An Introductory Approach to Operations Research

Robert J. Thierauf

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Dedicated to the memory of
D. J. O'CONOR, SR.,
a cofounder of the Formica
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About The Author

Robert J. Thierauf is a Professor of Quantitative Management and Information Systems at Xavier University in Cincinnati. He has also taught at the University of Cincinnati and The Ohio State University. Prior to coming to Xavier University he was staff consultant for several years with Coopers and Lybrand where he specialized in information systems and operations research studies for business and industry. Many of the studies and applications found in his publications are the direct result of consulting activities. His published works include books on operations research, data processing, management information systems, and management principles and practices.

A native of Cincinnati, Professor Thierauf earned his Ph.D. in management from The Ohio State University. He is a Certified Public Accountant and holds membership in several professional societies including The American Institute for Decision Sciences, The Institute of Management Science, The Academy of Management, The Association for Computing Machinery, and The American Institute of Certified Public Accountants. He is the first recipient of the D. J. O'Conor Memorial Professor of Business Administration endowed chair in Xavier's College of Business Administration.

Preface

Although this book owes much of its content to the highly successful *Decision Making Through Operations Research*, 2nd Edition, by Robert J. Thierauf and Robert C. Klekamp (Wiley/Hamilton, 1975), it possesses its own character and has been written to satisfy a particular need of those who teach the introductory course in Operations Research. Chiefly, it presents the basic techniques of operations research (OR) as well as the important requirements for formulating specifical OR models, resulting in a "model formulation" approach to an introductory study of operations research.

The level of presentation is geared toward the undergraduate business student. Within this framework, the book contains features that assist the student in learning about operations research as practiced within a typical business organization. These student-oriented learning activities are found in each chapter:

- study goals for the chapter, accompanied by a chapter outline
- the requirements for formulating operations research models to solve specific business problems
- applications of OR models to specific business problem areas
- summaries of operations research methodology or steps that are involved in a specific OR model
- questions, model formulation exercises, and mathematical exercises at the end of each chapter, with answers to the latter (Appendix E)

These pedagogical tools also assist the instructor whose time may be somewhat limited, because of large classes.

The structure of the book follows a logical sequence for presenting a comprehensive treatment of standard OR techniques. Part I, focuses on an overview of operations research and model formulation; Part II presents traditional business models; namely, breakeven and inventory, that rely on algebra for their solution. Decision theory, decision models treating a variable demand, and PERT (Program Evaluation and Review Technique) models that utilize probability and statistics are set forth in Part III. In Part IV, the underlying mathematics of matrix algebra is employed for linear programming, transportation methods, dynamic programming, and Markov analysis. Due to the importance of simulation techniques, queuing and simulation models are presented in Part V. Lastly, advanced operations research topics and an overview of the present and future directions of OR make up Part VI. In addition, background on vectors, matrices,

and determinants along with a calculus approach to selected inventory and queuing models are found in Appendix B. Thus, the level of mathematics builds from one chapter to another.

The book is designed for a one-quarter or one-semester undergraduate course where calculus may or may not be required. Chapters can be omitted without destroying the unity of the course since each chapter has been written to stand on its own. This allows for a wide variability of emphasis for each major part covered. Also, several of the problems can be solved by the computer, either in a batch-processing mode or a time-sharing mode.

For a project of this magnitude, I am indebted to many individuals who have helped, in particular, Professor Elwood S. Buffa, Advisory Editor of the Wiley/Hamilton Series in Management and Administration. Likewise, I am deeply grateful to Professors Richard Hall, Robert Klekamp, Marcia Ruwe, and Michael Thierauf of Xavier University for their constructive criticism. Also, I would like to thank the many students who participated in this project, in particular, Herbert Banks, John Benny, Donald Blose, B. Bossard, R. J. Gabelman, Steve Jarvis, Malven R. Johnson, Robert C. Monteith, J. N. Platt, and J. Toron.

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Part I

Overview of Operations Research

OPERATIONS RESEARCH—
AN INTRODUCTION

Operations Research—An Introduction

Introduction Objectives

- ☐ To survey the history of operations research and its relationship to computers.
- ☐ To present a method for formulating standard and custom-made OR (operations research) models.
- ☐ To examine the essential characteristics of operations research as a means of defining it.
- ☐ To formulate the six steps in the planned approach for undertaking (and resolving) operations research projects.
- ☐ To set the stage for the rest of the text by surveying the standard "tools of the trade" employed by OR practitioners.

Introduction Outline

History of Operations Research
Computers and Operations Research
Model Defined
Model Formulation
Role of Models in OR Projects
Advantages of Models
Disadvantages of Models
Major Characteristics of Operations Research
Operations Research Defined
Quantitative Models to be Covered
Introduction Summary
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In managing the affairs of modern business and government, managers need considerable assistance in coping with the complexity of their jobs. Unaided, the human mind cannot possibly weigh the manifold complexities involved in the development of a missile, the erection of a large office building or the operation of an enterprise producing hundreds of products for diverse customer needs. A multitude of decisions go into scheduling jobs, ordering supplies, managing inventories, negotiating with contractors, hiring labor, pricing goods and planning production facilities. Furthermore, managers are constrained by such uncertainties as the unpredictable tastes of consumers, the speculative nature of economic forecasts and research and development programs. Thus, all too often, they must act largely on hunches and intuition, wondering if the best decision was made.

In recent years, operations researchers have been showing managers how to avoid some of the perplexity that relates to decision making. They have developed various mathematical or operations research (OR) techniques for evaluating possible courses of action. Some of these techniques are best suited to situations in which all the factors are known or predictable, but the complexity is so great that the human mind cannot arrive at a wholly rational decision. Other techniques cope with "risks"—chances that can be accurately measured or calculated from past experience. Still others deal with "uncertainties" (which operations researchers distinguish from risks)—chances that can be estimated only roughly at best, because they depend on future events which cannot be controlled by the manager. Whatever the orientation of these OR techniques, their central focus is to improve the quality of the manager's final decision.

Although this book is devoted to the standard "tools of the trade" found in OR projects, this initial part serves as an introduction to operations research. First, the history of operations research and the essential aspects of model building are set forth. Next, the major characteristics of operations research are enumerated, followed by a definition of operations research. Also, the quantitative models to be covered are briefly noted. In essence, the main thrust of this introduction is to assist the reader in relating the essentials of model building to the requirements for formulating specific OR models found in the remainder of the text.

History of Operations Research

It is difficult to mark the official beginning of operations research. Many early pioneers performed work that today would be considered operations research. In England, as early as 1914, F. W. Lanchester published papers on the theoretical relationships between victory and superiority in manpower and firepower. In the United States, during World War I, Thomas Edison was given the task of finding the maneuvers of merchant ships that would be most effective in minimizing shipping losses to enemy submarines. Instead of risking ships in actual war conditions, he made use of a "tactical game board" for a solution. Around the same time (late 1910s), a Danish engineer, A. K. Erlang of the Copenhagen Telephone Company, was performing experiments involving the fluctuation of demand for telephone facilities upon automatic dialing equipment. His work is the foundation for mathematical models used in waiting line theory today.

In the 1930s, Horace C. Levenson applied sophisticated mathematical models to large amounts of data which would otherwise have been totally unmanageable. One of his most interesting and best-known studies involved customers refusing to accept C.O.D. packages from a relatively small order house. The rejection rate was about 30 percent of gross sales. Two categories of merchandise were most frequently rejected: more expensive orders, and merchandise shipped later than five days after the order was placed. On the average, orders older than five days were not profitable. With such data available, it was relatively easy for the mail order firm to compare the cost of rejection with the higher cost of fast shipping and thereby determine the optimum shipping effort.

Military Operations Research In 1937, British scientists were asked to help the military learn how to use the newly developed radar in locating enemy aircraft. The scientists, working on different aspects of this problem, were brought together