than 40 orders of magnitude, about the same as the range of distances shown in Table 1.2.

The Standard of Mass

The international standard of mass is a cylinder of platinum-iridium,[1] which is defined as 1 kilogram = 10^3 grams.

It would, of course, be highly desirable to have an atomic standard for mass just as we have for length and time. We do have such a standard that is used in the comparison of the masses of atoms and molecules, but, unfortunately, we have no precision method at present of utilizing this standard above

[1]Originally, the kilogram was defined as the mass of 1000 cm^3 of water.

Table 1.5 The Range of Time Intervals in the Universe

seconds	
10^{18}	Age of Universe
	Age of Earth
10^{15}	
	Earliest men
10^{12}	Age of Pyramids
10^{9}	Lifetime of a man
10^{6}	1 year = 3.156×10^7 s 1 day = 8.64×10^4 s
10^{3}	Light travels from Sun to Earth
1	Interval between heartbeats
10^{-3}	Period of a sound wave
10^{-6}	Period of a radio wave
10^{-9}	Light travels 1 ft
10^{-12}	Period of a molecular vibration
10^{-15}	Period of an atomic vibration
10^{-18}	Light travels an atomic diameter
10^{-21}	Period of a nuclear vibration
	Light travels a nuclear diameter
10^{-24}	

Table 1.6 The Range of Masses in the Universe

kg	
10^{50}	Universe
10^{40}	Our Galaxy
10^{30}	Sun
	Earth Moon
10^{20}	
10^{10}	
	Ocean-going ship
1	
	1 g
10^{-10}	
	Oil droplet
10^{-20}	
	Uranium atom Proton
10^{-30}	Electron

Table 1.7 Some Important Masses

Object	Mass (kg)
Sun	1.991×10^{30}
Earth	5.997×10^{24}
Moon	7.35×10^{22}
Proton	1.672×10^{-27}
Electron	9.108×10^{-31}

the level of individual atoms and molecules. When the technology has developed to the point that we can determine precisely the mass of the standard kilogram in terms of the atomic mass standard, we will certainly adopt an atomic unit of mass as our standard.

Table 1.6 shows some of the marker points on the gigantic range of masses that we find in the Universe. Values of some of the more important masses are given in Table 1.7.

In this book we will use only the metric system of units for mass. However, common and legal usage still refers to the British system of units for mass. Although certain ambiguities exist for this unit, it is sufficient for our purposes to point out that an object stated to have a weight of a pound (lb) has, by legal definition, a mass of exactly 0.45359237 kilograms (kg). Thus a student of weighing 150 lb would have a mass of 68 kg.

The Use of Units

All physical quantities have *dimensions* and *units.* When we make numerical statements or write numerical equations concerning physical quantities, we must include the units of the quantities. If we make the statement, "The distance traveled is equal to the speed multiplied by the time," we could express this more briefly as

$$\text{distance} = \text{speed} \times \text{time}$$

or, by giving arbitrary symbols to the quantities, as

$$d = s \times t$$

This equation does not explicitly contain the units of the various quantities; the equation is valid for any system of units as long as they are used consistently. For example,

$$120 \text{ mi} = 60 \frac{\text{mi}}{\text{h}} \times 2 \text{ h}$$

Not only must the *numbers* balance in such an equation, but so must the *units*. On the right-hand side of the equation, the time unit "hour" occurs in both numerator and denominator and therefore cancels, leaving "miles" as the unit on both sides of the equation.

We can alter the unit of any quantity by using *conversion factors*. In order to convert 60 mi/h to the corresponding number of m/s, we use the conversions

$$1 \text{ h} = 3600 \text{ s}, 1 \text{ mi} = 1.609 \text{ km, and}$$
$$1 \text{ km} = 1000 \text{ m}$$

Thus, we can form the ratios

$$\frac{1 \text{ h}}{3600 \text{ s}} = 1, \frac{1{,}609 \text{ km}}{1 \text{ mi}} = 1, \text{ and}$$
$$\frac{1000 \text{ m}}{1 \text{ km}} = 1$$

Because we can multiply any quantity by a factor of *unity* without altering the value (we only change the *scale*), we can write

$$\begin{aligned} 60 \frac{\cancel{\text{mi}}}{\cancel{\text{h}}} &= 60 \frac{\text{mi}}{\text{h}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}} \times \frac{1.609 \cancel{\text{km}}}{1 \cancel{\text{mi}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \\ &= \frac{60 \times 1.609 \times 1000 \text{ m}}{3600 \text{ s}} \\ &= 26.8 \text{ m/s} \end{aligned}$$

Note how "miles," "kilometers," and "hours" have canceled in this expression.

Any unit that appears in the denominator can always be written with a negative exponent; for example,

$$30 \text{ m/s} = 30 \text{ m s}^{-1}$$

An exponent $^{-1}$ always means *per*. It is sometimes convenient to use this notation.

Always give units when writing the numerical values of physical quantities. Be alert for the need in some instances to specify the algebraic sign (+ or −) for the calculated quantity. *Always* check equations to ensure that the units on both sides are the same (or are equivalent in the sense that they are related by a conversion factor such as demonstrated above); if the units are *not* the same, something is wrong!

Your pocket calculator is probably capable of giving the results of calculations to a large number of significant figures. For our purposes it will never be necessary to go beyond carrying calculations to a final result quoted to three significant figures, as in the above calculation. Assume that when a problem quotes a length to be 2 m for example, it is intended to be an exact value.

The Three Basic Units

The units of all mechanical quantities can be expressed in terms of the basic units of length, mass, and time. When we introduce such quantities as *force* and *energy*, for convenience we shall give special names to the units (newtons and joules), but these units are defined as certain combinations of length, mass, and time. These three units—meter, kilogram, and second—are all that we require; every mechanical quantity can be expressed in terms of these units.

Later, when treating heat and electricity, other basic units will have to be introduced.

Approximations and Estimates

Although the science of physics attempts to describe natural phenomena in terms that are as precise as possible, there are many occasions on which an approximation or even a crude estimate is quite adequate. For example, if we wish to describe the motion of the Earth around the Sun, it is not necessary to take account of the Earth's geologic features or its internal structure. Indeed, we obtain highly accurate results by making the approximation that the Earth is a *particle*, that its size is unimportant for the effects under consideration. If we shift our attention to the study of volcanoes or earthquakes, then, clearly, the internal structure of the Earth is of crucial importance.

Every situation should be examined to determine what approximations can be made to simplify the problem. We should always ask, "What features of this event or phenomenon can we neglect so that the calculations will be easier?" The features we find to be negligible in one situation may not be so in another. In one problem we may include the effects of gravity but neglect friction. In another problem friction may be important and the gravitational effect may be negligible.

Numerical approximations are also useful in judging whether a calculation is proceeding properly. Before carrying out a lengthy computation, it is often profitable to obtain an estimate of the answer by making such approximations as $3.7 + 5.4 \cong 10$, $\pi \cong 3$, $\sqrt{2} \cong 1.5$, and so forth. By quickly calculating the numerical coefficients and combining the powers of 10, you will be able to see whether your answer is going to be reasonable. If your result turns out to be that the size of an atom is 2×10^{-3} m, there is no point in improving the accuracy of the calculation until you correct the gross error!

You will also find it useful to develop your powers of estimation. Frequently, just a little thought will allow you to obtain an estimate (even a crude estimate) for something you may never have considered before. Try these:

How many cars are involved in a 2-mile traffic jam on a three-lane highway?

How many golf balls could be placed in your bedroom?

How many miles of interstate highway are there in the United States?

How many barrels of oil are required to supply the American public with gasoline for their automobiles for a year? (You need to know how many gallons of gasoline can be produced from a barrel of oil. The yield is about 20 gallons per barrel. Could you have estimated that figure?)

(Answers will be found at the end of this chapter.)

Scaling and the Sizes of Things

We see in the world around us a variety of living things that have vastly different sizes. The smallest cells have a size of about 10^{-6} m, whereas the largest living things, the giant sequoia trees, grow to heights in excess of 100 m. This range in size of living matter amounts to a factor of 10^8 or 100 million.

The characteristics and the function of an organism are related to its size. A rabbit scaled up to the size of an elephant would not be a viable creature. Nor could humans exist if they were the size of a mouse. To see how the life-style of an organism is determined by its size, we must examine the idea of *scaling*.

The biological properties of an organism depend to a remarkable extent on its geometrical properties of length, surface area, and volume. For regular geometrical objects, it is easy to relate these properties. To do this in a general way, we use the idea of a *characteristic length L* for an object. For a cube, the characteristic length is the length of a side; for a sphere, the characteristic length is the radius. Then, we know that

$$\text{surface area} \propto L^2; \quad \text{volume} \propto L^3$$

Moreover, if the object has a uniform density, the mass of the object is also proportional to the cube of the characteristic length.

To *scale* an object means to change its characteristic length by some factor. The area and the volume then scale according to the relations above. If the characteristic length is doubled, then the surface area increases by a factor of 4 and the volume increases by a factor of 8. Many biological properties depend upon the ratio of the surface area of an organism to its volume. This ratio is determined by the organism's characteristic length:

$$\frac{\text{surface area}}{\text{volume}} \propto \frac{L^2}{L^3} = \frac{1}{L}$$

The characteristic length of a regular geometrical object, such as a cube or a sphere, is easy to define. But what is the characteristic length associated with an ant, or a dog, or a human? Because we are interested only in approximate, not precise comparisons of the properties and functions of different organisms, we can use for the characteristic length any obvious or convenient length. Thus, the value of L for a human will be height (about 2 m); for a dog, $L \cong 1$ m; and for an ant, $L \cong 0.5$ cm.

Now, let us look at some of the biological properties of organisms that depend on the characteristic length.

Strength

How can we compare the strengths of organisms of different sizes? A full-grown human (mass ≅ 80 kg) can lift an object with a mass about equal to his or her own mass. A grasshopper (mass ≅ 1 g), on the other hand, can lift an object with a mass about 15 times its own mass. Does this mean that a grasshopper is "stronger" than a human? To answer this question, we need to refer to the features of the source of strength, that is, muscles. All muscle is composed of bundles of muscle fibers. These fibers are all quite similar and are packed with about the same density in the muscles of different organisms. The strength of a muscle is, to a sufficient approximation, directly proportional to the number of fibers in the muscle, that is, to the cross-sectional area of the muscle. The muscle area of an organism, again to a sufficient approximation, is proportional to a characteristic cross-sectional area, that is, to the square of its characteristic length. Thus,

$$\text{strength} \propto L^2$$

In order to compare the strengths of two organisms with different sizes, we should use the strength per unit mass. We can call this quantity the *specific strength:*

$$\text{specific strength} = \frac{\text{strength}}{\text{mass}} \propto \frac{L^2}{L^3} = \frac{1}{L}$$

where we have used the fact that the mass is proportional to the cube of the characteristic length. Now, to compare the specific strengths of the grasshopper ($L \cong 2$ cm) and a human ($L \cong 2 \text{ m} = 200$ cm), we write

$$\frac{\text{specific strength of grasshopper}}{\text{specific strength of human}} = \frac{\dfrac{1}{L(\text{grasshopper})}}{\dfrac{1}{L(\text{human})}} = \frac{L(\text{human})}{L(\text{grasshopper})} = \frac{200 \text{ cm}}{2 \text{ cm}} = 100$$

That is, simply because of its smaller size, the grasshopper should have a specific strength 100 times that of a human. But, as pointed out above, a human can lift a mass about equal to his or her own mass, whereas a grasshopper can lift a mass about 15 times its own mass. Thus, the actual strength of a grasshopper falls short of that expected on the basis of its size. A human is able to use his or her muscular capacity more efficiently than can a grasshopper.

Food Requirements

The rate at which heat is generated in an animal's body is proportional to the amount of food intake which, in turn, is proportional to the mass (or the volume) of the animal. The rate at which heat is lost by an animal to its surroundings is proportional to the surface area of the animal's body. (Moreover, for warm-blooded animals whose body temperature remains constant, the rate of heat loss is greater in cold climates than in hot climates). The heat loss of an animal per unit body mass is therefore inversely proportional to the animal's characteristic length. A very small animal must replace the loss of body heat by an almost continual intake of food. Such an animal must daily eat an amount of food that is a large fraction of its body mass. A mouse each day eats food with a mass equal to about one-quarter of its own mass. The tiny shrew will die of starvation if it goes without food for more than about 3 hours. A giant elephant, on the other hand, has the opposite problem. Its rate of heat generation taxes the ability of its skin area to dispose of the excess. For this reason, elephants take every opportunity to cool themselves at waterholes. Insects, whose ratio of surface area to volume is relatively much larger than that of any warm-blooded animal, cannot take in food at a rate sufficient to maintain a constant body temperature. For insects this does not constitute a problem, however, because they are cold-blooded creatures with a body temperature always equal to the ambient temperature. By this mechanism the heat losses, and consequently the food intake requirements, are greatly reduced. (We will examine this problem with a more sophisticated approach in the essay on page 187).

Terminal Velocity

If you fall to the ground from a third-floor window, you will probably suffer a serious injury. However, if a flea or other small insect experiences such a fall it will survive uninjured. Even a small animal, such as a mouse, will probably be able to walk away after a fall from this height. The reason that a small animal or insect can negotiate a substantial fall without serious effects is due to its relatively large surface-area-to-volume ratio. When an object or an organism falls through the air, the frictional resistance of the air retards the motion and prevents the speed of fall from increasing beyond a certain value.

The downward force on a falling object is due to gravity and is proportional to the mass (or the volume) of the object. The effect of air resistance is to produce a retarding force that depends on the cross-sectional area of the object and on its velocity. Therefore, as an object falls, its velocity increases until the retarding force becomes equal to the downward force. When this point is reached, the object continues its fall at constant velocity. This maximum velocity of free fall is called the *terminal velocity*. The greater the area-to-volume ratio of an object or organism, the smaller will be the terminal velocity. The terminal velocity of a falling human is about 65 m/s (about 145 mi/h) if he or she is "spread eagled" the way sky divers fall; if he or she assumes a ball-like shape, the terminal velocity will increase to about 105 m/s (about 235 mi/h). The maximum rate of fall for a small insect is a few meters per second; no significant injury will result from an impact with such a small velocity. (See Section 8.6 for more details concerning terminal velocity.)

References

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