

The Laser Doppler Technique

L. E. DRAIN

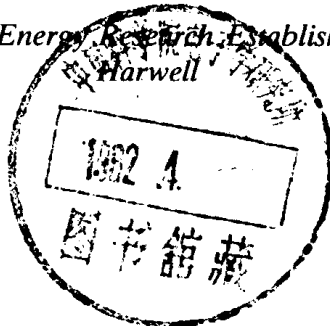


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Preface

The development of the high intensity coherent light sources known as lasers has made it possible to introduce many optical measurement techniques previously impracticable. The subject of this book is an optical technique for the measurement of velocity based on the determination of the Doppler shift of light scattered from moving particles or objects. This laser Doppler technique has undergone much development in recent years and is now much used, particularly in studies of fluid flow. This book is intended as an introduction to the principles of the technique for those wishing to employ it in measurements of velocity and turbulence.

Emphasis has been given to explaining the basic physical principles and, since some readers in the engineering field may not be very familiar with optics, a chapter has been devoted to summarizing basic optical concepts together with a discussion of the properties of lasers particularly relevant to laser Doppler measurements. Although a good deal of mathematical analysis is given, it is not essential that the reader follow the details. Provided the results are accepted, only an elementary knowledge of mathematics is required to understand the techniques.

The number of papers published in this field must now be well over a thousand and I have made no attempt to give a complete list of references. I hope that sufficient have been given to enable a reader to follow up any special topic.

I gratefully acknowledge the receipt of information and ideas from the following colleagues at Harwell: Dr P. J. Bourke, Mr C. G. Brown, Mr W. Dalzell, Mr A. B. Gillespie, Mr B. C. Moss, Dr C. R. Negus, Mr A. Taylor, Dr G. Wigley and Dr M. Y. Yeoman. I would particularly like to thank Dr P. Hutchinson, Dr B. T. M. Willis and Mr N. Lightfoot for reading and commenting on the manuscript.

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CHAPTER 1

Introduction

1.1 The Doppler Shift

The technique of using the Doppler shift of laser light to determine velocities was first demonstrated in 1964 by Yeh and Cummins,¹ who observed the shift of light scattered from particles carried in a water flow. Measurements of flow velocities in gases soon followed.² Lasers produce a very intense monochromatic light very suitable for this type of measurement. There are a number of variations of the technique which may be broadly described as laser Doppler anemometry, commonly abbreviated to 'LDA'.

In any form of wave propagation, frequency changes can occur owing to movement of the source, receiver, propagating medium, or intervening reflector or scatterer. These shifts are generally called 'Doppler' shifts after the Austrian physicist who first considered the phenomenon in 1842. The Doppler shift, familiar in acoustics, is due to the relative motion of source and receiver and this type of shift is also well known for electromagnetic radiation, including light. The 'red' shift to lower frequencies of light arriving at the earth from distant galaxies is attributed to their movement away from us at extremely high velocities. The shift of γ -rays from a moving radioactive source may be readily demonstrated in the laboratory using extremely well defined resonance absorption and emission lines that may be obtained in the Mössbauer effect.

In the experiments with which we shall be concerned there is no relative movement of the source and receiver. The shift is produced by the movement of a particle or larger body that reflects or scatters light from the source to receiver. This is of course exactly the same principle that is used in Radar in a much lower-frequency part of the electromagnetic spectrum. Since the velocities commonly encountered are very small compared with the velocity of light, the corresponding Doppler shifts are small. For example, scattering from an object moving with a velocity of 10 metres per second will produce a maximum shift of 32 MHz in the frequency of red light from a He-Ne laser, i.e. 7 parts in 10^8 in a light frequency of 4.7×10^{14} Hz. The resolution of shifts as small as this is beyond the capabilities of the highest resolution optical spectrometers and, unless exceptionally high (e.g. supersonic) velocities are involved, direct optical spectroscopy is not a practicable method of measurement.

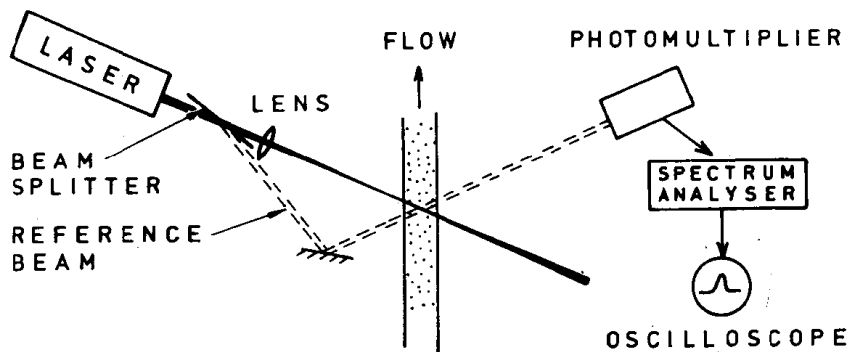


Figure 1.1 A basic laser Doppler experiment using a reference beam for heterodyning the scattered light

1.2 Optical Beating

The only technique suitable for measuring very small Doppler shifts uses the principle of heterodyning or 'beating' of two frequencies in a device having a non-linear response. This principle, so useful in radio circuits, may also be applied to light beams which may be heterodyned by presenting them simultaneously to a light detector. The output of the detector then contains a component of the difference or heterodyne beat frequency.

In the application to optical velocity measurement, Doppler shifted scattered light is heterodyned either with unshifted light obtained directly from the original source, or with further scattered light having a different shift because it is scattered through a different angle or from a different point.

A simple illustration of the use of laser-Doppler beating to measure the velocity of a fluid in a transparent pipe is shown in Figure 1.1. Light from the helium-neon laser is divided by the beam splitter, most of the light being focused by a lens to a point in the pipe where the velocity measurement is required. Light scattered by particles moving with the fluid is received by the photomultiplier tube. The weaker beam from the beam splitter, the unshifted reference, is directed to the detector by the mirror. It is desirable that it is incident at the detector along the same path as the scattered light. It is convenient to provide an adjustable attenuator in the reference path to ensure that the scattered and reference beams do not differ too greatly in intensity at the detector. The output from the photomultiplier contains a signal of the difference frequency between the two beams which is of course the Doppler shift. The presence and frequency of this signal may be conveniently shown by spectrum analysis, as illustrated.

For the experimental arrangement shown, the Doppler Shift frequency is given by:

$$\nu_D = \frac{2v}{\lambda} \sin\left(\frac{\alpha}{2}\right)$$

where v is the velocity in the tube, λ the wavelength of the light, and α the angle between the direction of illumination and the direction in which the scattered light

is received. (This formula is derived in Chapter 3.) The success of the experiment of course depends on the presence of particles to scatter light. Suitable particles may be added to the fluid but it is often found that enough is scattered from naturally occurring impurities.

It is important to realize that when the optical beating technique is used with good optical surfaces and/or with stationary or slow moving objects, the principle is familiar as interferometry — a long-established technique. In fact, no real distinction can be made between time-dependent interferometry and optical Doppler beating. Although, in principle, we may regard any laser Doppler beating arrangement as an interferometer, this term is usually reserved for arrangements using specular reflection from optically prepared surfaces rather than those using randomly scattered light.

1.3 The Differential Doppler Technique

Although the reference beam method is very useful, particularly in laboratory measurements with liquids and when using light scattered from a solid object, the most commonly used technique involves no reference beam. Instead two beams of equal intensity are focused and crossed at the point under investigation. The same basic idea was conceived at a number of laboratories³⁻⁷ and the method is generally known as the 'dual beam' or 'differential Doppler' technique. It has a considerable signal to noise advantage in circumstances where particles are few, e.g. in gas flows. Indeed, were it not for this second technique, laser Doppler velocity measurement would not be practicable in many situations of practical importance and applications would be comparatively limited.

A simple differential Doppler arrangement is illustrated in Figure 1.2. The two illuminating beams derived from the laser are focused into a small region conveniently by a single lens. Scattered light from this region is focused onto the photo-detector. Since light scattered from the beams reaches the detector simultaneously, a beat is obtained of frequency equal to the difference in Doppler shifts corresponding to the two angles of scattering. It may be shown that the beat frequency is independent of the receiving direction and is given by:

$$\nu_D = \frac{2v}{\lambda} \sin \left(\frac{\alpha}{2} \right)$$

where v is the velocity of particles passing through the active region, α the angle between the two illuminating beams, and λ the wavelength of the light. The output from the detector may be spectrum analysed or processed in more sophisticated ways.

The great advantage of this method is that light may be collected over a wide aperture and, given a low particle concentration, all the light contributes usefully to the signal. This is not true for the reference beam technique where the useful aperture is extremely restricted. The interrelation between optical arrangement, receiving aperture, particle density, and attainable signal-to-noise ratio is very important and is considered in detail in Chapters 4 and 5.

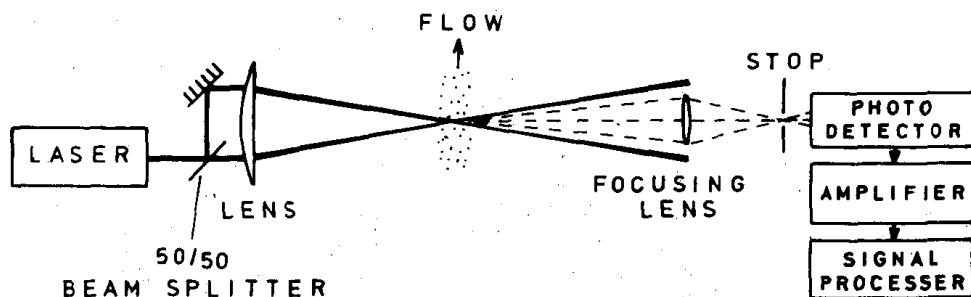


Figure 1.2 A simple differential Doppler velocity measurement system

Those familiar with optical interference will readily see that the differential Doppler 'beat' frequency permits simple interpretation. The region where the two beams cross is full of interference fringes. The modulation of the intensity at the detector is simply due to the variation in the illumination of particles as they cross light and dark fringes.

1.4 Survey of Applications

The laser Doppler technique has quickly found numerous applications. It is principally of value in studies of fluid dynamics where the great advantages are that there is no obstruction to the flow and spatial resolution is very high. Nothing but light is needed to be transmitted to the point of interest and light from a laser can be focused into a very small volume where the velocity is required. The consequent resolution, typically 20–100 μm exceeds that obtainable by any other method.

LDA has the advantage of linearity and can cover a wide range of flow velocities from millimetres per second to supersonic. The calibration can be calculated from optical geometry. In liquids and seeded gas flows continuous signals can be obtained and the response of LDA is essentially instantaneous and fast fluctuations from turbulence can be followed. The technique therefore cannot be bettered in this application.

Also, with the laser Doppler technique there are methods available that enable the direction as well as the magnitude of a velocity to be determined. This is not possible with most other techniques of velocity measurement and is an invaluable feature in studying complex flows.

It should be remembered that this is a high resolution fast response technique and thus it is in the field of turbulence that its potentialities are fully realized. Some measurements are not well suited to the technique. Measurement of total flow down a tube, for example, would have to be obtained by integration over the cross section, rather tedious for general application.

Very frequently velocity and turbulence profiles are required for design studies in wind tunnels and in flow situations, actual or modelled. It is here that the LDA technique is most useful and enables much detailed flow information to be rapidly obtained. The absence of material probes enables measurements to be made in hostile environments such as corrosive liquids or flames. The measurements are not

Table 1.1
Velocity measurement by the laser Doppler technique

Advantages	Disadvantages
Does not disturb the flow	Medium must be transparent
High spatial resolution	Needs scattering particles: artificial seeding may be necessary
Fast response	Optical access is required: windows may have to be installed
Response linear and easily calibrated	Expensive signal processing equipment may be required in difficult situations where the signal to noise ratio is poor
Directional discrimination possible	Not well suited for measurements of total flow as this requires a tedious integration over a cross section
Operation not usually seriously affected by temperature	

affected by temperature unless unusually long light paths through hot gases are involved.

The chief limitation of the laser Doppler technique is that it depends on the presence of particles in the flow. Insufficient particles may be present naturally, resulting in poor signal to noise ratios and the need for expensive processing equipment and/or long measurement times. The addition of particles may not always be possible or desirable. Fluids studied by LDA must of course be transparent. There is usually no difficulty here except for liquid metals of course or if long light paths through turbid water are required. Optical access to the fluid flow area must usually be provided by a transparent tube or window which does not necessarily need to be of very high optical quality.

A list of the advantages and disadvantages of the laser Doppler technique is given in Table 1.1.

CHAPTER 2

Optics and Lasers

In this chapter the fundamental principles of optics are recalled and the basic properties of lasers described paying special attention to the aspects relevant to the practical use of the laser Doppler technique. For a detailed and rigorous discussion the reader is referred to standard optical textbooks, but the treatment given here should enable a reader with some general knowledge of optics to appreciate the design and limitations of LDA instruments.

Readers familiar with optics may omit this chapter referring back to it for special results and formulae when necessary.

2.1 Light as Electromagnetic Radiation

Light is a special case of the general type of radiation arising from the interaction of electric and magnetic fields. The spectrum of electromagnetic waves extends from long radio waves to the γ -rays of nuclear physics and, in vacuo, all these waves travel with the same velocity, i.e. $c_0 = 2.99776 \times 10^8 \text{ m s}^{-1}$. The range of wavelengths or frequencies is conventionally divided into regions depending on the common methods of production and detection. The main divisions are shown in Figure 2.1. Visible radiation has wavelengths in the range approximately 0.40–0.70 μm , depending on colour.

The technique of velocity measurement by means of the Doppler shift is applied to the whole range of electromagnetic waves but the practical methods discussed in this book apply principally to wavelengths of around the visible region. The neighbouring ultra-violet and infra-red regions may also be included and these waves may be loosely referred to as 'light'.

The propagation of light, as of all electromagnetic waves, is determined by equations first derived by Maxwell and these are fundamental to the complete understanding of optics. However, in practice the exact solutions of Maxwell's equations are seldom necessary in optical problems and we shall not consider the full theory in this book. Approximations valid for distances of propagation large compared with the wavelength will be used. For many purposes the use of simple geometrical optics is adequate. Simple concepts of wave motion may be added and more precise calculations made with the use of Huygens's principle. It is assumed that the reader is fairly familiar with these concepts, but they will be summarized in this chapter and the optical theory most needed in laser Doppler work discussed.

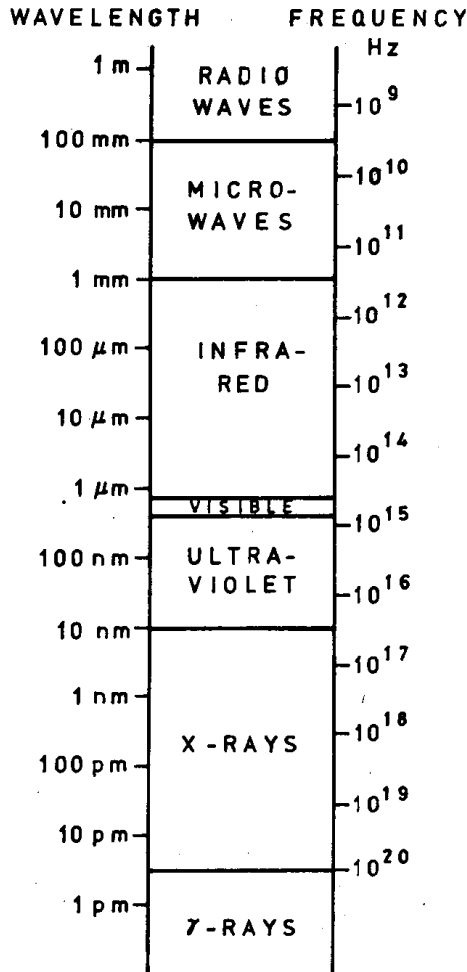


Figure 2.1 The electromagnetic spectrum

2.2 Geometrical Optics

In many situations the use of elementary geometrical optics is all that is needed to determine the propagation of light beams and much of the design of LDA systems is based on these principles. In this approximation the laws are as follows:

- (i) In a uniform medium light travels in straight lines along 'rays'.
- (ii) When rays are reflected at a smooth surface, the angle of incidence is equal to the angle of reflection.
- (iii) When passing from one medium to another, light is deviated or refracted, the angles of incidence and refraction θ_1 and θ_2 (see Figure 2.2) being related by Snell's law:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\mu_2}{\mu_1} \quad (2.1)$$

where μ_1 and μ_2 are the refractive indices of the media (usually referred to vacuum) for the particular wavelength used.

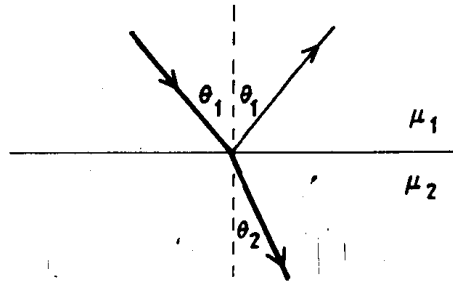


Figure 2.2 Reflection and refraction of light at a plane surface

Refraction from a dense to a less dense medium ($\mu_2 < \mu_1$) is not possible if $\theta_1 > \sin^{-1}(\mu_2/\mu_1)$. In this case the incident light is totally reflected. Total internal reflection inside prisms is often used for beam deflection.

The most familiar optical devices using the principle of refraction are lenses. They have many uses in LDA systems for expanding and focusing laser beams, image formation, and concentrating light onto detectors. The lens performance required is usually less demanding than in most optical instruments or in photography, and for most LDA applications simple one-component lenses are adequate. This is because of the small diameters of laser beams and monochromatic nature of the light.

The lens imperfection most likely to lead to difficulties in LDA is spherical aberration. This is particularly important in the differential Doppler technique and will be considered in detail in sections 5.16 and 5.17. In this application the use of a well corrected multi-element lens can considerably improve performance.

2.3 Wave Optics

The wave motion of light, being electromagnetic, involves fluctuations of both electric and magnetic fields, but in optics it is rare that the magnetic component is important. We are not usually concerned with magnetic materials. There are very few that are transparent anyway. The optical properties of materials are determined principally by the high frequency dielectric constant and electrical conductivity. Also, the coupling of the radiation to electrons in atoms leading to the absorption or emission of energy is, in this frequency range, almost always via the electric field component. Thus, in elementary descriptions of optical wave motion we will consider only the electric field.

In a wave of well defined frequency and constant intensity, the electric field at a point varies sinusoidally with time at the frequency, ν , of the wave motion. The phase of the motion varies with distance, x , along the direction of propagation. Thus, at any given time the variation of the electric field with x is also sinusoidal. The distance corresponding to a phase change of a full cycle (i.e. the separation of adjacent peaks or troughs) is known as the wavelength, usually denoted by λ . As the wave advances, the sinusoidal pattern moves along, as illustrated in Figure 2.3.

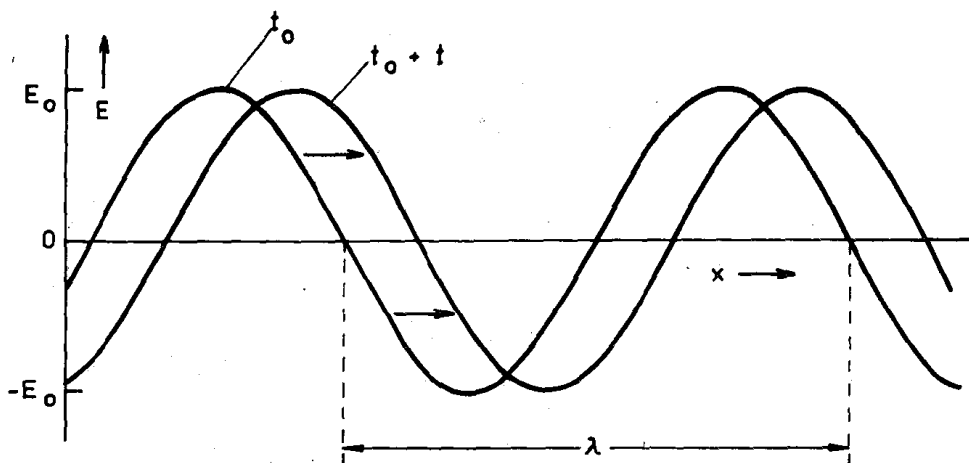


Figure 2.3 The variation of electric field with distance in an electromagnetic wave travelling in the x -direction at times t_0 and $(t_0 + t)$

It is evident that the wave pattern moves by one wavelength in one cycle of the oscillation frequency. Thus, the wave velocity, c , is given by:

$$c = \nu \lambda \quad (2.2)$$

The velocity of the wave depends on the medium of propagation, and the ratio of velocity in a medium to that in a vacuum, c_0 , is given by:

$$\frac{c}{c_0} = \frac{1}{\mu}$$

It is easy to show that the reduction in wave velocity in a denser medium leads to Snell's law of refraction and the constant μ is just the refractive index introduced in geometrical optics. Since the frequency of the wave is of course unchanged on entry into another medium, the wavelength is reduced from its value in vacuo by the factor μ . This fact should always be borne in mind when interpreting laser Doppler results in liquid media.

For a plane wave, the variation of electric field with distance x and time t may be represented mathematically as follows:

$$E = E_0 \cos \left(2\pi \left(\nu t + \frac{x}{\lambda} \right) + \phi \right)$$

where ϕ is a phase constant.

For a spherical wave, i.e. one that comes or appears to come from a point source:

$$E = \frac{A}{R} \cos \left(2\pi \left(\nu t + \frac{R}{\lambda} \right) + \phi \right)$$

where R is the distance from the source and A is a constant. The amplitude of the electric field fluctuation is inversely proportional to R . It follows that the intensity