MECHANICAL ENGINEERING SERIES

A.A. Shabana

Theory of Vibration

Volume II: Discrete and Continuous Systems

Springer-Verlag
世界图书出版公司

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With 147 Figures

Springer-Verlag
New York Berlin Heidelberg London
Paris Tokyo Hong Kong Barcelona
世界图书出版公司
北京·广州·上海·西安

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Library of Congress Cataloging-in-Publication Data
Shabana, Ahmed A., 1951–
Theory of vibration / A.A. Shabana.
p. cm.—(Mechanical engineering series)
Includes bibliographical references.
Contents: v. 1. An introduction—v. 2. Discrete and continuous systems
ISBN 0-387-97384-2 (alk. paper)
1. Vibration. I. Title. II. Series.
QA865.S49 1990
531'.32—dc20 90-9582

Reprinted by World Publishing Corporation, Beijing, 1992
for distribution and sale in The People's Republic of China only
ISBN 7-5062-1412-1

ISBN 0-387-97384-2 Springer-Verlag New York Berlin Heidelberg
ISBN 3-540-97384-2 Springer-Verlag Berlin Heidelberg New York

Mechanical Engineering Series

Frederick F. Ling
Series Editor

Dedicated to Professor M.M. Nigm

Series Preface

Mechanical engineering, an engineering discipline borne of the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions, among others. These solutions can only be obtained through engineering education, which requires new, updated approaches to teaching materials in general and to textbooks in particular. The Mechanical Engineering Series is a new series, featuring texts and research monographs, intended to address the need for information and teaching in contemporary areas of mechanical engineering.

The series is conceived as a comprehensive one which will cover a broad range of concentrations important to mechanical engineering education and research. We are fortunate to have a distinguished roster of consulting editors on the advisory board, each an expert in one of the areas of concentration. The names of the consulting editors are listed on the first page of the volume. The areas of concentration are: applied mechanics; biomechanics; computational mechanics; dynamic systems and control; energetics; mechanics of materials; processing; thermal science; and tribology.

Professor Marshek, the consulting editor for dynamic systems and control, and I are pleased to present the fifth volume of the series: *Theory of Vibration, Volume II: Discrete and Continuous Systems* by Professor Shabana.

Frederick F. Ling

Preface

The theory of vibration of single and two degree of freedom systems is covered in the first volume of this book. In the treatment presented in Volume I, the author assumed only a basic knowledge of mathematics and dynamics on the part of the student. Therefore, Volume I can serve as a textbook for a first undergraduate semester course on the theory of vibration. The second volume contains material for a one-semester graduate course which covers the theory of multi-degree of freedom and continuous systems. An introduction to the finite-element method is also presented in this volume. In the first and the second volumes, the author attempted to cover only the basic elements of the theory of vibration which students should learn before taking more advanced courses on this subject. Each volume, however, represents a separate entity and can be used without reference to the other. This gives the instructor the flexibility of using one of these volumes with other books in a sequence of two courses on the theory of vibration.

Chapter 1 of this volume covers some of the basic concepts and definitions used in the analysis of single degree of freedom systems. These concepts and definitions are also of fundamental importance in the vibration analysis of multi-degree of freedom and continuous systems. Chapter 1 is of an introductory nature and can serve to review the materials covered in the first volume of this book.

In Chapter 2, a brief introduction to Lagrangian dynamics is presented. The concepts of generalized coordinates, virtual work, and generalized forces are first introduced. Using these concepts, Lagrange's equation of motion is then derived for multi-degree of freedom systems in terms of scalar energy and work quantities. The kinetic and strain energy expressions for vibratory systems are also presented in a matrix form. Hamilton's principle is discussed in Section 6 of this chapter, while general energy conservation theorems are presented in Section 7. Chapter 1 is concluded with a discussion on the use of the principle of virtual work in dynamics.

Matrix methods for the vibration analysis of multi-degree of freedom systems are presented in Chapter 3 of this volume. The use of both Newton's second law and Lagrange's equation of motion for deriving the equations of motion of multi-degree of freedom systems is demonstrated. Applications related to angular oscillations and torsional vibrations are provided. The case of the undamped free vibration is first presented and the orthogonality of the mode shapes is discussed. The case of forced vibration of the undamped multi-degree of freedom systems is discussed in Section 7. The vibration of viscously damped multi-degree of freedom systems using proportional damping is examined in Section 8, and the case of general viscous damping is presented in Section 9. Coordinate reduction methods using the modal transformation are discussed in Section 10. Numerical methods for determining the mode shapes and natural frequencies are discussed in Sections 11 and 12.

Chapter 4 deals with the vibration of continuous systems. Free and forced vibrations of continuous systems are discussed. The analysis of longitudinal, torsional, and transverse vibrations of continuous systems is presented in this chapter. The orthogonality relationships of the mode shapes are developed and are used to define the modal mass and stiffness coefficients. The use of both elementary dynamic equilibrium conditions and Lagrange's equations in deriving the equations of motion of continuous systems is demonstrated. The use of approximation methods as a means of reducing the number of coordinates of continuous systems to a finite set is also examined in this chapter.

In Chapter 5 an introduction to the finite-element method is presented. The assumed displacement field, connectivity between elements, and the formulation of the mass and stiffness matrices using the finite-element method are discussed. The procedure for assembling the element matrices in order to obtain the structure equations of motion is outlined. The convergence of the finite element solution is examined and the use of higher order and spatial elements in the vibration analysis of structural systems is demonstrated.

I would like to thank many of the teachers, colleagues, and students who contributed, directly or indirectly, to this book. In particular I would like to thank my students D.C. Chen and W.H. Gau who have made major contributions to the development of this book. My special thanks to Ms. Denise Burt for the excellent job in typing the manuscript of this book. The editorial and production staff of Springer-Verlag deserve special thanks for their cooperation and their thorough professional work. Finally, I thank my family for the patience and encouragement during the time of preparation of this book.

Chicago, Illinois

Ahmed A. Shabana

Contents

1
Introduction

The purpose of this chapter is to present a brief introduction to the theory of vibration of single degree of freedom systems. The method of analysis of single degree of freedom systems serves as one of the fundamental building blocks in the theory of vibration of discrete and continuous systems. As will be shown in later chapters, the concepts introduced and the techniques developed for the analysis of single degree freedom systems can be generalized to study discrete systems with multi-degrees of freedom as well as continuous systems.

For this volume to serve as an independent text, several of the important concepts and techniques discussed in the first volume of this book are briefly discussed in this chapter. The free undamped and damped vibration of the single degree of freedom systems is covered in the first three sections of this chapter. Viscous, structural, and Coulomb damping are discussed and the significant effect of the damping on the free vibration of the single degree of freedom systems is demonstrated. Section 4 is devoted to the analysis of the forced vibrations of single degree of freedom systems subject to harmonic excitations. The impulse response and the response of the single degree of freedom system to an arbitrary forcing function are discussed, respectively, in Sections 5 and 6.

1.1 FREE VIBRATION

In this section, we study the effect of viscous damping on the free vibration of single degree of freedom systems. The differential equation of such systems will be developed, solved, and examined. It will be seen from the theoretical development and the examples presented in this section that the damping force has a pronounced effect on the stability of the systems.

Figure 1(a) depicts a single degree of freedom system. The system consists of a mass m supported by a spring and a damper. The stiffness coefficient of the spring is k and the viscous damping coefficient of the damper is c. If the system is set in motion because of an initial displacement and/or an initial velocity, the mass will vibrate freely. At an arbitrary position x of the mass

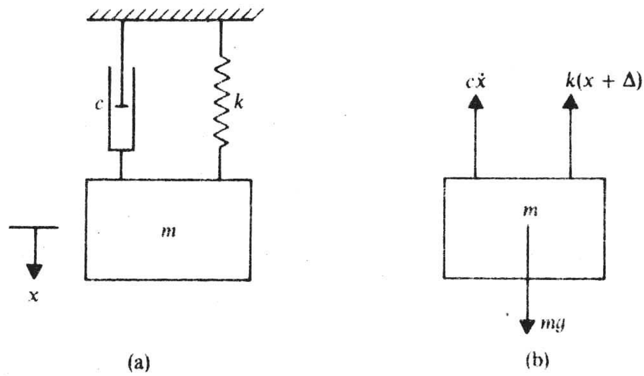

FIG. 1.1. Damped single degree of freedom system.

from the equilibrium position, the restoring spring force is equal to kx and the viscous damping force is proportional to the velocity and is equal to $c\dot{x}$, where the displacement x is taken as positive downward from the equilibrium position. Using the free body diagram shown in Fig. 1(b), the differential equation of motion can be written as

$$m\ddot{x} = mg - c\dot{x} - k(x + \Delta) \tag{1.1}$$

where Δ is the static deflection at the equilibrium position. Since the damper does not exert force at the static equilibrium position, the condition for the static equilibrium can be written as

$$mg = k\Delta \tag{1.2}$$

Substituting Eq. 2 into Eq. 1 yields

$$m\ddot{x} = -c\dot{x} - kx$$

or

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1.3}$$

This is the standard form of the second-order differential equation of motion that governs the linear vibration of damped single degree of freedom systems. A solution of this equation is in the form

$$x = Ae^{pt} \tag{1.4}$$

Substituting this solution into the differential equation yields

$$(mp^2 + cp + k)Ae^{pt} = 0$$

From which the characteristic equation is defined as

$$mp^2 + cp + k = 0 \tag{1.5}$$

The roots of this equation are given by

$$p_1 = -\frac{c}{2m} + \frac{1}{2m}\sqrt{c^2 - 4mk} \tag{1.6}$$

$$p_2 = -\frac{c}{2m} - \frac{1}{2m}\sqrt{c^2 - 4mk} \tag{1.7}$$

Define the following dimensionless quantity

$$\xi = \frac{c}{C_c} \tag{1.8}$$

where ξ is called the *damping factor* and C_c is called the *critical damping coefficient* defined as

$$C_c = 2m\omega = 2\sqrt{km} \tag{1.9}$$

where ω is the system *circular* or *natural frequency* defined as

$$\omega = \sqrt{k/m} \tag{1.10}$$

The roots p_1 and p_2 of the characteristic equation can be expressed in terms of the damping factor ξ as

$$p_1 = -\xi\omega + \omega\sqrt{\xi^2 - 1} \tag{1.11}$$

$$p_2 = -\xi\omega - \omega\sqrt{\xi^2 - 1} \tag{1.12}$$

Clearly, if ξ is greater than one, the roots p_1 and p_2 are real and distinct. If ξ is equal to one, the root p_1 is equal to p_2 and both roots are real. If ξ is less than one, the roots p_1 and p_2 are complex conjugates. The damping factor ξ is greater than one if the damping coefficient c is greater than the critical damping coefficient C_c. This is the case of an *overdamped* system. The damping factor ξ is equal to one when the damping coefficient c is equal to the critical damping coefficient C_c. In this case, the system is said to be *critically damped*. The damping factor ξ is less than one if the damping coefficient c is less than the critical damping coefficient C_c. In this case, the system is said to be *underdamped*. In the following, the three cases of overdamped, critically damped, and underdamped systems are discussed in more detail.

Overdamped System In the overdamped case, the roots p_1 and p_2 of Eqs. 11 and 12 are real. The response of the single degree of freedom system can be written as

$$x(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} \tag{1.13}$$

where A_1 and A_2 are arbitrary constants. Thus the solution, in this case, is the sum of two exponential functions and the motion of the system is non-oscillatory, as shown in Fig. 2. The velocity can be obtained by differentiating Eq. 13 with respect to time, that is,

$$\dot{x}(t) = p_1 A_1 e^{p_1 t} + p_2 A_2 e^{p_2 t} \tag{1.14}$$

The maximum displacement occurs at time t_m when the velocity $\dot{x}(t)$ is equal to zero, that is, the maximum displacement occurs when

$$p_1 A_1 e^{p_1 t_m} + p_2 A_2 e^{p_2 t_m} = 0$$

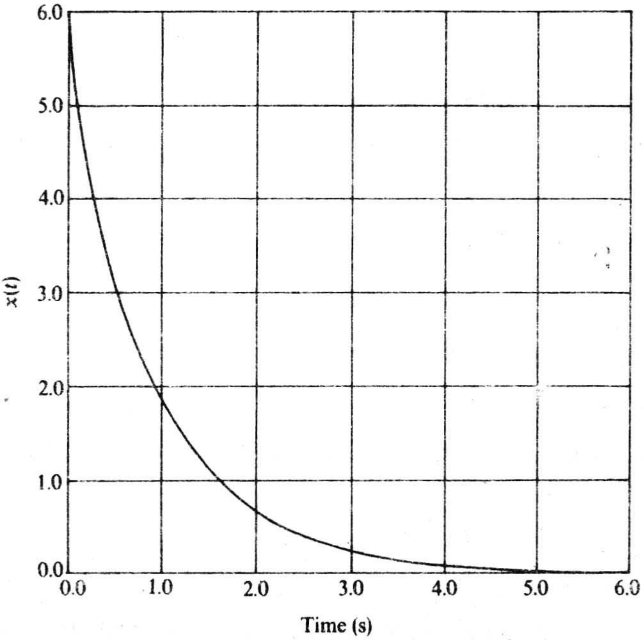

FIG. 1.2. Overdamped systems.

or

$$e^{(p_1 - p_2)t_m} = -\frac{p_2 A_2}{p_1 A_1}$$

This equation can be used to determine the time t_m at which the displacement is maximum as

$$t_m = \frac{1}{p_1 - p_2} ln\left(-\frac{p_2 A_2}{p_1 A_1}\right) \tag{1.15}$$

The constants A_1 and A_2 can be determined from the initial conditions. For instance, if x_0 and $\dot{x}_0$ are, respectively, the initial displacement and velocity, one has from Eqs. 13 and 14

$$x_0 = A_1 + A_2$$

$$\dot{x}_0 = p_1 A_1 + p_2 A_2$$

from which A_1 and A_2 are

$$A_1 = \frac{x_0 p_2 - \dot{x}_0}{p_2 - p_1} \tag{1.16}$$

$$A_2 = \frac{\dot{x}_0 - p_1 x_0}{p_2 - p_1} \tag{1.17}$$

provided that $(p_1 - p_2)$ is not equal to zero. The displacement $x(t)$ can then

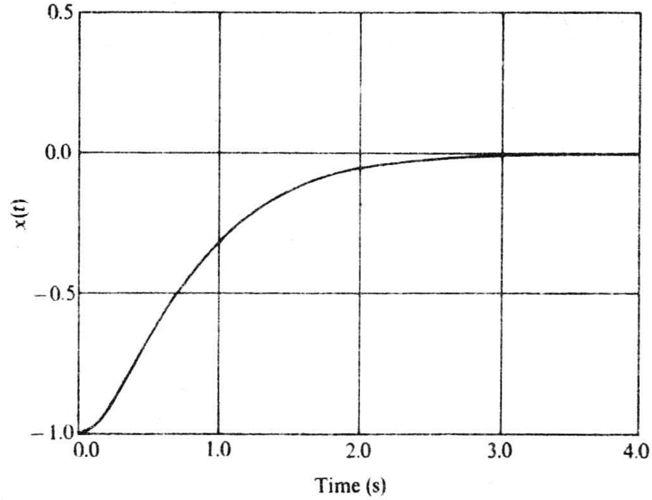

FIG. 1.3. Overdamped system with zero initial velocity.

be written in terms of the initial conditions as

$$x(t) = \frac{1}{p_2 - p_1}[(x_0 p_2 - \dot{x}_0)e^{p_1 t} + (\dot{x}_0 - p_1 x_0)e^{p_2 t}] \tag{1.18}$$

The time t_m at which the maximum displacement occurs can also be written in terms of the initial conditions as

$$t_m = \frac{1}{p_2 - p_1} ln \left[\frac{p_2(\dot{x}_0 - p_1 x_0)}{p_1(\dot{x}_0 - p_2 x_0)}\right] \tag{1.19}$$

provided that the natural logarithmic function ln is defined. That is, t_m of Eq. 19 is defined only when the argument of the natural logarithmic function in this equation is positive. If the system has initial displacement and zero initial velocity, t_m is given by

$$t_m = \frac{1}{p_2 - p_1} ln\ 1 = 0$$

That is, the maximum absolute displacement occurs at time $t = 0$, as in the example shown in Fig. 3. It is important, however, to emphasize that there are also cases in which the response curve does not have an extremum, as shown in Fig. 4. This case corresponds to the case in which the argument of the logarithmic function in Eq. 19 is negative.

Example 1.1

The damped mass–spring system shown in Fig. 1 has mass $m = 10$ kg, stiffness coefficient $k = 1000$ N/m, and damping coefficient $c = 300$ N · s/m. Determine the displacement of the mass as a function of time.

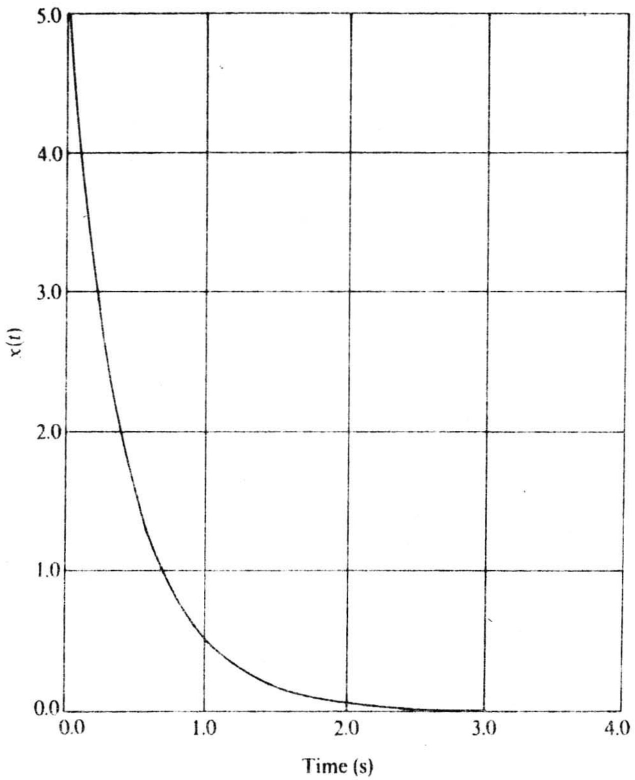

FIG. 1.4. Nonoscillatory motion of overdamped systems.

Solution. The natural frequency ω of the system is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{10}} = 10 \text{ rad/s}$$

The critical damping coefficient C_c is

$$C_c = 2m\omega = 2(10)(10) = 200 \text{ N}\cdot\text{s/m}$$

The damping factor ξ is given by

$$\xi = \frac{c}{C_c} = \frac{300}{200} = 1.5$$

Since $\xi > 1$, the system is overdamped and the solution is given by

$$x(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

where p_1 and p_2 can be determined using Eqs. 11 and 12 as

$$p_1 = -\xi\omega + \omega\sqrt{\xi^2 - 1} = -(1.5)(10) + (10)\sqrt{(1.5)^2 - 1} = -3.8197$$

$$p_2 = -\xi\omega - \omega\sqrt{\xi^2 - 1} = -(1.5)(10) - (10)\sqrt{(1.5)^2 - 1} = -26.1803$$