Heinz-Otto Peitgen Dietmar Saupe Editors

The Science of Fractal Images

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With Contributions by
Yuval Fisher Michael McGuire

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With 142 Illustrations in 277 Parts and 39 Color Plates

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Preface

This book is based on notes for the course *Fractals:Introduction, Basics and Perspectives* given by Michael F. Barnsley, Robert L. Devaney, Heinz-Otto Peitgen, Dietmar Saupe and Richard F. Voss. The course was chaired by Heinz-Otto Peitgen and was part of the SIGGRAPH '87 (Anaheim, California) course program. Though the five chapters of this book have emerged from those courses we have tried to make this book a coherent and uniformly styled presentation as much as possible. It is the first book which discusses fractals solely from the point of view of computer graphics. Though fundamental concepts and algorithms are not introduced and discussed in mathematical rigor we have made a serious attempt to justify and motivate wherever it appeared to be desirable. Basic algorithms are typically presented in pseudo-code or a description so close to code that a reader who is familiar with elementary computer graphics should find no problem to get started.

Mandelbrot's fractal geometry provides both a description and a mathematical model for many of the seemingly complex forms and patterns in nature and the sciences. Fractals have blossomed enormously in the past few years and have helped reconnect pure mathematics research with both natural sciences and computing. Computer graphics has played an essential role both in its development and rapidly growing popularity. Conversely, fractal geometry now plays an important role in the rendering, modelling and animation of natural phenomena and fantastic shapes in computer graphics.

We are proud and grateful that Benoit B. Mandelbrot agreed to write a detailed foreword for our book. In these beautiful notes the *Father of Fractals* shares with us some of the computer graphical history of fractals.

The five chapters of our book cover :

- an introduction to the basic axioms of fractals and their applications in the natural sciences,
- a survey of random fractals together with many pseudo codes for selected algorithms,
- an introduction into fantastic fractals, such as the Mandelbrot set, Julia sets and various chaotic attractors, together with a detailed discussion of algorithms,
- fractal modelling of real world objects.

Chapters 1 and 2 are devoted to random fractals. While Chapter 1 also gives an introduction to the basic concepts and the scientific potential of fractals, Chapter 2 is essentially devoted to algorithms and their mathematical background. Chapters 3, 4 and 5 deal with deterministic fractals and develop a dynamical systems point of view. The first part of Chapter 3 serves as an introduction to Chapters 4 and 5, and also describes some links to the recent chaos theory.

The Appendix of our book has four parts. In Appendix A Benoit B. Mandelbrot contributes some of his brand new ideas to create random fractals which are directed towards the simulation of landscapes, including mountains and rivers. In Appendix B we present a collection of magnificent photographs created and introduced by Michael Mc Guire, who works in the tradition of Ansel Adams. The other two appendices were added at the last minute. In Appendix C Dietmar Saupe provides a short introduction to rewriting systems, which are used for the modelling of branching patterns of plants and the drawing of classic fractal curves. These are topics which are otherwise not covered in this book but certainly have their place in the computer graphics of fractals. The final Appendix D by Yuval Fisher from Cornell University shares with us the fundamentals of a new algorithm for the Mandelbrot set which is very efficient and therefore has potential to become popular for PC based experiments.

Almost throughout the book we provide selected pseudo codes for the most fundamental algorithms being developed and discussed, some of them for beginning and some others for advanced readers. These codes are intended to illustrate the methods and to help with a first implementation, therefore they are not optimized for speed.

The center of the book displays 39 color plates which exemplify the potential of the algorithms discussed in the book. They are referred to in the text as *Plate* followed by a single number N. Color plate captions are found on the pages immediately preceding and following the color work. There we also describe the front and back cover images of the book. All black and white figures are listed as *Figure* $N.M$. Here N refers to the chapter number and M is a running number within the chapter.

After our first publication in the *Scientific American*, August 1985, the Mandelbrot set has become one of the brightest stars of amateur mathematics. Since then we have received numerous mailings from enthusiasts around the world.

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Foreword

People and events behind the "Science of Fractal Images"

Benoit B.Mandelbrot

It is a delight to watch Heinz-Otto Peitgen get together with several of our mutual friends, and tell the world the secrets of drawing fractals on the computer. A pedant would of course proclaim that the very first publication in each branch of fractals had immediately revealed *every* secret that matters. So let me rephrase what was said: this book's goal is to tell the world how to draw the basic fractals without painfully rediscovering what is already known.

The book needs no foreword, but being asked to provide one, without limitation of space, has unleashed a flood of recollections about some Men and some Ideas involved in the *Science of Fractal Images*, including both Art for Art's sake and Art for the sake of Science. A few of these recollections may even qualify as history, or perhaps only as what the French call *la petite histoire*. As some readers may already know, for me history is forever part of the present.

Perhaps as a reward for holding this belief, the very fact of writing down for this book my recollections concerning fractal forgery of landscapes has made me actually unhappy again about a feature of all past fractal forgeries, that they fail to combine relief with rivers. Eventually, we did something about this defect, as well as about other features of the subdivision forgeries described in the body of this book. The new directions are sketched in Appendix A and were added to this book at the last minute.

0.1 The prehistory of some fractals-to-be: Poincaré, Fricke, Klein and Escher

To begin, while fractal geometry dates from 1975, it is important in many ways to know that a number of shapes now counted as fractals have been known for a long time. But surprisingly few had actually been drawn before the computer era. Most were self-similar or self-affine and represent the artless work of the draftsmen on the payroll of science publishers. Also, there are renditions of physical and simulated Brownian motion in the book by Jean Perrin, *Les Atomes*, and William Feller's *Introduction to Probability*. These renditions have helped me dream in fruitful ways (as told in my 1982 book *The Fractal Geometry of Nature* [68] p. 240), but they are not beautiful. Fractals-to-be occur in the work of Fatou and Julia circa 1918, but they led to no illustration in their time.

However, Poincaré's even earlier works circa 1890 do include many sketches, and two very different nice stories are linked with illustrations that appeared shortly afterwards, in the classic book titled *Vorlesungen über die Theorie der automorphen Funktionen* [43], which Fricke & Klein published in 1897. This book's text and its preface are by the hand of Fricke, R. Robert Fricke, but (see p. vi) the great Felix Klein, "a teacher and dear friend" seems to have graciously consented to having his name added on the title page. The illustrations became even more famous than the text. They have been endlessly reproduced in books on mathematics, and for the better or for the worse have affected the intuition of countless mathematicians.

A tenacious legend claims that students in industrial drawing at the Technische Hochschule in Braunschweig, where Fricke was teaching mathematics, drew these figures as assignment, or perhaps even as an exam. Unkind words have been written about some of the results. In fact, I have done my share in detailing the defects of those which claim to represent the fractal-to-be limit sets of certain Kleinian groups (leading some to wonder which of Fricke's students should be failed posthumously). These dubious figures were drawn with the help of the original algorithm of Poincaré, which is very slow, too slow even for the computer. However, my paper [70] in *The Mathematical Intelligencer* in 1983 has given an explicit and quick new algorithm for constructing such limit sets, as the complements of certain " sigma-discs", and has compared Fricke's Figure 156 with the actual shape drawn by a computer program using the new algorithm. The comparison is summarized in *The Fractal Geometry of Nature*, page 179. As was to be expected, the actual shape is by far the more detailed and refined of the two, but this is not all: against all expectations, it is *not* nec-

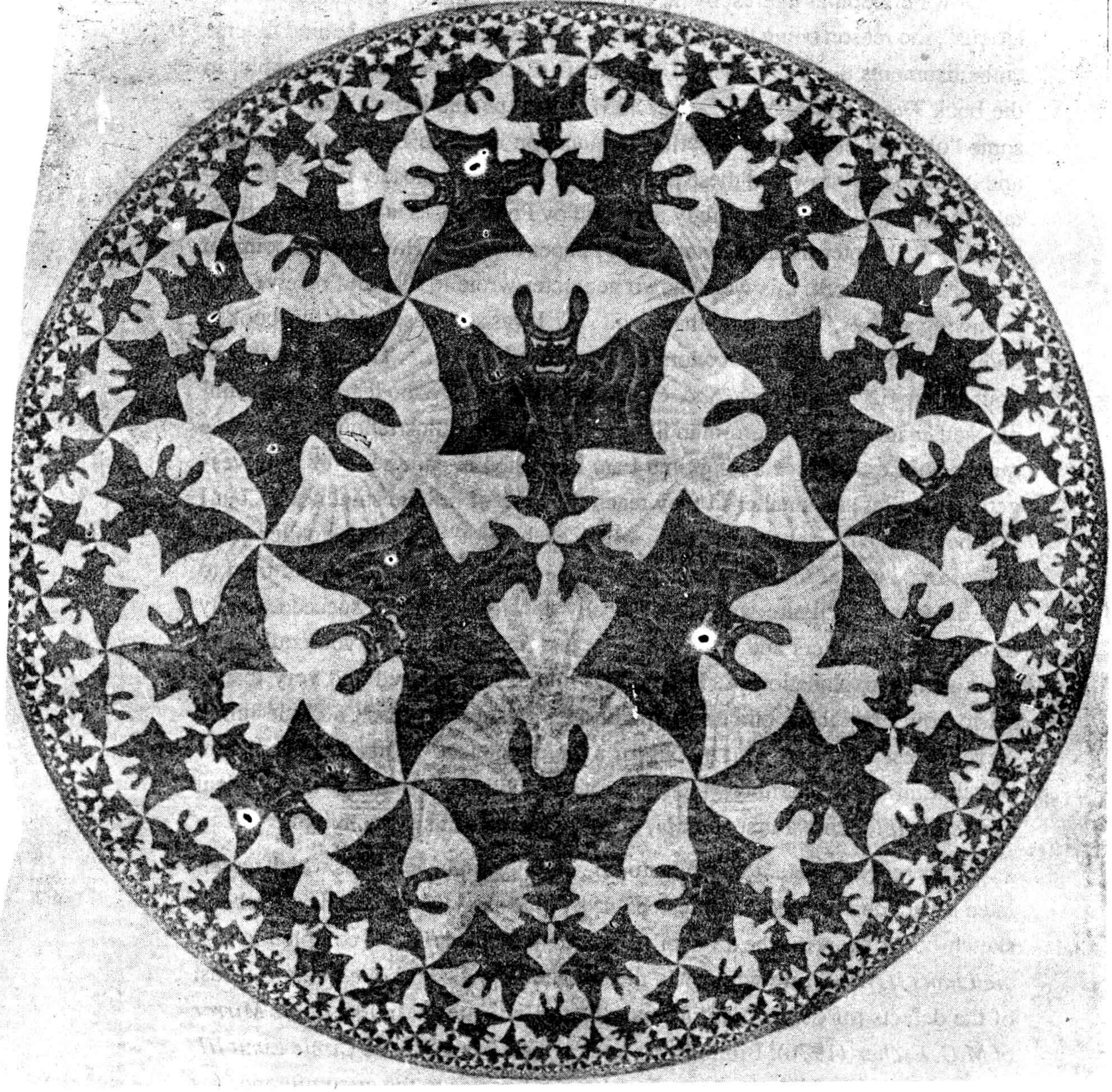

Fig. 0.1: Circle Limits IV by M.C. Escher, ©1988 M.C. Escher c/o Cordon Art – Baarn – Holland

essarily perceived as being more complicated. I feel it is more harmonious, and can be comprehended as a whole, therefore it is perceived as far simpler than the clumsy old pictures. However, a famous mathematician (15 years my senior) has expressed dismay at seeing the still vibrant foundation of his intuition knocked down by a mere machine.

Of wider popular interest by far are Fricke's drawings of "hyperbolic tessellations", the reason being that they have become widely popular behind diverse embellishments due to the pen of Maurits C. Escher, as seen, for example, in the book *The World of M.C. Escher* [33]. Many people immediately perceive some "obvious but hard to describe" connection between Escher and fractals, and it is good to know that these tessellations are indeed closely related to fractals. In fact, they were knowingly triggered by Poincaré, as is well documented by H.S.M. Coxeter in his *Leonardo* [22] paper of 1979. Having seen some of Escher's early work, this well-known geometer wrote to him and received the following answer: "Did I ever thank you . . . ? I was so pleased with this booklet and proud of the two reproductions of my plane patterns!. . . Though the text of your article [in *Trans. Royal Soc. Canada*, 1957] is much too learned for a simple, self-made plane pattern-man like me, some of the illustrations . . . gave me quite a shock. . . . Since a long time I am interested in patterns with "motives" getting smaller and smaller til they reach the limit of infinite smallness. . . but I was never able to make a pattern in which each "blot" is getting smaller gradually from a center towards the outside circle-limit, as [you] show. . . . I tried to find out how this figure was geometrically constructed, but I succeeded only in finding the centers and radii of the largest inner-circles. If you could give me a simple explanation. . . , I should be immensely pleased and very thankful to you! Are there other systems besides this one to reach a circle-limit? Nevertheless,. . . I used your model for a large woodcut". This was his picture 'Circle Limit I', concerning which he wrote on another occasion: "This woodcut *Circle Limit I*, being a first attempt, displays all sorts of shortcomings".

In his reply, Coxeter told Escher of the infinitely many patterns which tessellated a Euclidean or non-Euclidean plane by black and white triangles. Escher's sketch-books show that he diligently pursued these ideas before completing *Circle Limits II, III, IV.* He wrote: "In the coloured woodcut *Circle Limit III* most of the defects [of *Circle Limit I*], have been eliminated". In his *Magic Mirror of M.C. Escher* (1976), Bruno Ernst wrote: "best of the four is *Circle Limit III*, dated 1959. . . In addition to arcs placed at right angles to the circumference (as they ought to be), there are also some arcs that are not so placed". [Now going back to Coxeter], "In fact all the white arcs 'ought' to cut the circumference at the same angle, namely 80° (which they do, with remarkable accuracy). Thus Escher's work, based on his intuition, without any computation, is perfect, even though his poetic description of it was only approximate".

The reader is encouraged to read Coxeter's paper beyond these brief quotes, but an important lesson remains. As already stated, the Coxeter pictures which made Escher adopt the style for which he became famous, hence eventually affected the esthetics of many of our contemporaries, were not the pure creation of an artist's mind. They came straight from Fricke & Klein, they were largely inspired by Henri Poincaré, and they belong to the same geometric universe as fractals. Note also that the preceding story is one of only two in this paper to involve a person who had been professionally trained as an artist.

0.2 Fractals at IBM

The first steps of the development of a systematic fractal geometry, including its graphic aspects, were taken at the IBM T.J. Watson Research Center, or wherever I happened to be visiting from this IBM base. The next task, therefore, in historical sequence, is to reminisce about the IBM fractals project.

This project has always been an example of very small science, in fact it had reduced to myself for the first ten of my thirty years at IBM. Since then, it has in principle included one full-time programmer; actually, there were short overlaps and long periods with no programmer. The assistants of J.M. Berger (whom I had "borrowed' in 1962), as well as my project's first assistant, Hirsh Lewitan, were "career" IBM employees, but all the others were recent graduates or even students on short contract. Here is a complete chronological list of those who stayed for over a few weeks: G.B.Lichtenberger (part-time), M.S.Taqqu, J.L.Oneto, S.W.Handelman, M.R.Laff, P.Moldave (part-time), D.M.McKenna, J.A.Given, E.Hironaka, L.Seiter, F.Guder, R.Gagné and K. Musgrave. The grant of IBM Fellowship in 1974 also brought a half-time secretary: H.C.Dietrich, then J.T. Riznychok, and later V.Singh, and today L.Vasta is my full-time secretary.

R.F.Voss has been since 1975 an invaluable friend, and (as I shall tell momentarily) a close companion when he was not busy with his low-temperature physics. The mathematicians J.Peyriére, J.Hawkes and V.A.Norton, and the meteorologist S. Lovejoy (intermittently) have been post-doctoral visitors for a year or two each, and two "IBM'ers", the hydrologist J.R.Wallis and the linguist F.J.Damerau, have spent short periods as de facto inter-project visitors. As for equipment, beyond slide projectors, terminals and P.C.'s and (of course) a good but not lavish allotment of computer cycles, my project has owned one high-quality film recorder since 1983. Naturally, a few IBM colleagues outside of my project have also on occasion briefly worked on fractals.

These very short lists are worth detailing, because of inquiries that started coming in early in 1986, when it was asserted in print, with no intent to praise, that "IBM has spent on fractals a perceptible proportion of its whole research budget". The alumni of the project are surprised, but endlessly proud, that the bizarre perception that fractals ever became big science at IBM should be so widely accepted in good faith. But the almost threadbare truth is even more interesting to many observers of today's scientific scene. To accept it, and to find it deserving gratitude, was the price paid for academic freedom from academia.

The shortness of these lists spanning twenty years of the thirty since I joined IBM also explains my boundless gratitude for those few people.

0.3 The fractal mountains by R.F. Voss

My next and very pleasant task is to tell how I met the co-authors of this book, and some other people who matter for the story of the *Science of Fractal Images*.

During the spring of 1975, Richard F. Voss was hopping across the USA in search of the right job. He was soon to become Dr. Voss, on the basis of a Berkeley dissertation whose contents ranged from electronics to music, without ever having to leave the study of a widespread physical phenomenon (totally baffling then, and almost equally baffling today), called $\frac{1}{f}$-*noise*. Other aspects of this noise, all involving fractals, were favorites of mine since 1963, and my book *Les objets fractals,* which was to be issued in June 1975, was to contain primitive fractal mountains based on a generalization of $\frac{1}{f}$-noise from curves to surfaces. One of the more striking parts of Voss's thesis concerned (composed) music, which he discovered had many facets involving $\frac{1}{f}$-noises. He had even based a micro-cantata on the historical record of Nile river discharges, a topic dear to my heart.

Therefore, Voss and I spoke after his job-hunting talk at IBM Yorktown, and I offered a deal: come here and let us play together; something really nice is bound to come out. He did join the Yorktown low-temperature group and we soon became close co-workers and friends. Contrary to what is widely taken for granted, he never joined my tiny project, and he has spent the bulk of his time on experimental physics. Nevertheless, his contribution to fractals came at a critical juncture, and it has been absolutely essential. First, we talked about writing a book on $\frac{1}{f}$-noise, but this project never took off (and no one else has carried it out, to my knowledge). Indeed, each time he dropped by to work together, he found me involved with something very different, namely, translating and revising *Les objets fractals*. The end result came out in 1977 as *Fractals,* and preparing it kept raising graphics problems. Voss ceaselessly inquired about what Sig Handelman and I were doing, and kept asking whether we would consider better ways, and then he found a sure way of obtaining our full attention.

He conjured a computer graphics system where none was supposed to exist, and brought along pictures of fractals that were way above what we had been dealing with until then. They appeared in *Fractals*, which is why the foreword describes him as the co-author of the pictures in that book.

Color came late at Yorktown, where it seems we fractalists continued to be the only ones to use demanding graphics in our work. We first used color in my next book, the 1982 *Fractal Geometry of Nature*. In late 1981, the text was already in the press, but the color pictures had not yet been delivered to the publishers. The film recorder we were using was ours on a short lease, and this fact and everything else was conspiring to make us rush, but I fought back. Since "the desire is boundless and the act a slave to limit" ([68], p. 38), I fought hardest for the sake of the *Fractal Planetrise* on the book's jacket. It was soon refined to what (by the standards of 1981) was perfection, but this was not enough. Just another day's work, or another week's, I pleaded, and we shall achieve something that would not need any further improvement, that would not have to be touched up again when "graphic lo-fi" will go away, to be replaced by "graphic hi-fi". To our delight, this fitted Voss just fine.

Fractal illustrations had started as wholly unitarian, the perceived beauty of the old ones by Jean-Louis Oneto and Sig Handelman being an unexpected and unearned bonus. But by 1981 their beauty had matured and it deserved respect, even from us hard scientists, and it deserved a gift of our time. Many people have, since those days, showed me their fractal pictures by the hundreds, but I would have been happier in most cases with fewer carefully worked out ones.

Everyone experiences wonder at Voss's pictures, and "to see [them] is to believe [in fractal geometry]". Specialists also wonder how these pictures were done, because, without ever drawing specific attention to the fact, Voss has repeatedly conjured technical tricks that were equivalent to computer graphics procedures that did not officially develop until much later. This brings to mind a philosophical remark.

Watching Voss the computer artist and Voss the physicist at work for many years had kept reminding me of the need for a fruitful stress between the social and the private aspects of being a scientist. The only civilized way of being a scientist is to engage in *the process* of doing science primarily for one's *private pleasure* . To derive pleasure from the *public results* of this process is a much more common and entirely different matter. The well-known danger is that, while *dilettare* means *to delight* in Italian, its derivative *dilettante* is a term of contempt. While not a few individually profess to be serious scientists, yet motivated primarily by personal enjoyment of their work, very few could provide