CONTENTS

SURVEY

By Arthur von Hippel

The book begins with an orienting look at the procedures of classical science and engineering (Chap. 1). Here we find axiomatic facts and their quantitative formulation, matter classified by macroscopic criteria, and materials described by numerical coefficients of response to external probing. Mathematical logic takes over and covers this framework with a theory of streamlined elegance; inquiry gives way to application until the horizon proves too limited, and new adventure begins. In our specific discussion we refer to electrical science and engineering because questions about the electrical structure of matter lead to the particles of the molecular world.

Chapter 2 introduces the atoms with their electron clouds of standing-wave modes described by quantum mechanics, the response of atoms to electric and magnetic fields, causing the Stark and Zeeman effects, and the gyroscopic nature of magnetic moments. A dilemma comes into focus with the build-up scheme of the periodic system using hydrogenlike wave functions Quantum mechanics has to deal with strong interactions between particles and, like classical physics, can handle its multibody problems only by approximation. This fact, added to the principal limitation imposed on observability by the quantum of action, requires critical examination of any mathematical solution as to its actual range of validity. The remainder of Chap 2 sketches the interaction of atoms binding and repulsion effects, release or attachment of electrons, various bond types, the stereostructure and isomerism of molecules permanent electric dipole moments, and the radii of atoms and ions.

This acquaintance with atoms and molecules suffices for a discussion of the electrical behavior of gases (Chap 3) A gas at normal temperature should be an ideal insulator The observation of conduction at low field strength therefore leads to exciting discoveries concerning the initiating charge carriers and their effects In stronger fields, charge-carrier multiplication sets in as described by Townsend's avalanches Breakdown itself requires a feedback mechanism causing instability; a variety of effects may contribute to the sudden collapse of the gas insulation. Space-charge formation contracts the field into a cathode fall, electrons are liberated from the cathode by ion bombardment, photoeffect, the impact of radicals, or by field emission, short-ranging ultraviolet radiation may ionize the gas as an avalanche develops, and so on. Visual evidence is needed to interpret the electrical data. Even the simple turning of a camera provides useful information about the mechanism of sparks and lightning strokes, Kerr cell shutters allow photographing momentary development stages; Lichtenberg figures show in most beautiful detail the unfolding of events The Wilson cloud chamber and, finally, direct recording succeeded in measuring individual avalanches and their progeny

With this preview, the stage is set for the inquiry of Chap 4 concerning our present understanding of thunderstorms and the electrical state of the atmosphere This is an age-old problem, with ever new ramifications, in contrast to a very recent field, the microwave discharges. Here, as Chap 5 reports, everything is simplified and well understood Charge generation in the gas balances diffusion, the walls of the container are limiting boundaries instead of electrodes, and a quantitative theory finds quantitative confirmation.

In Chap. 6, the theme of gas discharges is handed over to the molecular engineer for a review of present appli-

cations and future trends. Chapter 7 turns from electrical action to chemical reaction in gases, especially to processes leading by cumulative acceleration to explosion. This crescendo ends our preoccupation with gases.

Chapter 8 broadens the basis for a general understanding of all states of aggregation. Thermodynamic systems, their macroscopic treatment and statistical interpretation, come into focus. The idealized models of the perfect gas, the perfect electron gas and the perfect phonon gas, leading to Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein statistics, are contrasted with the perfect crystal embodying complete order and with the imperfect gas leading to phase equilibria. The problem of nucleation arises, and the extension of thermodynamics to systems containing electric and magnetic energy is indicated. This long-distance view is complemented in Chap. 9 by a close-up inquiry about the actual building laws for condensed phases. In sequence there appear the results of macroscopic and molecular crystallography; the concepts of dense packing, radius ratio, and interstitial position in their application to a variety of crystal structures; the transition from finite to infinite molecules in lattices; the building of structures from tetrahedral and octahedral groups; polymers and networks showing all gradations between short-range and long-range order; and the imperfections inherent in ordered systems.

In the subsequent five chapters, a fill-in and real mixing of diversified knowledge takes place. First the chemist speaks about our present insight into successive steps of reaction in liquid systems (Chap. 10). Then the physical chemist takes over in a review of polymer formation (Chap. 11), followed by the physicist with a detailed report on crystal growth, the techniques used and results achieved (Chap. 12). After this account of the formation of structures, the discussion shifts to disturbances of structure caused in solids by irradiation (Chap. 13) and by mechanical stress (Chap. 14). These five chapters survey a Disneyland of molecular action: from molecules, turning inside out like umbrellas, to nuclear shooting galleries and to dislocation mills spinning crystals to destruction. The reader should not despair if it takes time to visualize this molecular spectacle.

With Chap. 15, we return to the world of electric and magnetic phenomena. Electric and magnetic dipoles are considered in parallel presentation: First their individual contribution to polarization and magnetization, for static fields and in resonance or relaxation response, then coupled dipole systems come under scrutiny. Short-range energy terms prove decisive, not long-range dipole forces, in producing piezoelectrics, ferroelectrics, ferromagnetics, and their antipodes. The principles that make their behavior understandable are discussed; with this background secured, the attention turns to present-day ferroelectric and ferromagnetic materials and devices (Chaps. 16 to 19). Here the molecular scientist, building crystal structures, and the molecular engineer, searching for new applications, come into their own. Chapter 20 finishes the dipole story by presenting the currently most provocative devices: the masers, parametric amplifiers, and thin-film magnetic memories, and by giving some speculative implications for the future.

Once more the spotlight swings to a new subject: the action of charge carriers in liquids and solids. A surveying section (Chap. 21) provides general briefing on charge-carrier stabilization in condensed systems; on ions, their surroundings and motion in liquids and crystals; on electrons in crystals and their behavior under the rules of quantum mechanics. Chapters 22 to 24 follow up with subjects of special interest. The first two sections discuss ions in peculiar and related situations: as polyelectrolytes, i.e., galaxies of cations or anions appended to polymer molecules and immersed in solutions of compensating countercharges, and as ion-exchange resins, which are actually polyelectrolytes frozen into a plastic framework. Chapter 24 analyzes the action of the most important and best understood semiconductor devices: rectifier diodes and transistors.

A short closing chapter focuses attention on the challenges of this space age and calls molecular engineering to the rescue.

1 · MACROSCOPIC LAWS AND MOLECULAR INTERPRETATIONS

By Arthur von Hippel

Materials and macroscopic theory — Classical concepts of electrical science and engineering — Molecular portraits of C, R, and L — Complex permittivity and permeability — Electromagnetic field and matter — Testing of materials

Materials and macroscopic theory

Mathematics and art can live in a world of abstract symbols, but science and engineering draw their substance from our material surroundings. The history of civilization can be written in terms of the transformation and use of materials. Until recently this conversion of nature's resources has been an empirical one. Materials were found, their macroscopic properties evaluated, their chemical constituents analyzed, and applications made after proper shaping and processing. The results of macroscopic tests were tabulated in handbooks for ready reference. Matter thus became classified according to its macroscopic characteristics, as insulating or conducting, hard or soft, strong or weak, brittle or plastic, opaque or transparent, etc., with numerical coefficients establishing the range of such properties.

The prerequisite of this cataloguing effort was the formulation of macroscopic laws through which the coefficients could acquire accurate meaning. Galileo (1564–1642) originated the scientific method of combining experimental inquiry with mathematical deduction, which has guided us ever since. His keen observation of weights falling along inclined planes established the fact that not motion itself but change of motion requires external force. This insight and Galileo's astounding discoveries with the telescope[1] brought the Copernican system to practical test and final victory.[2] The angels supposed to push the planets through their orbits (Fig. 1.1) were dethroned by the force of gravitation. And thus the principle became established that facts have to be accepted, irrespective of the human desire to make nature conform to preconceived rationalization. Newton's mechanics[3] resulted with its quantitative concept of force, mass, and later of energy. Ambition to construct a perpetuum mobile gave way to the recognition that heat is a form of energy[4] and is of limited convertibility.[5] The conservation laws of energy and momentum assumed new significance in the frameworks of thermodynamics and statistics. Experiments with Leyden jars and lightning rods, frogs' legs and compass needles ushered in the age of electricity. Electrical and optical concepts culminated in Maxwell's equations of the electromagnetic field.[6]

Thus we learned how macroscopic systems respond to external manipulation. Matter became characterized by coefficients of elasticity, viscosity, conductivity,

[1] Galileo Galilei, *Sidereus Nuncius*, Venice, 1610; *Dialogue*, Florence, 1632.

[2] For a penetrating account of this struggle cf. G. de Santillana, *The Crime of Galileo*, The University of Chicago Press, Chicago, 1955.

[3] Isaac Newton, *Principia*, London, 1687.

[4] J. R. Mayer, "Bemerkungen über die Kräfte der unbelebten Natur," *Liebig's Annalen 42*, 233 (1842); B. Thompson (Count Rumford), "Enquiry Concerning the Source of Heat," *Phil. Trans. Roy. Soc. (London)*, 1798.

[5] N. L. S. Carnot, *Réflexions sur la puissance motrice du feu*, Bachelier, Paris, 1824.

[6] J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Clarendon Press, Oxford, 1873.

Fig. 1.1. Angel propelling globe (Tarocchi playing-card engraving, anonymous, North Italy, 15th century. (Courtesy of Museum of Fine Arts, Boston.)

permittivity, permeability, etc. This integrating kind of information by itself leads only to relatively simple surmises about underlying microscopic events. Decades of study of elementary particles and their interaction in gases, liquids, and solids were required before the exciting possibilities became apparent of shaping properties and directing action by molecular strategy. It is therefore not surprising that the engineer, oblivious of the microcosm at this stage, concentrated on macroscopic laws and their applications.

A short discussion of the classical concepts of electrical science and engineering may illustrate how macroscopic theory built up its amazing edifice of logical thinking and why its camouflaged molecular content failed to challenge the imagination. At the same time, this survey provides a logical introduction to the main theme of our book, since the properties of the molecular world arise through the actions of electric charges.

Classical concepts of electrical science and engineering[7]

At the outset, we encounter experimental facts defying, as yet, any deeper explanation:

(1) Positive and negative *electric charges* exist and can be identified qualitatively and quantitatively by their force of interaction $\mathbf{F}$ with a detector charge Q'. The *force law*

$$\mathbf{F} = Q'\mathbf{E} \tag{1.1}$$

leads to the concept of an electric field of the strength $\mathbf{E}$, generated by charges. The total electric flux emerging from a closed surface A measures the uncompensated charge contained in the enclosed volume (*Gauss's law*); hence a charge Q placed in vacuo (dielectric constant or permittivity ϵ_0) can be measured by the surface integral

$$\oint \epsilon_0 \mathbf{E}\cdot\mathbf{n}\, dA = Q, \tag{1.2}$$

with $\mathbf{n}$ a unit vector normal to A. For a charge Q spread uniformly over a sphere of radius r, *Coulomb's law* results:

$$\mathbf{E} = \frac{Q}{\epsilon_0 4\pi r^2}\mathbf{r}^0, \qquad -\int_\infty^r \mathbf{E}\cdot d\mathbf{r} \equiv \varphi = \frac{Q}{\epsilon_0 4\pi r}, \qquad \mathbf{F} = \frac{Q'Q}{\epsilon_0 4\pi r^2}\mathbf{r}^0, \tag{1.3}$$

the force law (Eq. 1.1) for spherical geometry; $\mathbf{r}^0$ signifies a radial unit vector.

(2) Charges NQ traversing an area A with a velocity $\mathbf{v}$ represent an electric current I of the density $\mathbf{J}$,

$$I = \int \mathbf{J}\cdot\mathbf{n}\, dA = \int NQ\mathbf{v}\cdot\mathbf{n}\, dA; \tag{1.4}$$

such a current is surrounded by a *circular magnetic field* of the strength $\mathbf{H}$ (Fig. 1.2),

$$\oint \mathbf{H}\cdot d\boldsymbol{l} = I \ (\textit{Ampère's circuital law}).^8 \tag{1.5}$$

[7] A systematic development of the macroscopic theory may be found in A. von Hippel, *Dielectrics and Waves*, John Wiley and Sons, New York, 1954.

[8] This is a *relativistic* phenomenon: A charge moving relative to the observer is surrounded by a magnetic field. Hence, in place of Ampère's circuital law we could introduce relativity theory.

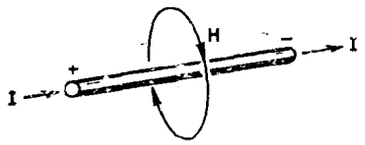

Fig. 1.2. Magnetic field circling electric current.

(3) The *sources of magnetic fields* are *currents* or *magnetic dipoles;* magnetic monopoles are not observed. A circular current I encircling an area A produces at a

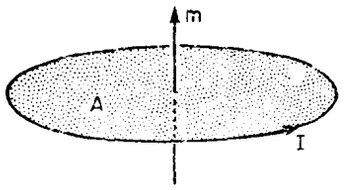

Fig. 1.3. Magnetic dipole created by ring current.

distance, large in comparison to its radius, a magnetic field identical to that of a magnetic dipole of the moment (Fig. 1.3)

$$\mathbf{m} = IA\mathbf{n}. \tag{1.6}$$

Hence, elementary ring currents may be the source of dipolar magnetism.

(4) Magnetic fields can be measured by the torque $\mathbf{T}$ exercised on a detector dipole $\mathbf{m}$; in vacuo (permeability μ_0)

$$\mathbf{T} = \mathbf{m} \times \mu_0\mathbf{H} \equiv \mathbf{m} \times \mathfrak{B}. \tag{1.7}$$

An alternative observation: Moving charges are deflected by a magnetic field perpendicularly to the field and their direction of motion (Fig. 1.4). Accordingly, the force law of Eq. 1.1 can be generalized for charges

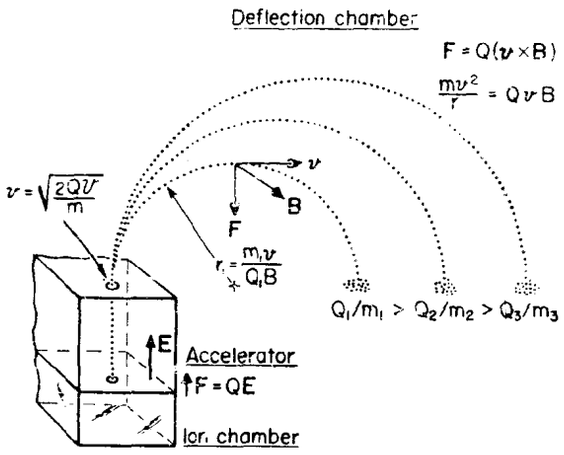

Fig. 1.4. Mass spectrograph, based on Eq. 1.8, separating particles according to their ratio of charge to mass.

Q moving with a velocity v, as

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1.8}$$

where $\mathbf{B}$ is the *magnetic induction* or *magnetic flux density*.

The appearance of the permittivity ϵ_0 in Eq. 1.2 and of the permeability μ_0 in Eq. 1.7 is required by dimensional considerations. In this book, as in the two preceding ones, we use (in addition to temperature) the *four dimensions*: length, mass, time, and electric charge in a rationalized system of primary units:[9] meter [m], kilogram [kg], second [sec] and coulomb [coul]. Equation 1.1 prescribes for the electric field strength the dimension

$$[\mathbf{E}] = \left[\frac{\text{force}}{\text{charge}}\right] = \left[\frac{\text{kg m}}{\text{sec}^2\,\text{coul}}\right] \equiv \left[\frac{\text{volt}}{\text{m}}\right], \tag{1.9}$$

while Eq. 1.2 gives $\epsilon_0\mathbf{E}$ the dimension of surface charge density or [coul/m^2]. Hence, the permittivity fills the role of an adjusting coefficient with the dimensions

$$[\epsilon_0] = \left[\frac{\text{sec}^2\,\text{coul}^2}{\text{kg m}^3}\right]. \tag{1.10}$$

Similarly, Eq. 1.5 defines the dimensions of the magnetic field strength

$$[\mathbf{H}] = \left[\frac{\text{current}}{\text{length}}\right] = \left[\frac{\text{coul}}{\text{sec m}}\right] \equiv \left[\frac{\text{amp}}{\text{m}}\right], \tag{1.11}$$

while Eq. 1.7 prescribes for $\mu_0\mathbf{H}$ the dimension: torque per unit magnetic dipole moment. Equation 1.6 specifies for the magnetic dipole moment the dimensions

$$[\mathbf{m}] = \left[\frac{\text{coul m}^2}{\text{sec}}\right]. \tag{1.12}$$

Thus, magnetic phenomena are traced back to electric phenomena, and the coefficient "permeability" has the dimensions

$$[\mu_0] = \left[\frac{\text{kg m}}{\text{coul}^2}\right]. \tag{1.13}$$

(5) The temporal change of a magnetic flux

$$\Phi = \int_A \mathbf{B}\cdot\mathbf{n}\, dA \tag{1.14}$$

traversing a loop induces an electric voltage along the rim of the loop

$$\int_a^b \mathbf{E}\cdot d\mathbf{l} = \mathfrak{V}_i = -\frac{d\Phi}{dt} \quad (\textit{Faraday's induction law}). \tag{1.15}$$

Thus far we have described the origin of electric and magnetic fields, their force and torque action, and their interrelations. No explicit reference has been made to matter, but it is obvious that *materials* are normally needed to store charges and their countercharges, and to permit current flow. Hence, matter now enters the discussion, in order to realize experimental arrangements based on the preceding laws; simultaneously the effects of electric and magnetic fields on matter come

[9] Cf. Ref. 7, pp. 21 ff.

under scrutiny. The circuit engineer summarizes both aspects in the three observations:

(1) Charges can be stored in capacitors

$$Q = C\mathcal{V},$$

or

$$\mathcal{V} = \frac{1}{C}\int I\,dt. \tag{1.16}$$

(2) Currents flow through resistors

$$\mathcal{V} = RI. \tag{1.17}$$

(3) Current changes create opposing voltages in inductors, hence must be sustained by countervoltages

$$\mathcal{V} = L\frac{dI}{dt}\cdot \tag{1.18}$$

Thus, classical electrical engineering introduces properties of matter through the linear circuit elements: *capacitance* C, *resistance* R (or *conductance* $G = 1/R$), and *inductance* L.

When a sinusoidal voltage

$$\mathcal{V} = \mathcal{V}_0 e^{j\omega t} \tag{1.19}$$

is applied to these circuit elements, charging, loss, and magnetization currents result,

$$\begin{aligned} I_c &= j\omega C\mathcal{V}, \\ I_l &= G\mathcal{V} \equiv \frac{1}{R}\mathcal{V}, \\ I_m &= \frac{1}{j\omega L}\mathcal{V}. \end{aligned} \tag{1.20}$$

They are advanced by 90° in phase and retarded by 90° in relation to the driving voltage, respectively

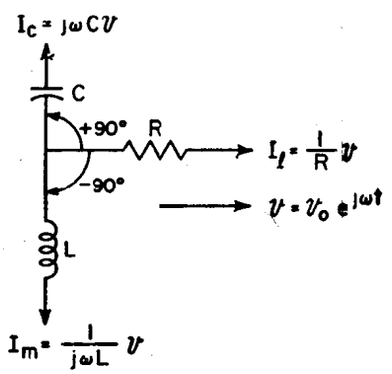

Fig. 1.5. Phase relations of driving voltage $\mathcal{V}$ to charging current I_c, loss current I_l, and magnetization current I_m.

(Fig. 1.5). Hence, by combining these elements into more or less intricate networks, any desired *impedance*

$$Z \equiv \frac{\mathcal{V}}{I} \tag{1.21}$$

may be created. Simultaneously, since the linear circuit elements represent elastance, friction, and acceleration coefficients in the differential equations of electrical engineering, any kind of resonance or relaxation response becomes possible.

The quantities C, G, and L contain a geometrical and a material factor:

$$\begin{aligned} C &= C_0\frac{\epsilon'}{\epsilon_0} \equiv C_0\kappa', \\ G &= \frac{A}{d}\sigma \equiv \frac{A}{d}\frac{1}{\rho}, \\ L &= L_0\frac{\mu'}{\mu_0} \equiv L_0\kappa_m'. \end{aligned} \tag{1.22}$$

Here, ϵ' and ϵ_0 are the *permittivity* (dielectric constant) of the material filling the capacitor and of vacuum, respectively; κ' is the *relative* permittivity. Similarly, μ' is the *permeability* of the medium embedding the coil, μ_0 the permeability of vacuum, and κ_m' the *relative* permeability. Finally, σ is the *conductivity* of the resistor material and its inverse, ρ, the *resistivity*.

The conductance G in Eq. 1.22 is that of a rod of area A and length d. By referring similarly to the geometrical capacitance C_0 of a plate capacitor and to the geometrical inductance L_0 of a long coil of N windings (fringing neglected),

$$\begin{aligned} C_0 &= \frac{A}{d}\epsilon_0, \\ L_0 &= N^2\frac{A}{d}\mu_0, \end{aligned} \tag{1.23}$$

we obtain for these simple geometrical shapes

$$\begin{aligned} C &= \frac{A}{d}\epsilon', \\ G &= \frac{A}{d}\sigma, \\ L &= N^2\frac{A}{d}\mu'. \end{aligned} \tag{1.24}$$

The permittivity ϵ', conductivity σ, and permeability μ' are the real *parameters of matter* which concern the circuit engineer.

Molecular portraits of C, R, and L

The question now arises: Are we still free agents, unbiasedly inquiring about the nature of these properties, or captives committed by the preceding assumptions to a molecular portrait of matter? To decide

this issue, let us reread the equations for their molecular content.

Equations 1.20 and 1.22 prescribe the loss-current density by *Ohm's law*

$$\mathbf{J} = \sigma\mathbf{E}. \tag{1.25}$$

The current density results from the *drift velocity* $\mathbf{v}$ of N charge carriers per unit volume transporting charges e (cf. Eq. 1.4),

$$\mathbf{J} = NQ\mathbf{v}. \tag{1.26}$$

Hence, the drift velocity is proportional to the driving field strength,

$$\begin{aligned} \mathbf{v} &= b\mathbf{E}, \\ \sigma &= NQb, \end{aligned} \tag{1.27}$$

where b is the *mobility* of the charge carriers of the conductor. Thus, our molecular picture of conduction is that charge carriers fall through resistors in an electric field, as spheres fall through oil in the gravitational field; the motion is dominated by friction (*Stokes's law*).

Turning to the capacitance, we rewrite the storage equation (Eq. 1.16) as

$$\mathcal{V} = \frac{Q}{C} = \frac{Q}{\kappa'}\frac{1}{C_0} \tag{1.28}$$

and read: Only a fraction of the total charge, the *free* charge Q/κ', sets up an electric field and thus a voltage $\mathcal{V}$ toward the outside. The remainder, the *bound*

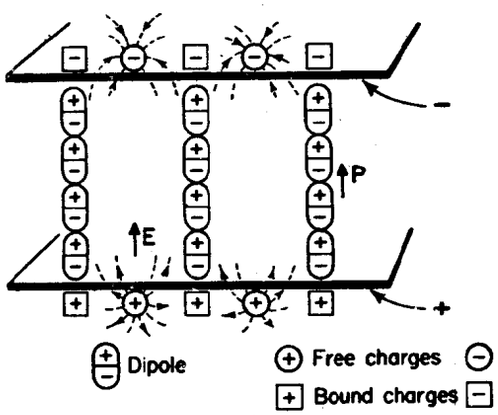

Fig. 1.6. Schematic representation of polarization by dipole chains and bound charges.

charge $Q(1 - 1/\kappa')$, has been neutralized by countercharges, by a *polarization* of the dielectric (Fig. 1.6). The *total electric flux density* $\mathbf{D}$ traversing the material is thus the sum of the densities of electric field lines and dipole chains,

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon'\mathbf{E}. \tag{1.29}$$

The *polarization* $\mathbf{P}$ represents the surface charge density of the bound charge or, alternatively, the dipole moment per unit volume of the material,

$$\mathbf{P} = N\bar{\boldsymbol{\mu}}. \tag{1.30}$$

Thus, the polarization is visualized as created by the appearance of elementary electric dipole moments, N dipoles per unit volume, adding their average moments $\bar{\boldsymbol{\mu}}$.

According to Eqs. 1.20 and 1.24, the charging-current density traversing a plate capacitor is

$$\mathbf{J}_c = j\omega\epsilon'\mathbf{E}. \tag{1.31}$$

The contribution of the dielectric results from the temporal change of its polarization,

$$\frac{d\mathbf{P}}{dt} = j\omega(\epsilon' - \epsilon_0)\mathbf{E} = j\omega(\kappa' - 1)\epsilon_0\mathbf{E} = N\frac{d\bar{\boldsymbol{\mu}}}{dt}. \tag{1.32}$$

Hence, the capacitive behavior of a material is described macroscopically by its *electric susceptibility*

$$\chi \equiv \kappa' - 1 = \frac{\mathbf{P}}{\epsilon_0\mathbf{E}}, \tag{1.33}$$

the ratio of bound to free surface charge density.

Microscopically, the polarization is composed of average moments $\bar{\boldsymbol{\mu}}$ which have to vary proportionally to the electric field strength,

$$\bar{\boldsymbol{\mu}} = \alpha\mathbf{E}, \tag{1.34}$$

because the capacitance and with it the susceptibility are introduced as independent of $\mathbf{E}$. The *polarizability* α measures the electric pliability of the elementary particles in this linear range. The molecular picture requires that the moments follow the field instantaneously, as otherwise the charging current could not be advanced by 90° with respect to voltage. Thus,

$$\frac{d\mathbf{P}}{dt} = j\omega N\alpha\mathbf{E}, \tag{1.35}$$

and

$$\chi = \frac{N\alpha}{\epsilon_0}. \tag{1.36}$$

The influence of matter on the magnetization currents leads to an analogous interpretation. A magnetizing field $\mathbf{H}$ creates magnetic dipole chains in matter. In consequence, the total *magnetic flux density* $\mathbf{B}$ is composed of the densities of field lines and dipole chains as

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu'\mathbf{H}. \tag{1.37}$$

A comparison of the electric and magnetic flux equations (Eqs. 1.29 and 1.37) reveals a discrepancy in formulation caused by historical development and creating a great deal of confusion: Electric flux density $\mathbf{D}$ and polarization $\mathbf{P}$ have the identical dimension of dipole moment per unit volume, and the electric

field strength plays the role of force per unit charge (Eq. 1.9) or torque per unit dipole moment. In the magnetic case, **H** and **M** have the dimension of magnetic dipole moment per unit volume, while the magnetic flux density **B** represents the torque per unit dipole moment (Eq. 1.7).

The magnetic dipole moment per unit volume, the *magnetization*

$$\mathbf{M} = N\overline{\mathbf{m}} = N\alpha_m\mathbf{H}, \tag{1.38}$$

is built up from average elementary magnetic moments $\overline{\mathbf{m}}$. These moments are proportional to the magnetizing field and follow it instantaneously. A material contributes to the induced voltage by its temporal change in magnetization

$$\frac{d\mathbf{M}}{dt} = j\omega\left(\frac{\mu' - \mu_0}{\mu_0}\right)\mathbf{H} = j\omega(\kappa_m' - 1)\mathbf{H}$$
$$= j\omega N\alpha_m\mathbf{H}. \tag{1.39}$$

This effect is measured macroscopically by the *magnetic susceptibility*

$$\chi_m \equiv \kappa_m' - 1 = \frac{\mathbf{M}}{\mathbf{H}} = N\alpha_m \tag{1.40}$$

and described on molecular scale by the *density* N and *magnetizability* α_m of the elementary constituents.

Hence, by introducing the linear circuit elements C, R, and L, one visualizes charge carriers drifting in oil and dipole moments appearing like jack-in-the-boxes at the call of electric and magnetic fields (Fig. 1.7). Assuming that there is nothing further to explore, the electrical engineer refers to tables of permittivities, conductivities, and permeabilities and retreats into his private world of field vectors and equivalent circuits.

Complex permittivity and permeability

Let us break loose at this point and create some chance for adventure. The molecular statements just made are obviously of limited validity. They served the electrical engineer relatively well because of his preoccupation with a limited frequency range, where metals can make ideal resistors and coils, while dielectrics (such as air or mica) approximate ideal capacitors. But dipole moments will need time to appear and disappear; similarly, charge carriers have to be mobilized and accelerated before they settle down to a constant drift velocity, if ever they do.

A time lag between creating field and induced moment causes a phase shift between voltage and current, reducing the phase angle of the polarization

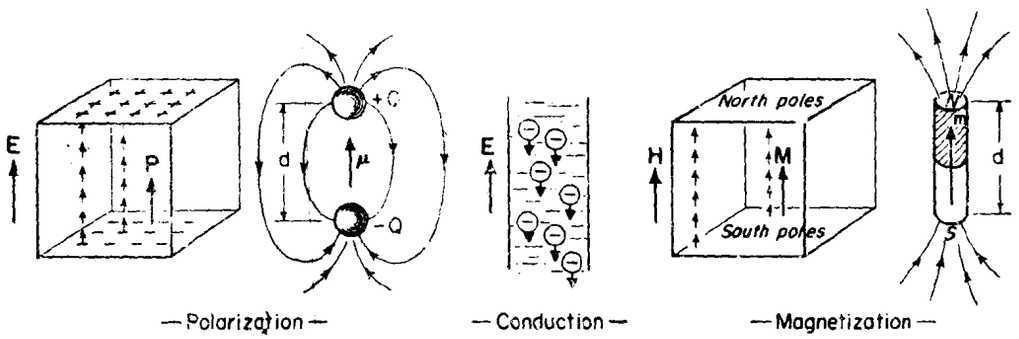

Fig. 1.7. Molecular portraits of polarization, conduction, and magnetization.

and magnetization currents below 90° (Fig. 1.8). Loss currents thus appear that are not caused by charge-carrier migration. The storage of electric and of magnetic energy is usually accompanied by energy dissipation. A mathematical representation of this

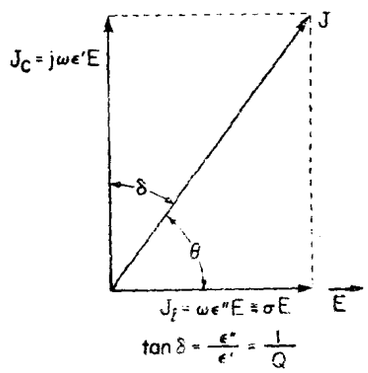

Fig. 1.8. Charging- and loss-current density of polarization current.

fact, without a commitment as to the origin of such loss, is provided by the introduction of a *complex* permittivity and permeability,

$$\epsilon^* = \epsilon' - j\epsilon'',$$
$$\mu^* = \mu' - j\mu''. \tag{1.41}$$

In principle, we should also introduce a complex con-

ductivity, but the *loss factors* ϵ'' and μ'' are able to represent conduction phenomena as well. For example, in the generalized expression for the current density of a plate capacitor (cf. Eq. 1.31)

$$\mathbf{J} = j\omega\epsilon^*\mathbf{E} = j\omega\epsilon'\mathbf{E} + \omega\epsilon''\mathbf{E}, \tag{1.42}$$

the *dielectric conductivity*

$$\sigma = \omega\epsilon'' \tag{1.43}$$

can stand for an actual ohmic conductivity caused by migrating charge carriers as well as for a loss associated with the formation or orientation of electric dipoles. Similarly, the charging current may indicate dipole polarization or result from a field distortion by space-charge formation. Thus nothing would be gained by the additional introduction of a complex conductivity, while the ease of mathematical formulation would be sacrificed.

Electromagnetic field and matter

Maxwell expanded Ampère's circuital law (Eq. 1.5) by stating that not only is a conduction current surrounded by a magnetic field but a temporal change of the electric flux density **D** (Eq. 1.29) in a material also creates a magnetic turbulence, quite analogous to that of a true current of density **J**. This addition of a *displacement current*, demanded by the principle of charge conservation,[10] transforms Ampère's circuital law into the *integral form of Maxwell's first field equation*

$$\oint \mathbf{H}\cdot d\boldsymbol{l} = \int_A \mathbf{J}\cdot\mathbf{n}\, dA + \int_{A'} \frac{\partial \mathbf{D}}{\partial t}\cdot\mathbf{n}\, dA. \tag{1.44}$$

By rewriting Faraday's induction law (Eq. 1.15) as a generalized statement that the temporal change of a magnetic flux density creates an electric turbulence,

$$\oint \mathbf{E}\cdot d\boldsymbol{l} = -\int_A \frac{\partial \mathbf{B}}{\partial t}\cdot\mathbf{n}\, dA, \tag{1.45}$$

we obtain the *integral form of Maxwell's second field equation*. Transformation of the line integral at the left side of these equations into a surface integral (by composing it of differential line integrals around elementary areas according to *Stokes's theorem*[11]) leads to the differential formulation of the field equations

$$\begin{aligned} \nabla\times\mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \\ \nabla\times\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}. \end{aligned} \tag{1.46}$$

[10] Cf. Ref. 7, p. 257.

[11] Cf. Ref. 7, pp. 15 ff.

When dealing with sinusoidal fields and isotropic linear[12] dielectrics, we can introduce the complex material parameters of Eq. 1.41 and arrive at a differential formulation of Maxwell's field equations which is symmetrical save for a negative sign:

$$\begin{aligned} \nabla\times\mathbf{H} &= \epsilon^*\frac{\partial \mathbf{E}}{\partial t}, \\ \nabla\times\mathbf{E} &= -\mu^*\frac{\partial \mathbf{H}}{\partial t}. \end{aligned} \tag{1.47}$$

This new formulation of the field equations actually goes beyond the content of Maxwell's original equations. It states that not only a dielectric charging current but also a dielectric loss current of any origin produces a magnetic turbulence and that not only the change of an inductive magnetization component but also a temporal change of a resistive magnetization component induces an electric turbulence. Hence, for example, it proves possible, by the proper selection of magnetic materials and frequency ranges, to produce transformers of resistive impedance.

By separating the field vectors **E** and **H** in Eqs. 1.47, we arrive at the *wave equations* of the electromagnetic field[13]

$$\begin{aligned} \nabla^2\mathbf{E} &= \epsilon^*\mu^*\frac{\partial^2 \mathbf{E}}{\partial t^2}, \\ \nabla^2\mathbf{H} &= \epsilon^*\mu^*\frac{\partial^2 \mathbf{H}}{\partial t^2}. \end{aligned} \tag{1.48}$$

The solution of these differential equations, if the field vectors depend only on x and t, represents a *plane wave*

$$\begin{aligned} \mathbf{E} &= E_0 e^{j\omega t-\gamma x}, \\ \mathbf{H} &= H_0 e^{j\omega t-\gamma x}, \end{aligned} \tag{1.49}$$

varying periodically in time with the *frequency*

$$\nu = \frac{\omega}{2\pi} \tag{1.50}$$

and advancing in the $+x$ direction through space with a *complex propagation factor*

$$\gamma = j\omega\sqrt{\epsilon^*\mu^*} = \alpha + j\beta. \tag{1.51}$$

The *attenuation factor* α describes the exponential attenuation of the wave amplitude (Fig. 1.9); the *phase*

[12] The relation between **D** and **E** and between **B** and **H** is assumed to be a "linear" one; i.e., permittivity and permeability are independent of field strength. This condition is not fulfilled in ferroelectrics and ferromagnetics.

[13] Cf. Ref. 7, pp. 20 ff.

factor β determines the space period of the wave, the *wavelength*

$$\lambda = \frac{2\pi}{\beta}. \tag{1.52}$$

Thus, for a medium without loss ($\epsilon'' = \mu'' = 0$), the wave equations (Eqs. 1.48) can be written in the form

$$\nabla^2\mathbf{E} + \left(\frac{2\pi}{\lambda}\right)^2 \mathbf{E} = 0,$$
$$\nabla^2\mathbf{H} + \left(\frac{2\pi}{\lambda}\right)^2 \mathbf{H} = 0. \tag{1.53}$$

We will return to this formulation in Chap. 2 when introducing quantum mechanics.

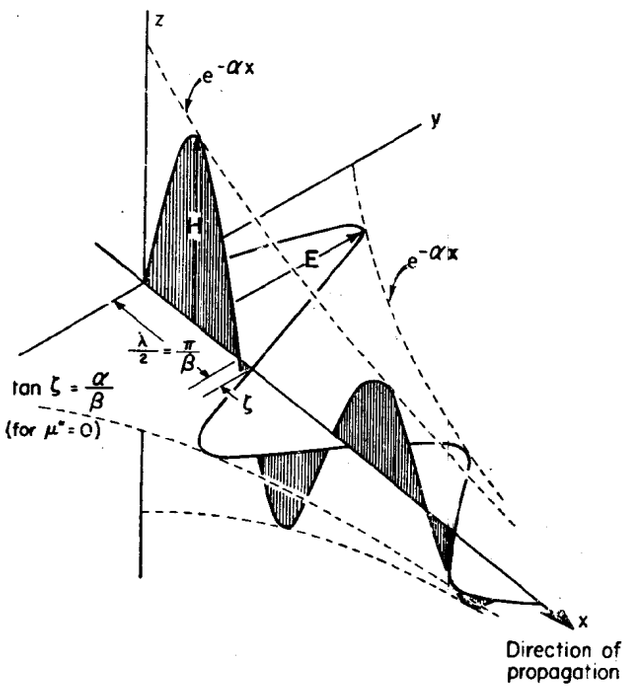

Fig. 1.9. Electromagnetic wave train in an absorbing medium.

This illustration of the evolution of macroscopic theory must suffice for the present. A detailed discussion of the electrical field and circuit theory based on ϵ^* and μ^* has been given elsewhere,[7] and we will draw on that information whenever needed. We shall refer, when speaking about the electric and magnetic properties of matter, to the complex parameters of Eq. 1.41 or to the *relative* permittivities and permeabilities

$$\kappa^* = \frac{\epsilon^*}{\epsilon_0} = \kappa' - j\kappa'',$$
$$\kappa_m^* = \frac{\mu^*}{\mu_0} = \kappa_m' - j\kappa_m'', \tag{1.54}$$

measured in reference to free space, where

$$\epsilon_0 = \frac{10^{-9}}{36\pi} = 8.854 \times 10^{-12} \quad \left[\frac{\text{farad}}{\text{m}}\right],$$
$$\mu_0 = 4\pi \times 10^{-7} \quad \left[\frac{\text{henry}}{\text{m}}\right], \tag{1.55}$$

and the velocity of light

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3 \times 10^{-8} \quad [\text{m sec}^{-1}]. \tag{1.56}$$

Alternative sets of parameters, the complex propagation factor (Eq. 1.51), the *complex index of refraction*

$$n^* = n(1 - j\kappa) = \sqrt{\kappa^*\kappa_m^*}, \tag{1.57}$$

and the *intrinsic impedance of the dielectric*

$$Z \equiv |Z|e^{j\zeta} = \sqrt{\frac{\mu^*}{\epsilon^*}} \tag{1.58}$$

will be invoked as required.

Testing of materials

The electromagnetic theory, as just summarized, formulates quantitatively the macroscopic response of matter to electric and magnetic fields in the linear range. Reflection, transmission, and absorption are determined by measuring the complex permittivity and permeability; frequency-response characteristics of materials from direct current to the ultraviolet region can be obtained by using special measurement techniques.[14] A prototype study of such broad-band "dielectric spectroscopy" has recently been carried out on ferromagnetic semiconductors.[15]

In addition, engineers need quantitative data on the high-field-strength behavior of materials (breakdown, surface tracking, deterioration, nonlinear response, saturation), on mechanical and chemical properties, moisture absorption, handling, and aging. Such information is a prerequisite for the proper matching of materials to applications, for property control and substitution, for handling in manufacture, specification writing, and guarantee of equipment performance.

For the standardization of test methods and specifications, a very complex and unwieldy pattern has grown up in the United States under the conflicting impact of producer, consumer, and general-interest organizations (Table 1.1).[16]

[14] *Dielectric Materials and Applications*, A. von Hippel, Ed., The Technology Press of M.I.T. and John Wiley and Sons, New York, 1954.

[15] P. A. Miles, W. B. Westphal, and A. von Hippel, *Revs. Mod. Phys. 29*, 279 (1957).

[16] K. N. Mathes, "ASTM Tests—Their Interpretation and Use," *Molecular Engineering*, Summer Session Notes, Laboratory for Insulation Research, Massachusetts Institute of Technology, Aug., 1956.