Introduction to Logic

PREDICATE LOGIC

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Student's Preface

I have three aims for this book. The *first* goal is to teach you the vocabulary and grammar of predicate logic so that you will be able to translate the sentences of English (or other natural languages) into the notation of this important branch of symbolic logic. The *second* goal concerns three techniques for evaluating predicate arguments: formal proofs, logic diagrams, and interpretations. I aim to help you become proficient in employing these logical methods. The *third* goal of the book is to develop your ability to identify and assess those predicate arguments you encounter daily as you read books and newspapers, carry on conversations, and watch television. Most of the examples and exercises in the text involve arguments of this everyday variety.

I enjoyed writing the book. If you enjoy studying it (as I hope you will), I think my goals will be achieved.

Teacher's Preface

This text presupposes familiarity with propositional logic and, in particular, acquaintance with the natural-deduction approach to formal proofs in propositional logic.[1] Appendix One contains a review of this material. It will refresh the memory of students previously exposed to the subject, but it is too compact to be fully intelligible to the complete novice.

Predicate logic is developed gradually in this volume, starting with the simplest monadic symbolizations and proceeding through multiple quantification to the logic of relations. Students learn to symbolize *and* evaluate arguments of a given degree of complexity before addressing themselves to the symbolization of more complex problems. This graduated approach has worked well in my logic classes.

The formal-proof system presented here excludes quantifier-introduction rules. I owe the idea to Stephen F. Barker's fine text, *The Elements of Logic*. The main advantages of this system over the more common systems which incorporate quantifier-introduction rules are (1) that the quantifier rules can be stated more simply, and (2) that proofs (although often longer) are generally easier to devise. The set of quantifier rules presented here contains fewer rules than Barker's set. For the sake of deductive completeness, Barker is required to include two (not alto-

[1] Natural-deduction proofs are treated extensively in my *Introduction to Logic: Propositional Logic* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1974). See Chapters Two through Nine.

gether intuitive) rules which sanction changes in quantifier scope.[2] In my system, these rules are obviated by the adoption of natural-deduction propositional inference rules. An expanded set of inference rules which incorporates quantifier-introduction rules is presented in the second appendix to accommodate teachers who prefer the customary approach.

Most of the examples and exercises center around arguments similar to those encountered by students. The majority of these arguments are *natural*, rather than contrived; many are presented by direct quotation from newspapers and other sources. My purposes in employing natural everyday arguments are (1) to evoke the reader's interest, (2) to counter the common but mistaken view that formal logic is an impractical academic diversion, and (3) to improve the reader's capacity to notice and assess the arguments he encounters. The final chapter explicitly addresses the problems which arise when predicate logic is applied to natural arguments.

[2]See *The Elements of Logic* (2nd ed.; New York: McGraw-Hill Book Company, Inc., 1974), p. 177.

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I am grateful to J. M. Duich for being a professor of turfgrass science, a fact which helped me begin section 5.2.

Predicate Logic

Contents

chapter one

Introduction

1.1 Predicate Logic and Propositional Logic

When my son, Michael, was in the first grade he brought home an issue of *My Weekly Reader*[1] featuring mammals. In large type it proclaimed:

MAMMALS HAVE HAIR

Several photos of mammals followed. Beneath a picture of apes were these words:

Apes have hair. Are apes mammals? Why?

The author of the pictorial essay was encouraging his youthful audience to reason as follows:

[1]*My Weekly Reader Picture Reader,* Vol. 49, Issue 1 (September 15, 1971), pp. 2a–2d.

Mammals have hair.

Apes have hair.

Therefore, apes are mammals.

Let's call this the "ape" argument.

During that school year Michael was given a standardized intelligence test which included this item:

> *All school buses are yellow. The bus that goes downtown is green. Greyhound buses are blue and silver.*

3. *What can you tell from this story?*
   a. *School buses cannot go downtown.*
   b. *A green bus is not a school bus.*
   c. *Yellow buses do not look good.*
   d. *Some school buses are blue and silver.*

The grading key identified (b) as the proper answer. (Michael chose this answer.) Pretty clearly, the test constructor was inviting students to reason:

All school buses are yellow.

A green bus is not yellow. [Unstated premise]

Thus, a green bus is not a school bus.

We will call this the "bus" argument.

One of these arguments is valid; that is, its conclusion follows with necessity from its premises.[2] The other argument is invalid; its conclusion does not follow. Which one is valid? Why is the other argument invalid? *Predicate logic* is a discipline which provides techniques for answering these questions.[3] Of course, it also enables us to answer similar questions about arguments which are much more complex than the "ape" and "bus" inferences—arguments such as this one advanced by Fran Tarkenton before Super Bowl VI:

[As Dallas is facing Miami] either Roger Staubach or Bob Griese will win a championship.

Both men are scrambling quarterbacks.

It follows that the axiom that a scrambling quarterback will never win a championship is mistaken.[4]

[2]Another (equivalent) definition of *validity:* a valid argument is one having a form such that it is impossible that its premises are all true and its conclusion false.

[3]The "ape" and "bus" arguments are assessed in section 6.2.

[4]This argument is exercise 41 in Chapter Five.

So far I have indicated that predicate logic is a branch of logic which can be applied to the arguments displayed in the preceding three paragraphs. To achieve a better understanding of what predicate logic is we contrast it with *propositional logic.* Propositional logic is the logic of the five expressions 'not', 'and', 'or', 'if . . . then', and 'if and only if'. This volume presupposes a knowledge of propositional logic. The first appendix provides a review of this branch of logic and describes the specific techniques of propositional logic that are used in the book. If your knowledge of this part of logic has become "rusty," it will be to your advantage to study Appendix One before addressing Chapter Two.

The "ape" and "bus" arguments may be symbolized in propositional logic as follows:

(ape) B, C $\vdash$ D	B = Mammals have hair
	C = Apes have hair
	D = Apes are mammals
(bus) E, $\sim$F $\vdash$ $\sim$H	E = All school buses are yellow
	F = Some green buses are yellow
	H = Some green buses are school buses

If these two symbolized arguments are assessed with the techniques of propositional logic, the verdict "invalid" will be rendered twice. This is hardly surprising in view of the fact that no capital letter occurs more than once in either symbolized argument. In each English argument there are several recurring elements; for example, the term 'mammals' occurs twice in the "ape" argument. With the exception of the term 'not' in the "bus" argument, the recurring elements are *not* represented in the propositional symbolization; and they are not represented because they are not propositions (statements) or statement connectives. The feeling that these symbolizations ignore important aspects of the English arguments is strengthened by the following consideration. One of the arguments expressed in English is valid. Neither of the symbolized arguments is valid. Therefore, at least one of the symbolizations is inadequate. In fact, both symbolizations are inadequate; both ignore aspects of the English arguments that are crucial for their validity or invalidity.

The English arguments contain double occurrences of general terms (which we shall call *predicates*), expressions such as 'apes' and 'yellow'. A logic which is adequate to the task of evaluating such arguments must be capable of representing these general terms. Because general terms are not statements or statement connectives, they cannot be represented by propositional logic. Predicate logic, by contrast, contains the symbolic equipment for representing general terms. In propositional logic simple statements are the smallest units of analysis. In predicate logic simple statements are analyzed into parts (some of which are general terms). Thus, predicate logic provides a deeper analysis than does propositional logic. For some arguments the latter branch of logic is an adequate tool of analysis, but for other arguments it is insufficient. Many arguments falling beyond the scope of propositional logic can be treated successfully in predicate logic. The "ape" and "bus" inferences are two such arguments. We can define 'predicate logic' roughly as the logic of general terms. Your conception of the nature and scope of this branch of logic will become clearer as you work through the book.

Predicate logic and propositional logic are intimately connected. All of the symbols of propositional logic appear in the formulas of predicate logic, and all of the propositional inference rules are employed in constructing formal proofs in predicate logic. Obviously it is absolutely essential that a person studying this book know the logic of propositions. In the chapters which follow, we shall develop a formal system of predicate logic by grafting new "branches" onto the "trunk" of propositional logic. The symbols of predicate logic will be added to our vocabulary in Chapters Two and Ten. We will add just three predicate inference rules to the eighteen propositional rules listed on pages 203 and 204. Two rules are introduced in Chapter Three and one in Chapter Four.

chapter two

Symbolization

2.1 Singular Statements

Many English sentences can be viewed as consisting of two parts: an expression which is used to refer to an individual, and an expression which is used to ascribe some property to the individual. Let's call expressions of the former sort ***singular terms*** and expressions of the latter kind *predicates*. Sentences composed of singular terms and predicates are known as *singular statements*. Some examples:

SINGULAR STATEMENT	SINGULAR TERM	PREDICATE
Ted Kennedy is a Democrat.	Ted Kennedy	is a Democrat
David's tumor is an acoustic neuroma.	David's tumor	is an acoustic neuroma
She sings poorly.	she	sings poorly
The first man to walk on the Moon is American.	the first man to walk on the Moon	is American

The key characteristic of a singular term is that it is customarily used to refer to an individual. I use *individual* broadly, counting not only people but pets, rivers, rocks, cities, planets, numbers, and so on as individuals. Singular terms are expressions that function like proper nouns, but the concept is a logical, not a grammatical one. Singular terms may be proper nouns ('Shakespeare'), pronouns ('he'), or noun phrases ('David's tumor', 'the janitor'). In predicate logic we abbreviate singular terms with lower-case letters of the alphabet from *a* through *w* (the letters *x*, *y*, and *z* are reserved for another use which is explained in the next section). Normally the letter chosen as an abbreviation will be the first letter of a prominent word occurring in the singular term; for example, *j* will abbreviate 'the janitor'. Let's call the letters that abbreviate singular terms *names.*

A predicate or general term is an expression which may be used to ascribe a property (such as *being fat*) to an individual or to assert that several individuals stand in some relationship (like *hating*). At present we will concentrate on property predicates, postponing our treatment of relational predicates until Chapter Ten. Predicates may be composed of various parts of speech. Some examples:

PREDICATE	PART OF SPEECH
sleeps	verb
sleeps poorly	verb + adverb
speaks German	verb + noun
is greedy	copula[1] + adjective
is a Texan	copula + noun phrase

It will become clear as we proceed that our concept of "predicate" does not correspond exactly to the grammarian's notion. Predicates are abbreviated in our logic by capital letters. The letter selected will usually be the first letter of one of the words comprising the predicate; for example, *T* will abbreviate 'is a Texan'. We call the letters that abbreviate English predicates *predicate letters* (or just *predicates*).

To symbolize an affirmative singular statement in the notation of predicate logic, we write the capital which abbreviates the predicate followed by the lower-case letter which serves as the abbreviation of the singular term. S1 is symbolized by F1.

(S1) David's <u>tumor</u> is an acoustic NEUROMA.

(F1) Nt

[1] A copula is a word or expression (such as a form of the verb 'to be') that links the subject of a sentence with its grammatical predicate without asserting action.