Gauge Theories of Strong and Electroweak Interactions

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Preface

Our understanding of the basic elements of which matter is constituted has made considerable progress over the last decade. In theory, this led to the concept of gauge theories to describe the various interactions. The aim of the present book is to summarize this area of work for a wider range of interested parties in the realm of physics. It is in fact the elaboration of a manuscript containing lectures given to young students of elementary particle physics at the Autumn School for high energy physics in Maria Laach arranged by Professor J. K. Bienlein. The book is, therefore, mainly suited to more advanced students who want to become familiar with this area of work. Naturally, we hope that anyone with a general interest in gauge theories will learn something from it too.

As the detailed list of contents shows, the area covered is very extensive. It is a field of rapid development and no final conclusions are as yet in sight. The question then is whether a textbook can be justified at this stage. In the light of these considerations, the present book should be regarded as an experiment, surrounded by many loose ends. Over the course of time, many aspects will be seen in a different light, so that details which now seem important and which do in fact make the book difficult to read may eventually be omitted entirely. We tried to reduce the problem somewhat by giving relatively detailed summaries in the individual subsections, which could form a good guideline in themselves. On a first reading, technically complicated parts may be skipped over without detracting too much from a general understanding.

A serious problem was presented by the number of references selected from the gigantic number of original works. Apart from standard works, we have tried to include all review articles and papers which helped us originally towards a better understanding of the subject. Collective references are given for each chapter and these are placed before the actual full reference details in the bibliography section at the back, making for easier identification.

Conventions used are given on page 298.

This book owes much of its existence to the support offered by Deutsches Elektronen-Synchrotron DESY and its cooperation with the German universities. Many colleagues, from DESY and Würzburg University in particular, have assisted us both in word and deed: our thanks go to Dr A. Ali, Professor H. D. Dahmen, Professor H. Fraas, Dr W. Hollik, Dr M.

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January 1981

P. Becher, M. Böhm, H. Joos

Preface to the Second Edition

Since the appearance of the first German edition, the ideas on the physics of elementary particles formulated with help of quantum gauge field theories have not changed very much. However, there has been some progress in details. The appendix to the second edition (p. 299) is a short report of these. The original text was changed only very little. Reference to new literature is marked by an N and collected in an appendix (p. 300). The experimental data as well as the conventions of the parton model have been changed according to [Pa 82]. Misprints in the first edition have been corrected.

Hamburg/Würzburg
January 1983

P. Becher, M. Böhm, H. Joos

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CHAPTER 1

Phenomenological Basis of Gauge Theories of Strong, Electromagnetic, and Weak Interactions

The aim of elementary particle physics is to discover the fundamental law which will establish the dynamics of matter. Everything else remains a type of verbal painting, a 'super-review of particle properties' which is of interest to no-one for long. As this law combines the theory of relativity with quantum mechanics, it should take the form of a unified field theory for the strong, electromagnetic, weak, and ultimately gravitational interactions. This claim was made for elementary particle physics by W. Heisenberg at the Spring Conference of the German Physical Society in Munich in 1975 [He 76]. He tried to illustrate his point by presenting a speculative outline of such a theory [He 67].

Has research in the field of elementary particle physics brought us any nearer to this objective? Any answer to this question must take into earnest consideration the widespread hope that a theory, based on principles similar to those which have been so successful in quantum electrodynamics, can be built for the other interactions. The aim of this book is to give some foundation to these hopes, to describe the aforementioned dynamical principles and to discuss first applications of these theories.

In its purest form, quantum electrodynamics describes the interaction of electrons and positrons with photons. One special feature of this quantum theory of the electromagnetic and the Dirac-electron field is that the interaction of the field is uniquely defined by the principle of *minimal gauge-invariant coupling of the electric charge*. This important principle, first encountered in quantum mechanics in the form of the substitution rule $\partial_\mu \rightarrow \partial_\mu - ieA_\mu$, will be discussed in detail later on. It can be generalized to the interaction of complicated charge structures. One talks then of Yang–Mills field theories or non-Abelian gauge theories. One such Yang–Mills theory is quantum chromodynamics, which describes the interaction of hadrons by the gauge-invariant coupling of gluon fields to quark fields. A non-Abelian gauge theory can also describe the phenomena of the weak interaction when weak and electromagnetic fields are combined into a unified field theory, e.g. the Glashow–Salam–Weinberg theory. Thus, local gauge invariance may turn out to be an important general principle of the dynamics of matter.

The gauge theories for the interactions of elementary particles are based on simple models which supply a phenomenological explanation of experimental results. Among them the simple quark model is of special importance. The various aspects of the quark concept are the following:

(a) Quarks as hypothetical constituents of the strongly interacting particles explain the meson and baryon spectrum.
(b) Quark and lepton fields form the currents which are the sources of the universal electromagnetic and weak interaction.
(c) Quarks are seen as point like scattering centres—partons—in the deep inelastic lepton scattering.

The first section of this book deals with the phenomenological quark model of the hadron spectrum, the Fermi model of the weak interaction, and the simple parton model, plus a short introduction to the most simple concepts of group and field theory, by way of preparation for the formulation of gauge theories. [EP]

1.1 The hadron spectrum in the quark model

The quark model [Ko 69] was proposed by G. Zweig [Zw 64] and M. Gell-Mann [Ge 64] in 1964. In spite of an intensive search, quarks have not yet been found as free particles [Mo 79]. But pointlike scattering centres—partons—with quark properties have been seen in the deep inelastic lepton scattering. This apparent contradiction will, within the realm of quantum chromodynamics, form the basis of a discussion of the nature of quarks and especially their confinement in hadrons. First of all, a naive explanation of the properties of hadrons as determined by their quark composition is given.

1.1.1 Quantum numbers and wavefunctions of hadrons in the quark model

(a) *Quantum numbers*

It is well known that the many meson and baryon resonances observed can be classified by means of (conserved) *quantum numbers* [Lo 78, Ro 71a]. Currently, the flavour quantum numbers isospin I, I_3, strangeness S, charm C, baryon number B, hypercharge $Y = S + B - C/3$, electric charge $Q = I_3 + Y/2 + 2C/3$ and the geometric quantum numbers spin j, j_3, parity π, and charge conjugation parity π_c are known. Figs 1.1 and 1.2 give a survey of the quantum numbers of the observed hadrons [Pa 82].

The remarkable experimental fact, that only certain values of these quantum numbers occur, can be explained with help of the quark model by

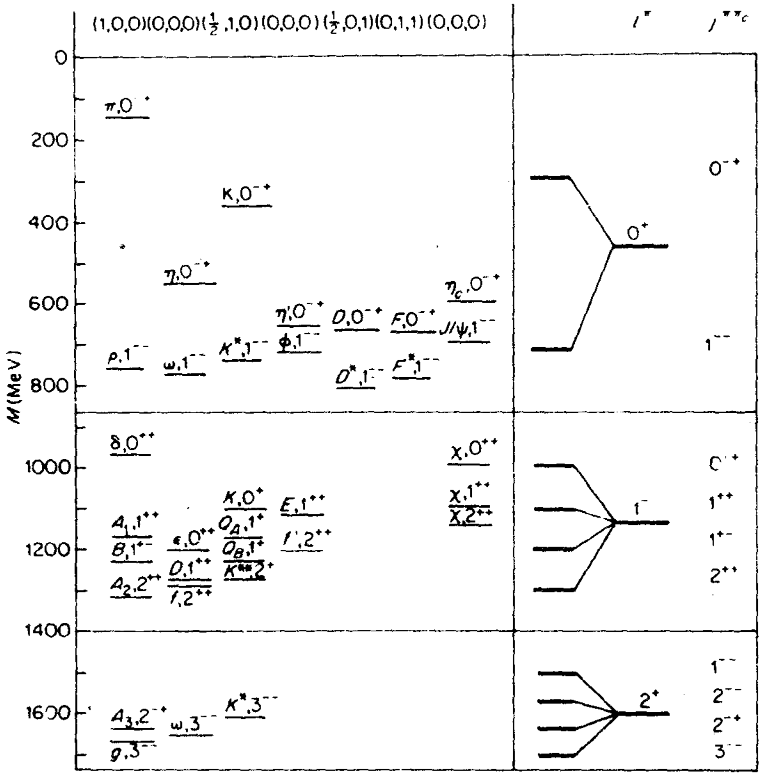

Figure 1.1 The meson spectrum in the quark model with four flavour degrees of freedom. The spectrum found experimentally is shown on the left. The number triplets in the first line stand for the quantum numbers (I, Y, C), $M - (150\,\text{MeV}) \cdot n_s - (1200\,\text{MeV}) \cdot n_c$ is written as mass, $n_s \equiv$ number of s and $\bar{s}$ quarks, $n_c \equiv$ number of c and $\bar{c}$ quarks in the meson. The low-lying multiplets expected in theory are shown on the right

the following hypothesis: mesons consist of one quark and one antiquark; baryons consist of three quarks.

In correspondence with the four quantum numbers I, I_3, S, and C, four quarks u, d, s, and c with flavour quantum numbers according to Table 1.1 are introduced. All quarks have baryon number 1/3 and spin 1/2.

The discovery of the Y (9.46) particle [He 77] indicates that there is another family of hadrons with a new flavour quantum number, bottom, and thus another quark b. For this reason, the number of flavour degrees of freedom N_F will be left open in the following.

The quantum numbers and wavefunctions of hadrons are formed from the quark degrees of freedom, according to the rules for two- and three-particle bound states of non-relativistic quantum mechanics. The composition of

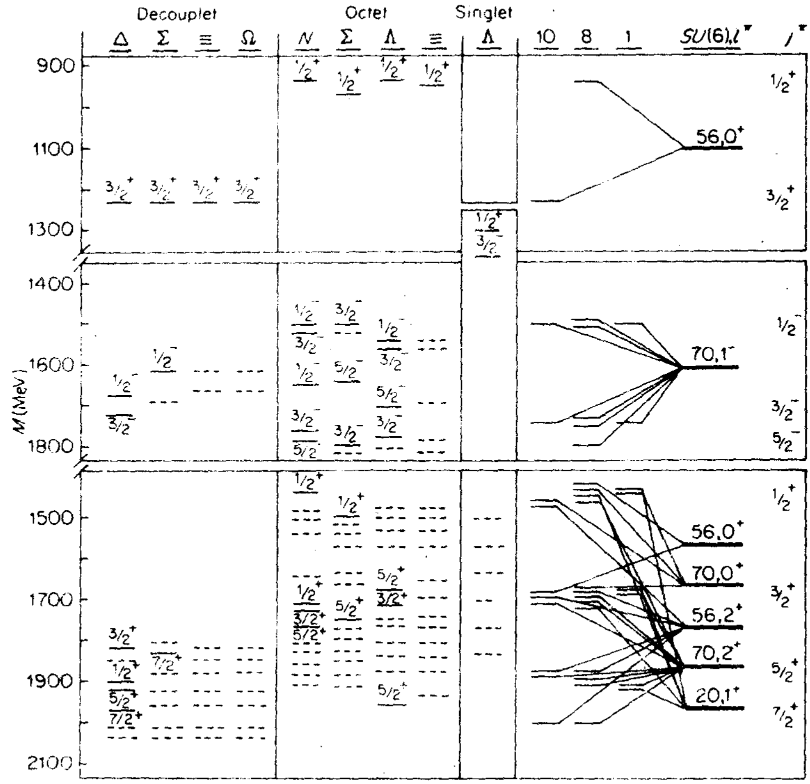

Figure 1.2 The baryon spectrum in the quark model with three flavour degrees of freedom. The spectrum observed experimentally is shown on the left. $M-(150\ \text{MeV})\cdot n_s$ is written as mass, $n_s \equiv$ number of s quarks in the baryon. On the right are shown the low-lying particle multiplets predicted, arranged in order of total angular momentum values j^π. The particles still missing from the quark model are marked with dashed lines

Table 1.1 Quantum numbers of the quarks

Quark	Quantum number					
	I	I_3	S	C	Y	Q
u	1/2	+1/2	0	0	+1/3	+2/3
d	1/2	−1/2	0	0	+1/3	−1/3
s	0	0	−1	0	−2/3	−1/3
c	0	0	0	1	0	+2/3

wavefunctions from flavour, spin, and orbital parts is sketched in the following [Li 78, Cl 79].

(b) *Wavefunctions of* mesons *as quark–antiquark systems*

Construction of the flavour part $|I, I_3, S, C\rangle$ *from the quark degrees of freedom*

$$|1,1,0,0\rangle = -\bar{d}u, \quad |1,0,0,0\rangle = (\bar{u}u - \bar{d}d)/\sqrt{2}, \quad |1,-1,0,0\rangle = \bar{u}d,$$
$$|0,0,0,0\rangle = (\bar{u}u + \bar{d}d)/\sqrt{2};$$
$$|\tfrac{1}{2},\tfrac{1}{2},1,0\rangle = \bar{s}u, \quad |\tfrac{1}{2},-\tfrac{1}{2},1,0\rangle = \bar{s}d,$$
$$|\tfrac{1}{2},-\tfrac{1}{2},-1,0\rangle = \bar{u}s, \quad |\tfrac{1}{2},+\tfrac{1}{2},-1,0\rangle = -\bar{d}s,$$
$$|0,0,0,0\rangle = \bar{s}s;$$
$$|\tfrac{1}{2},\tfrac{1}{2},0,1\rangle = -\bar{d}c, \quad |\tfrac{1}{2},-\tfrac{1}{2},0,1\rangle = \bar{u}c, \tag{1.1.1}$$
$$|\tfrac{1}{2},-\tfrac{1}{2},0,-1\rangle = \bar{c}d, \quad |\tfrac{1}{2},\tfrac{1}{2},0,-1\rangle = \bar{c}u;$$
$$|0,0,1,1\rangle = \bar{s}c, \quad |0,0,-1,-1\rangle = \bar{c}s,$$
$$|0,0,0,0\rangle = \bar{c}c;$$

The structure of multiplets of two quarks (u, d), three quarks (u, d, s), four quarks (u, d, s, c), and possibly more can be read off directly.

There are several particles with quantum numbers $|0,0,0,0\rangle$ which can therefore mix, for example, pseudoscalar mesons show roughly SU(3) mixing:

$$|\eta(549)\rangle \simeq -(\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}, \quad |\eta'(958)\rangle \cong (\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}.$$

Spin part: $|s, s_3\rangle$. Quark and antiquark spins have the possible orientations $\uparrow$ and $\downarrow$. They can be combined to form a spin triplet and a spin singlet with the following wavefunctions:

$$|1,1\rangle = \uparrow\uparrow, \quad |1,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad |1,-1\rangle = \downarrow\downarrow \tag{1.1.2}$$
$$|0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow);$$

Orbital part: $|l, m, n\rangle$. After separating off the centre of mass motion, the relative motion between quark and antiquark is described by a Schrödinger wavefunction of the relative coordinate r, which is made up of a spherical function $Y_{l,m}(\hat{r})$ and a radial part $f_{l,n}(r)$

$$|l, m, n\rangle = Y_{l,m}(r) f_{l,n}(r). \tag{1.1.3}$$

Spin and orbital angular momentum are added to meson spin j. This gives $j = l+1,\ l,\ l-1$ for the triplet and $j = l$ for the singlet states. The parity π and charge conjunction parity π_c of this fermion–antifermion system have the values $\pi = (-1)^{l+1}$ and $\pi_c = (-1)^{l+s}$. Fig. 1.1 shows the structure of the meson spectrum as it results from this composition of degrees of freedom.

(c) Baryons *as three-quark systems*

When three 'identical' particles are involved, the permutation symmetry of the wavefunction gives an important quantum number. In addition to the well-known symmetrical (*Sy*) and the antisymmetrical (*An*) there are the mixed-symmetrical wavefunctions ($\overline{Mi}$, $\underline{Mi}$), all of which can be constructed from the unsymmetrical wavefunctions as follows [Jo 70]:

$$|\mathrm{Sy}\rangle = \frac{1}{\sqrt{6}}(|abc\rangle + |bca\rangle + |cab\rangle + |acb\rangle + |cba\rangle + |bac\rangle),$$
$$|\mathrm{An}\rangle = \frac{1}{\sqrt{6}}(|abc\rangle + |bca\rangle + |cab\rangle - |acb\rangle - |cba\rangle - |bac\rangle). \quad (1.1.4)$$

For $a \neq b$, there exist two 2-dimensional, mixed symmetrical representations $Mi_{\pm}$:

$$|\overline{Mi}\rangle_{\pm} = \frac{1}{\sqrt{6}}((|abc\rangle \pm |bac\rangle) + \varepsilon(|bca\rangle \pm |acb\rangle) + \varepsilon^*(|cab\rangle \pm |cba\rangle)),$$
$$|\underline{Mi}\rangle_{\pm} = \frac{1}{\sqrt{6}}((|abc\rangle \pm |bac\rangle) + \varepsilon^*(|bca\rangle \pm |acb\rangle) + \varepsilon(|cab\rangle \pm |cba\rangle)), \quad (1.1.4)$$

where $\varepsilon := \exp(2\pi i/3) = (-1 + i\sqrt{3})/2$; in '*Mi*', the cyclic permutation (123) leads to multiplication with the phase ε, ε^* and the transposition (12) causes an exchange of both components. Mi_- is zero for $a = b$.

Flavour part. The number of states in the flavour multiplets can be obtained by decomposing the quark product states into symmetry types. Table 1.2 shows the result for N_F flavour degrees of freedom.

The flavour wavefunctions of a baryon can be constructed explicitly by using its quark content and (1.1.4) for its symmetry type; for instance $|uud\rangle$

Table 1.2 Number of states in the flavour multiplets

	N_F	$N_F = 2$	$N_F = 3$	$N_F = 4$
Sy	$\binom{N_F+2}{3}$	4	10	20
Mi 2×	$2\binom{N_F+1}{3}$	2	8	20
An	$\binom{N_F}{3}$	0	1	4

is the quark content and

$$|\tilde{P}\rangle = (|uud\rangle + \varepsilon\,|udu\rangle + \varepsilon^*\,|duu\rangle)/\sqrt{3}$$
$$|\underline{P}\rangle = (|uud\rangle + \varepsilon^*\,|udu\rangle + \varepsilon\,|duu\rangle)/\sqrt{3}$$

the flavour part of the proton wavefunction with mixed symmetry.

Spin part. The spins of the three quarks can be combined to form the total spin $(3/2)_{Sy}$ and $(1/2)_{Mi}$ by the same method. As the spin has only two possible orientations, there is no antisymmetrical combination:

$$|\tfrac{3}{2}, +\tfrac{3}{2}\rangle = \uparrow\uparrow\uparrow, \quad |\tfrac{3}{2}, +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow),$$

$$|\tfrac{3}{2}, -\tfrac{3}{2}\rangle = \downarrow\downarrow\downarrow, \quad |\tfrac{3}{2}, -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow);$$

$$\overline{|\tfrac{1}{2}, +\tfrac{1}{2}\rangle} = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \varepsilon\uparrow\downarrow\uparrow + \varepsilon^*\downarrow\uparrow\uparrow),$$

$$\overline{|\tfrac{1}{2}, -\tfrac{1}{2}\rangle} = \frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \varepsilon\downarrow\uparrow\downarrow + \varepsilon^*\uparrow\downarrow\downarrow); \qquad (1.1.5)$$

$$\underline{|\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle} = \overline{|\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle} \quad \text{complex conjugate}$$

Orbital part. The relative motion of the three quarks is a function of two relative coordinates $\mathbf{z}_1 = (2\mathbf{x}_3 - \mathbf{x}_1 - \mathbf{x}_2)/\sqrt{6}$, $\mathbf{z}_2 = (\mathbf{x}_2 - \mathbf{x}_1)/\sqrt{2}$. Thus, there are many different combinations of internal angular momenta leading to one total orbital momentum l. In addition, Schrödinger wavefunctions

$$|l, m, n, \eta\rangle = F_l(\mathbf{z}_1, \mathbf{z}_2) \qquad (1.1.6)$$

of all three symmetry types can be systematically constructed using the mixed-symmetrical, complex, relative coordinates $\mathbf{z}_1 = (\mathbf{z}_1 + i\mathbf{z}_2)/\sqrt{2}$, $\mathbf{z}^* = (\mathbf{z}_1 - i\mathbf{z}_2)/\sqrt{2}$. The structural details of Schrödinger wavefunctions are determined by the quark–quark interaction potentials (cf. [Gr 76, Bö 80]).

Composition of the baryon wavefunctions. When flavour, spin, and orbital parts are combined to give a total wavefunction, its symmetry character is founded upon the following composition rules

$$Sy \otimes Sy = Sy, \quad An \otimes An = Sy, \quad Mi \otimes Mi = Sy \otimes Mi \otimes An,$$
$$Sy \otimes An = An, \quad Sy \otimes Mi = Mi, \quad An \otimes Mi = Mi. \qquad (1.1.7)$$

In this way, the flavour and spin parts (Table 1.2 or Eqn. (1.1.5)) combine to give the mutliplets with defined symmetry as shown in Table 1.3. If the interaction shows only a slight flavour and spin dependence, it is advantageous to amalgamate multiplets of equal symmetry to form supermultiplets of the higher symmetry group $SU(2N_F)$ [Cl 79].

Table 1.3 (Flavour, spin)-multiplets with definite symmetry

	$N_F=2$	$N_F=3$	$N_F=4$
Sy	$20=(4,\frac{3}{2})+(2,\frac{1}{2})$	$56=(10,\frac{3}{2})+(8,\frac{1}{2})$	$120=(20,\frac{3}{2})+(20,\frac{1}{2})$
Mi $2\times$	$20=(4,\frac{1}{2})+(2,\frac{3}{2})$ $+(2,\frac{1}{2})$	$70=(10,\frac{1}{2})+(8,\frac{3}{2})$ $+(8,\frac{1}{2})+(1,\frac{1}{2})$	$168=(20,\frac{3}{2})+(20,\frac{1}{2})$ $+(20,\frac{1}{2})+(4,\frac{1}{2})$
An	$4=(2,\frac{1}{2})$	$20=(8,\frac{1}{2})+(1,\frac{3}{2})$	$56=(20,\frac{1}{2})+(4,\frac{3}{2})$

The final stage consists of the combination with the orbital wavefunction to give states with definite total angular momentum. If the total wavefunction is to be symmetrical under permutations, then the classification shown in Fig. 1.2 follows. This is a good description of the experimental spectrum for baryon resonances with strangeness zero. The many blanks which occur in the case of resonances with strangeness -2 or -3 are due to the impossibility of carying out phase shift analyses for this case. The first baryons with charm were discovered recently [Ab 79]. The mass of $\Lambda_c = udc$ is (2.285 ± 0.006)GeV, i.e. at the expected level [De 75].

The fact that a symmetrical total wavefunction must be chosen to explain the experimental baryon spectrum is a problem for the simple, phenomenological quark model: as quarks with spin 1/2 obey Fermi statistics, the total wavefunction must be antisymmetric. Accordingly, the orbital part of the wavefunction of the Δ^{++} resonance with $I=3/2$, $I_3=+3/2$ and $j=3/2$ must be antisymmetric, since the flavour and spin components are necessarily symmetric. On the other hand, the Δ^{++} is the ground state of all particles made up of three u quarks. It is possible to show that for relatively general potentials the orbital wavefunction of the ground state has no nodes and this is in contradiction to the antisymmetry required. The solution of this problem of quark statistics by using a quark model with colour was an important discovery on the way to quantum chromodynamics.

1.1.2 Quark model with colour

The antisymmetrical total wavefunction of baryons with symmetrical flavour, spin, and orbital parts, required by the Pauli principle, can be achieved by extending the quark degrees of freedom. This solution to the problem of quark statistics was first proposed by O. W. Greenberg [Gr 64] and M. Y. Han and Y. Nambu [Ha 65]; the present formulation originates in that of M. Gell-Mann [Ge 72a]. According to him, quarks have an additional degree of freedom, colour, which can assume three values (red, green, and blue) and in which the baryon wavefunction is antisymmetrical.

Thus the constituents of hadrons are quarks in three different colours:

$$q_{f,c}\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \\ c_r & c_g & c_b \\ \vdots & \vdots & \vdots \end{pmatrix}\downarrow N_F \text{ flavour degrees of freedom}$$

$\rightarrow$ 3 colour-degrees of freedom

Hadrons are constructed according to the rule 'hadrons are colourless'. This means that the meson and baryon wavefunctions have the following flavour and colour content:

$$\text{meson} = \sum_{c,c'} \bar{q}_{f,c} q_{f',c'}\, \delta_{cc'}/\sqrt{3}, \tag{1.1.8}$$

$$\text{baryon} = \sum_{c,c',c''} q_{f,c} q_{f',c'} q_{f'',c''} \varepsilon_{cc'c''}/\sqrt{6}; \tag{1.1.9}$$

here, $\varepsilon_{cc'c''}$ is the totally antisymmetric tensor of rank 3 with $\varepsilon_{123} = +1$.

The complete wavefunctions for the ρ^+ mson and the Δ^{++} resonance are given explicitly by way of illustration. A Gaussian wavefunction is a suitable ansatz for many dynamical problems. Therefore the radial wavefunction was chsoen of this form:

$$\rho^+ = -\frac{1}{\sqrt{3}}(\bar{d}_r u_r + \bar{d}_g u_g + \bar{d}_b u_b)\cdot|\uparrow\uparrow\rangle\cdot(\alpha_M/\pi)^{3/4}\exp(-\alpha_M \mathbf{r}^2/2),$$

$$\Delta^{++} = \frac{1}{\sqrt{6}}(u_r u_g u_b + u_b u_r u_g + u_g u_b u_r - u_g u_r u_b - u_b u_g u_r - u_r u_b u_g) \tag{1.1.10}$$
$$\times|\uparrow\uparrow\uparrow\rangle\cdot(\alpha_B/\pi)^{3/2}\exp[-\alpha_B(\mathbf{z}_1^2+\mathbf{z}_2^2)/2].$$

The colour degree of freedom, which was introduced because of the statistics problem, is dynamically relevant. According to the colour rule, only colourless combinations of quarks form bound states, i.e. hadrons with conventional masses. This is a first look at the relationship between colour and quark confinement.

The introduction of colour was found to be of decisive importance for the formulation of the theory of strong interactions. Therefore, further phenomenological information on the colour degree of freedom is essential. Leaving the statistics problems aside, the classical examples [Fr 73] are the size of the total cross-section for e^+e^- annihilation into hadrons (see Section 1.4.4) and the decay rate for $\pi^0 \rightarrow 2\gamma$ (see Section 1.3.2). The structure of the colour part of the hadron wavefunctions indicates that there is an exact symmetry connected with colour. This aspect will be considered further in Section 1.2.