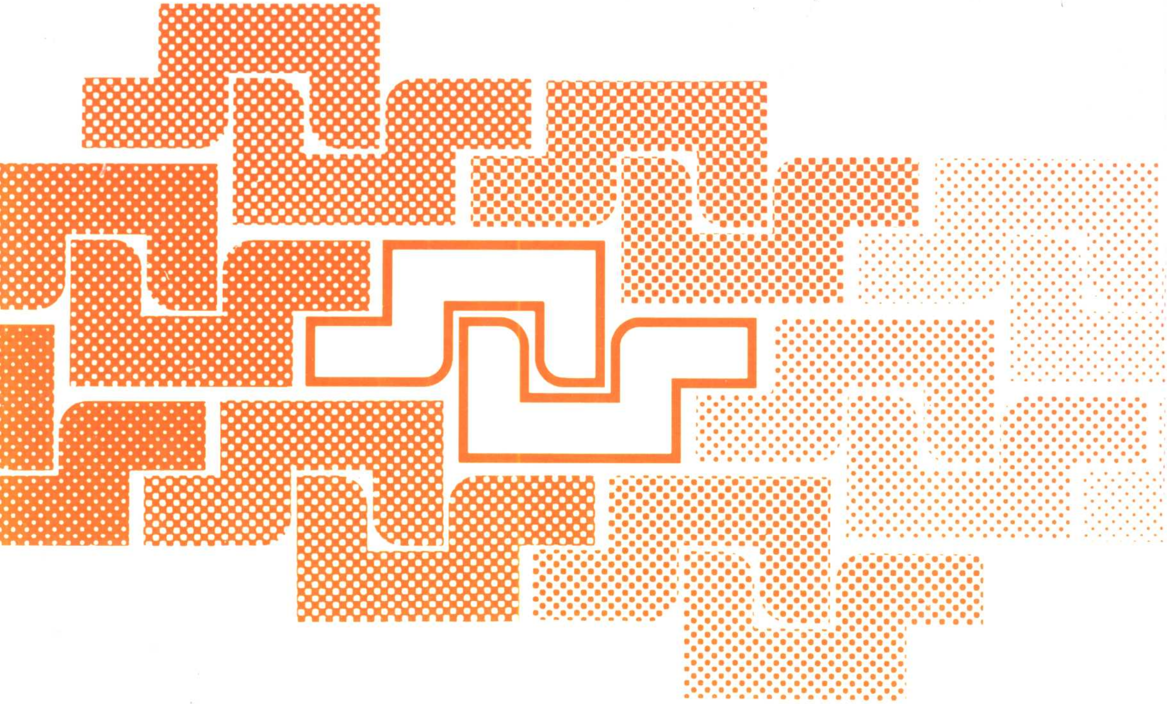

Microwave Superconductivity

Edited by

Harold Weinstock and Martin Nisenoff

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Microwave Superconductivity

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INTRODUCTION

During the first two years following the discovery of superconductors with transition temperatures greater than 77 K (the boiling point of liquid nitrogen at atmospheric pressure), many fanciful predictions were made about the immediate applications of these ceramic materials. After more than a decade of often-feverish worldwide research activity, most of those euphoric hopes have yet to be realized. This is true despite considerable progress in understanding the fundamental mechanisms responsible for superconducting behavior in perovskite ceramics and in significant advances in processing technology to produce thin-film, thick-film and tape structures. However, if there is one application area that already has had a significant impact in the commercial and military markets, it is the area of passive microwave components. Specifically, thin-film and thick-film HTS microwave resonators with extremely high values of the quality factor, Q, have been used to fabricate filters that provide very low insertion loss and very sharp frequency roll-off characteristics ("skirts") that only can be realized using conventional filters that are about two orders of magnitude larger in volume and one order of magnitude larger in mass. A complete filter system, containing either six or twelve filters, *with its associated closed-cycle cooling system*, can be packaged to fit into a conventional electronic instrument rack, with the height of the system only 10 to 20 cm. The operator need not know that the key components are operating at very cold temperatures, typically, close to the boiling point of liquid nitrogen (77 K).

Currently, there are many - possibly eight to ten – (mostly) small companies throughout the world manufacturing such filter systems. It is estimated that the total number of such systems manufactured to date is about 1,000 units. Many units are undergoing beta-site testing in the base stations of numerous cellular communications systems. There have been published reports from the HTS filter vendors that the use of HTS filter systems reduces the number of "dropped calls" and, correspondingly, this should produce increased revenue as a result of this more efficient operation – see the chapter in this volume by Balam Willemsen for details. Despite the obvious improvement in system performance, the service providers are still performing cost-benefit trade-off studies before committing to large-scale introduction of HTS filter systems to their operating systems.

It is clear also that the world's military forces are considering the insertion of HTS filters into their surveillance and communications systems. Although it is difficult to predict the degree to which military usage will drive this industry, it is probably correct to say that military adaptation of the technology is a certainty; only the magnitude is uncertain.

To summarize, there are numerous emerging markets for passive HTS microwave filters and related components. While it is difficult to predict the ultimate size of these markets, under the most optimistic scenarios, the market could exceed 100,000 units per year globally. Within the next two to three years, it should be far more certain what the actual size of the market will be for these products. At this time, however, there is considerable interest in learning how to improve the microwave properties of HTS materials, how to characterize the materials and evaluate circuit parameters, how to improve the design of HTS microwave and radio-frequency circuits, and how to provide efficient, compact cryogenic packaging.

This volume, based upon lectures presented at the NATO Advanced Study Institute (ASI) held August 29 to September 10, 1999 in Millau, France, has been specifically planned to provide the basis for a graduate-level course on microwave superconductivity. Additionally, it is designed to serve as a convenient reference for people conducting research in this field. The inspiration for the NATO ASI itself was a desire on the part of university researchers to teach a class on this subject, but who found there wasn't a suitable textbook upon which to base such a course. Although the discussion above speaks only of the major application of HTS microwave filters, a course in microwave superconductivity must be comprehensive. It should cover the relevant fundamentals of electromagnetism, fabrication and characterization of materials, a variety of applications, and cryocooler technology. Although the focus is primarily on HTS superconductors, some applications involving liquid-helium-based (or LTS) superconductivity have been included. Applications or structures covered (either HTS or LTS) involve antennas, lumped elements, RF accelerators, RF SQUIDs, RF SQUID-based amplifiers, long-Josephson-junction circuits, and (of course) filters. While it is impossible to claim all applications have been covered, those that have been covered are the ones that already have been found to be (or show promise to be) significant with regard to the commercial, military and research communities. Additionally, measurement techniques and cryogenic systems have been considered.

Arlington, Virginia — Martin Nisenoff
October 2001 — Harold Weinstock

ACKNOWLEDGMENTS

This book comprises the official proceedings of the NATO Advanced Study Institute (ASI) held 29 August to 10 September 1999 at the International Hotel in Millau, France. The Institute would not have been possible without generous financial support from the Scientific and Environmental Affairs Division of NATO. Hence, we first must express our gratitude to NATO for making the Institute and these proceedings possible. Its support of six sequential ASIs that were proposed and directed by one of us (HW) in the field of superconductivity since 1988 is greatly appreciated. It is worth noting that the other co-director (MN) of the 1999 ASI also was a co-director of the first ASI in 1988; thus completing the cycle as it had begun. The (United States) Air Force Office of Scientific Research (AFOSR) through its European Office of Aerospace Research and Development provided substantial additional financial support. Furthermore, HW must thank AFOSR for allowing him to work on activities related to the ASI during the course of his normal duties and for providing other institutional resources. It is a pleasure, as well, to acknowledge financial support from the (United States) National Science Foundation, which provided travel support to US researchers, primarily graduate students.

The ASI would not have been possible without the dedicated and superior efforts of Mrs. Sandra Ronayne, a Management Assistant at AFOSR. She handled all of the ASI administrative tasks, created the ASI Web site and handled most correspondence with participants. It is regretful that Mrs. Ronayne was unable to attend the ASI itself due to health problems that fortunately have had a happy ending. She was sorely missed, although we would like to thank Mrs. Linda Weinstock for her service at the ASI registration desk and library, and for doing whatever else helped ease the burden of two rather busy co-directors.

We reserve our greatest thanks for the fifteen distinguished scientists who were ASI lecturers and who (in some cases together with their collaborators) subsequently became the authors of the chapters in these proceedings. During the ASI all of them devoted from one to two weeks of their valuable time to lectures, workshops and informal discussions with about eighty mostly young researchers from fifteen countries. Surely their most difficult task was to prepare a manuscript - two manuscripts for six of the lecturers - for these proceedings and to deal with two rather fussy editors (MN and HW). For all of these reasons, we are very much in their debt. Thanks are due also to the aforementioned ASI participants who provided the enthusiasm for microwave superconductivity that helped motivate the efforts of the lecturers.

As always, we must acknowledge the patience and good will of our families for excusing the many distractions caused by the ASI and the editing of these proceedings.

Arlington, Virginia	Martin Nisenoff
October, 2001	Harold Weinstock

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1. Introduction

This chapter presents a fundamental analysis of the interaction of electromagnetic waves with superconducting materials. This presentation is based on simple electromagnetic theory and provides a basic understanding of the physical phenomenon involved. The model developed is of considerable practical use and is the basis for many calculation of the performance of superconducting devices at high frequency.

The chapter first introduces the concept of surface impedance followed by the two fluid model. Once these have been understood, basic electromagnetic theory is used to examine the propagation of plane electromagnetic waves in superconductors. Examination of this process leads to an understanding of the basic properties of superconductors at high frequencies and develops expressions for use in practical situations. Power and energy considerations are examined through a discussion of Poynting's theorem in relation to superconducting materials. This is followed by examples of experimental measurements of the important material parameters.

2. Surface impedance

Surface impedance is probably the most important concept when considering the interaction of electromagnetic waves with almost any material and is particularly important when dealing with superconductors. Consider the scenario in Figure 1, where an electromagnetic wave with electric field, represented by E_i, is incident on a boundary between two materials. The material on the left could be free space and the material on the right could be a block of superconductor. The incident wave produces a reflected wave, E_r and a transmitted wave E_t, the amplitude and phase of which are determined by the material properties of the two media.

H. Weinstock and M. Nisenoff (eds.), Microwave Superconductivity, 1–20.

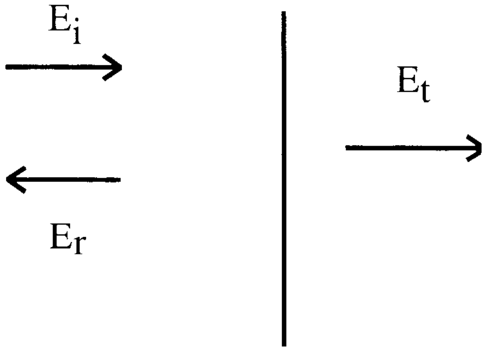

Figure 1 Electromagnetic waves at a boundary.

Assuming the waves are sinusoidal, they can be written as,

$$E_i = E_{oi} e^{-\gamma_1 z} \qquad H_i = \frac{E_{oi}}{Z_1} e^{-\gamma_1 z}$$

$$E_t = E_{ot} e^{-\gamma_2 z} \qquad H_t = \frac{E_{ot}}{Z_2} e^{-\gamma_2 z} \tag{1}$$

$$E_r = E_{or} e^{+\gamma_1 z} \qquad H_r = -\frac{E_{or}}{Z_1} e^{+\gamma_1 z}$$

E_{0i}, E_{0t}, and E_{0r} are the amplitudes of the waves, z is the usual Cartesian co-ordinate and is in the same direction of the propagated wave. Z_1 and Z_2 are the intrinsic impedance of the two materials and, γ_1 and γ_2 are the propagation constants of the two materials.

The tangential electric and magnetic fields at the boundary must add as follows

$$E_i + E_r = E_t$$

$$H_i + H_r = H_t$$

We define the surface impedance of a material as the ratio of the tangential electric to magnetic fields, hence,

$$Z_s = \frac{E_t}{H_t} = \frac{E_i + E_r}{H_i + H_r} \tag{2}$$

If one substitutes values for the fields from (2) and take $z = 0$ then

$$Z_s = Z_2 = Z_1\left(\frac{E_{oi} + E_{or}}{E_{oi} - E_{or}}\right)$$

The importance of this relationship is that the intrinsic impedance of a material Z_2 is equal to the surface impedance of the material Z_s. Of course, a transmission and reflection coefficient can be defined as the ratio of the transmitted and reflected fields to the incident field respectively are simply shown to be

$$\frac{E_{0t}}{E_{0i}} = \frac{2Z_2}{Z_1 + Z_2} = T\ ,$$

$$\frac{E_{0r}}{E_{0i}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = R\ .$$

Both reflection and transmission coefficients are strongly dependant upon the surface impedance of medium 2. Knowing the surface impedance allows simple calculations of the reflected and transmitted waves to be made.

The concept of surface impedance can be generalised to non-planar boundaries. Figure 2 shows an enclosed cavity, having either metallic or superconducting walls. From the point of view of analysing the structure, it can be assumed that the boundary has a surface impedance Z_s. Analytical calculations can be made of the resonant frequencies and quality factor of the cavity, and then the value of Z_s can be substituted at the end. Thus, metallic, superconducting or more complex boundaries can be considered after the main calculation.

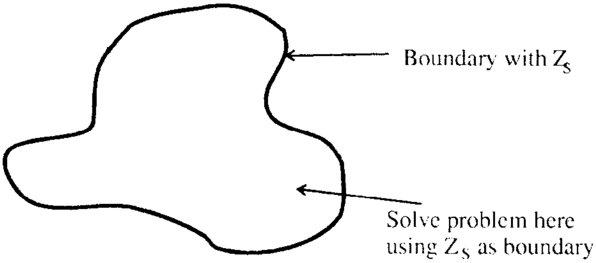

Figure 2 Generalised enclosed structure with a boundry with surface impedance Z_s.

For example, the walls of the cavity may be made up of the multi-layer structure shown in Figure 3. By simple calculations, the surface impedance of the structure can be calculated with the effective surface impedance being the combined surface impedances of the HTS, dielectric and air. Of course this impedance can then be used for the calculation of the properties of the cavity of Figure 2[1].

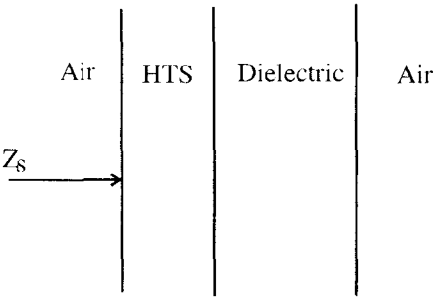

Figure 3 Multi-layer structure

The main point here is that the concept of surface impedance is extremely useful and this point will be enlarged upon later in this text.

The surface impedance can also be considered as the impedance of a square piece of material. Consider Figure 4 where a piece of material length l and width w has a current I flowing through it generating a potential V across the length l.

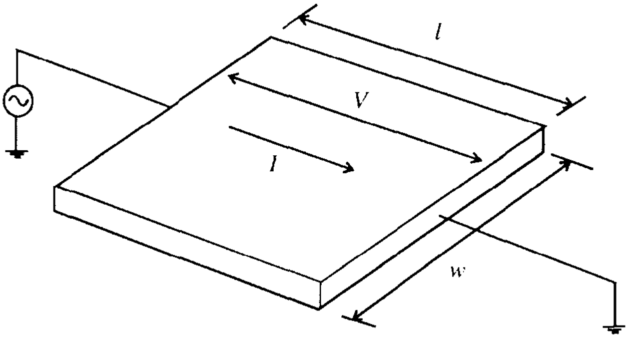

Figure 4 Applied voltage and induced current across the surface of a piece of material.

The surface impedance is given as the ratio of the voltage to current. This can be written as the ratio of the electric field E multiplied by the length l to the surface current density K multiplied by the width w. If the width is equals to the length, then this results in the ratio of the surface electric to magnetic fields since the surface current density is equal to the magnetic field at the surface.

$$Z_s = \frac{V}{I} = \frac{El}{Kw} = \frac{E_t}{H_t}.$$

---

[1] It is acceptable to use this concept provided the skin depth or penetration depth are small, compared with the curvature of the object under investigation.

A second concept will now be considered before we can proceed with the basic calculation of electromagnetic field with superconductors.

3. Two fluid model

In normal metals, current is carried by electrons, while in superconductors current is carried by a combination of normal electrons and electron pairs. This is depicted in Figure 5. This is a well-known phenomena and examined thoroughly in theoretical and experimental studies. The electron pairs are bound together by lattice interactions and travel without loss.

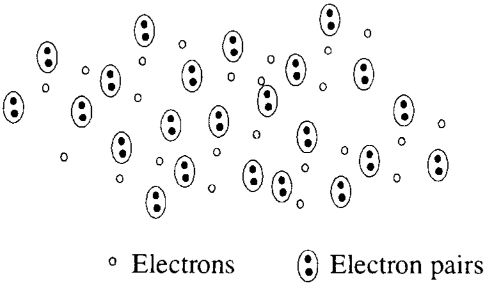

Figure 5 Electron and Electron pairs flowing inside a superconductor

The ratio of normal electrons to electron pairs depend upon the temperature, with the concentration of paired electrons decreasing as the temperature increases and going to zero at the transition temperature. We will now use this picture to form some fundamental expression and ideas.

For an electron pair, the force acting on the paired electrons is given by

$$2m\frac{d\underline{v}_s}{dt} = -2e\underline{E} \tag{3}$$

Here, m is the mass, $\underline{v}_s$ the velocity of the paired electrons, e the electron charge and $\underline{E}$ applied electric field.

For a normal electron there is also a force due the electrons scattering off the lattice.

$$m\frac{d\underline{v}_n}{dt} + m\frac{\underline{v}_n}{\tau} = -e\underline{E} \tag{4}$$

It is interesting to note that the term $m\frac{\underline{v}_n}{\tau}$ provides for the decay of current. If the electric field is removed, putting $\underline{E}$ to zero, the equation becomes

$$m\frac{d\underline{v}_n}{dt} = -m\frac{\underline{v}_n}{\tau} \ .$$

This has a solution

$$v_n = ke^{-t\tau} \qquad \underline{v}_n \propto e^{-t/\tau} \ .$$

Here τ is the momentum relaxation time and k a constant. It can be seen that the velocity decays exponentially once the field is removed.

The current density due to the electron pairs can be written as

$$\underline{J}_s = -n_s e \underline{v}_s \qquad (5)$$

While the current density due to the normal electrons is

$$\underline{J}_n = -n_n e \underline{v}_n \qquad (6)$$

Here n_s and n_n are the paired and normal electron densities. (3) to (6) now define current density in the superconductor. We will now look at two particular cases.

In the first case, it will be assumed that the normal, unpaired electron density is very small. This is the case at low temperatures where almost all the electrons are paired. This is also an approximation for a low frequency analysis as will be seen below.

Substituting (5) into (3) gives:

$$\Lambda\frac{\partial \underline{J}_s}{\partial t} = \underline{E} \qquad (7)$$

Where

$$\Lambda = \frac{m}{n_s e^2} \ .$$

(7) is called the first London equation [1]. Now take the curl of (7), giving

$$\Lambda\frac{\partial}{\partial t}(\underline{\nabla} x \underline{J}_s) = \underline{\nabla} x \underline{E} \qquad (8)$$

Faradays Law is given by

$$\underline{\nabla} x \underline{E} = -\frac{\partial \underline{B}}{\partial t} \ .$$

Making a substitution for the curl of E into (8), and integrating with respect to time gives

$$\Lambda \underline{\nabla} x \underline{J}_s = -\underline{B} .$$

This is the second London equation. It should be noted that there should be a constant of integration in the above step. This has been arbitrarily set to zero. The main reasoning for this is that the second London equation has been verified experimentally and no constant has been deemed necessary for agreement. Equation (7) together with Maxwell's equations gives a simple method for making calculations on superconducting materials.

Now let us look at a second case. Here it will be assumed that all the electromagnetic fields vary in a sinusoidal manner. Hence

$$\underline{J}_s = \underline{J}_{so} e^{j\omega t} \qquad \underline{J}_n = \underline{J}_{no} e^{j\omega t} \qquad \underline{E} = \underline{E}_o e^{j\omega t} .$$

Using these sinusoidal time dependencies and substituting (5) and (6) into (4) and (3) results in expressions for the super and normal current densities. By summing these, a total current can be deduced.

$$\underline{J}_o = \underline{J}_{no} + \underline{J}_{so} = \left\{ \frac{n_n e^2 \tau}{m(1+\omega^2\tau^2)} - j\left(\frac{n_s e^2}{\omega m} + \frac{n_n e^2 \tau^2 \omega^2}{\omega m(1+\omega^2\tau^2)} \right) \right\} \underline{E} \qquad (9)$$

This expression for the total current can be written

$$\underline{J}_o = (\sigma_1 - j\sigma_2)\underline{E}_o , \qquad (10)$$

where

$$\sigma_1 = \frac{n_n e^2 \tau}{m(1+\omega^2\tau^2)} \qquad \text{and} \qquad \sigma_2 = \frac{n_s e^2}{\omega m} + \frac{n_n e^2 \tau^2 \omega^2}{\omega m(1+\omega^2\tau^2)} . \qquad (11)$$

Equation (10) defines a complex conductivity. We can now use Maxwell's equations with sinusoidal time dependencies and the complex conductivity for calculations. That is, replace $\underline{J} = \sigma \underline{E} by J = (\sigma_1 - j\sigma_2)\underline{E}$.

Figure 6 depicts the current flow in a superconductor which is represented as a simple circuit. There is both an inductive term and a resistive term, as we would expect from the complex conductivity. At low frequencies, the inductive part provides a short and there is little effect from the resistive term. In fact, when a non-time varying signal is applied, the resistor has no effect and the superconductor has no loss as is well known. The inductive part of the circuit becomes highly reactive at high frequencies and then losses occur due to current flowing through the resistor.

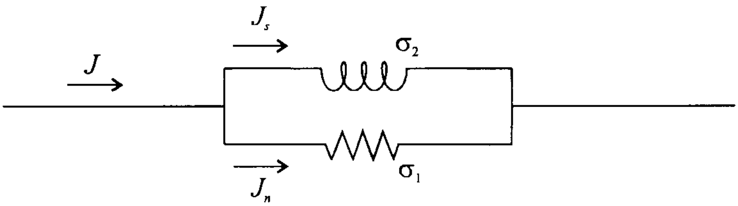

Figure 6 Circuit reresentation of current flow in superconductors.

The calculation resulting in (9) and (10) is very basic. However the main aim here has been to show that a superconductor can be represented by a complex conductivity. There are much more accurate methods based on the BCS theory to calculate this complex conductivity in terms of fundamental material parameters [2,3,4,5,6]. However, for practical use of these materials, these expressions are seldom required.

As stated above, as the temperature increases, the number of electron pairs decrease resulting in more electrons. The commonly used expressions for the number of carriers are given by [7].

$$\begin{aligned} n_s &= n_o(1-t^4) \\ n_n &= n_o t^4 \end{aligned} \tag{12}$$

Here $t=T/T_c$, where T is the actual temperature and T_c is the superconducting transition temperature.

4. Maxwell's equations, the wave equation and superconductors

Now that we understand surface impedance and complex conductivity, it is possible to examine fundamental electromagnetic properties of superconductors. This will be done by looking firstly at Maxwell's equations for sinusoidal time dependence and seeing that a wave equation can be developed from them. Using the wave equation, a propagation constant and impedance can be extracted and then these parameters will be examined using complex conductivity.

For sinusoidal time dependencies, Maxwell's equations are:

$$\underline{\nabla} x \underline{H} = \underline{J} + j\omega \underline{D} \,, \tag{13}$$

$$\underline{\nabla} x \underline{E} = - j\omega \underline{B} \,, \tag{14}$$

$$\underline{\nabla} \cdot \underline{B} = 0 \,, \tag{15}$$

$$\underline{\nabla} \cdot \underline{E} = 0 \,. \tag{16}$$

Here we have assumed a region free of static charges. Now take the curl of (14) and put $\underline{B}=\mu\underline{H}$

$$\underline{\nabla} x \underline{\nabla} x \underline{E} = -j\omega\mu \underline{\nabla} x \underline{H} .$$

Dividing by $-j\omega\mu$ and substitute for $\underline{\nabla} x \underline{H}$ from (13) gives

$$\frac{\underline{\nabla} x \underline{\nabla} x \underline{E}}{-j\omega\mu} = \underline{J} + j\omega\underline{D} .$$

Now put $\underline{J}=\sigma\underline{E}$,.

$$\underline{\nabla} x \underline{\nabla} x \underline{E} = -j\omega\mu(\sigma + j\omega\varepsilon)\underline{E} . \qquad (17)$$

The conductivity σ is complex for a superconductor and real for a normal metal but will be left just as σ for the moment.

Using the vector identity

$$\underline{\nabla} x \underline{\nabla} x \underline{E} = \underline{\nabla}(\underline{\nabla} \cdot \underline{E}) - \nabla^2 \underline{E} ,$$

but with $\underline{\nabla}(\underline{\nabla} \cdot \underline{E}) = 0$ from (16), gives

$$\underline{\nabla} x \underline{\nabla} x \underline{E} = -\nabla^2 \underline{E} .$$

Substitution into (17) gives

$$\nabla^2 \underline{E} = j\omega\mu(\sigma + j\omega\varepsilon)\underline{E} .$$

This is the wave equation for the electric field. A similar one for $\underline{H}$ exists and is calculated in a similar way. This equation can be rewritten as

$$\nabla^2 \underline{E} = \gamma^2 \underline{E} \qquad (18)$$

Where γ is the propagation constant. This equation is very general, and will be used to get a better understanding of the propagation of electromagnetic waves in superconductors that will now be simplified by assuming plane waves.

5. Plane waves in superconductors

For a plane wave travelling in the *z* direction the simplest expressions for the electromagnetic fields are.