

HARVEY M. WAGNER

School of Organization and Management
Yale University;
Consultant to the Firm
McKinsey & Company, Inc.

Second Edition

PRINCIPLES OF MANAGEMENT SCIENCE

With Applications
to
Executive Decisions

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Preface

This book is written primarily for college students who have no previous background in management science and who intend careers as administrators, executives, managers, or consultants in business, nonprofit enterprises, or government. The book is used in half-year or full-year introductory courses for undergraduates or graduates in business, economics, engineering, liberal arts, and public administration curricula.

The central goal of the book is to answer the question, "What are the *fundamental ideas* of management science?" The text does not presuppose any *advanced* training in business administration, industrial engineering, mathematics, statistics, probability theory, or economics. Therefore, the main ideas do not rely on the reader's being expert in these areas. The text does assume, however, that the reader is not entirely naive about such subjects.

PRINCIPAL OBJECTIVES

Beginning students frequently ask, "What must I learn about management science if I intend to become an executive rather than a specialist?" and "What must I learn about management science given that I want to apply it to real problems?" Regrettably, a single introductory management science course cannot completely answer either question. But such a course can better answer the first rather than the second question.

In the context of these two questions, the book's principal objectives are

- To introduce the important ideas in management science which are both fundamental and long lasting.
- To provide those students not going beyond a single introductory course with enough understanding and confidence to appreciate the strengths and inherent limitations of the management science approach.
- To prepare and motivate future specialists to continue in their study by having an insightful overview of management science.
- To demonstrate the cohesiveness of management science methodology.

Students who have been assigned this book as a text have increased their skills in formulating and building formal models of complex decision environments, in perceiving the critical issues to be resolved, and in isolating the basic phenomena that comprise the key elements of real situations. When faced with actual managerial problems, these students have learned from their grasp of the analytic fundamentals how to achieve sound and incisive evaluations of the important alternatives and how to attain the crucial insights.

COVERAGE AND EMPHASIS

Successful practitioners of management science are quick to point out that characterizing the subject by means of its component mathematical techniques can be highly misleading. Their reason is that real-life decisions do not arise with technical labels attached, such as *linear programming* or *risk analysis simulation*. A manager must be able to recognize what existing methods, if any, can yield worthwhile insights for taking appropriate courses of action. Granting the truth of these observations, we comment on why we organized the text by standard mathematical techniques rather than by various management problems.

Accumulated teaching experience of hundreds of instructors indicates that beginning students with only limited time for this subject better understand the model-building approach when they can *readily* discern the formal structure of the decision problem. Although realism is sacrificed in the classroom by adopting this pedagogic view, students so trained effectively bridge the gap between the techniques and their applications.

To warn students, however, of the trap of thinking that management science is synonymous with a collection of mathematical formulas, we take pains to *verbalize* the contribution that each model and technique make to decision-making. Students previously trained in science, engineering, or mathematics sometimes are surprised and even impatient that the text material is not written in a succinct mathematical style. But we suggest that *thinking beyond the details of the formulas to the deeper significance of each approach, over the long pull*, better prepares future managers and analysts to make successful applications.

Obviously, we had to exercise considerable judgment in selecting topics and in choosing the depth of treatment. In this regard, we were influenced considerably by more than 20 years of first-hand experience in applying management science to actual situations in business and government. The examples in the text often are scaled-down versions of real problems that we have encountered. The specific models that we chose, their complexity, and sequential development all reflect our observations of how most students come to appreciate and understand the pivotal concepts in management science.

Throughout, the book puts into focus the value of information derived from a management science solution and attempts to give an accurate representation of how a hypothetical example would be applied in a real situation. The text develops the student's skill in formulating and building models and, specifically, in

translating a verbal description of a decision problem into an equivalent mathematical model. The book also explains the importance and the degree of severity of a model's assumptions, the connection between the starting assumptions and the derived results, and the seriousness of the assumptions for practical purposes. Finally, the text demonstrates by means of important examples the process and usefulness of constructing analogies, finding multiple interpretations of models, and deriving significant special cases from general models.

MATHEMATICS PREREQUISITES

The calculus is employed only in Chap. 15. Nevertheless, much of the text assumes a mathematical sophistication comparable to that acquired in standard college-level introductory calculus or a no-nonsense finite mathematics course. Elementary probability theory is first needed in Chap. 12.

SUGGESTIONS TO STUDENTS

If you have taken courses in both the calculus and elementary statistics or probability theory, you will recall that you could understand much of the calculus text without much help from the instructor whereas you needed the guidance of the instructor to fully understand the statistics or probability theory text. This is a very common occurrence among students. You will find management science to be more like statistics and probability theory than the calculus in that you can expect, as a matter of course, to rely on your instructor to amplify the ideas and techniques in this book. Since the concepts can be intricate, you may have to study several pages before comprehending the full idea. So be prepared for patient reading and some rereading. We have tried to avoid giving numerical examples that are so *misleadingly* simple that you are not sufficiently prepared for the exercises at the end of each chapter. As a result, you will find it advantageous to examine carefully the numerical illustrations in the chapters.

If your instructor assigns the exercises at the back of the chapters, do not wait until the last minute to begin the problems. The numerical exercises, although straightforward, can be fatiguing if attempted in a single evening session. Most students find that formulation problems require a "gestation period," so allow yourself a few days to ponder such exercises.

Surprisingly, it is difficult to write a *verbal* description of a management science problem that is completely unambiguous. More than once, thoughtful students have discovered vague wording in problems that we had assigned previously to other classes that experienced no difficulty in obtaining the intended solutions. Consequently, if you believe that an exercise displays a troublesome ambiguity, try to resolve it in a sensible way and make explicit on your paper the specific assumption that you adopted. The purpose of the exercises (namely, to give you practice in mathematical formulation) is well served by this procedure.

SUGGESTIONS TO INSTRUCTORS

The topic coverage has been purposely designed to give you considerable flexibility in choosing the subjects that you want to stress. The logical organization of the chapters is shown in Exhibit A.

In a one-term introductory course on mathematical programming models, you can include Chaps. 1 through 8, 10, 11, and 17, skipping all the starred (*) sections and optional material in smaller print. In a one-term introductory course on probabilistic models, you can include Chaps. 1, 12 through 17, also skipping optional and advanced material.

To assist you in selecting other combinations, the "Immediate Predecessors" of each chapter are shown in Exhibit B. In general, the chapters become progressively more difficult. Thus, although you could cover dynamic programming (Chaps. 8, 9, and 10) prior to linear programming, the reverse sequence ordinarily would be preferable, because the dynamic programming chapters assume the student already is somewhat familiar with multivariate constrained optimization models and solution techniques. *Any material designated by an asterisk or appearing between the symbols ► and ◄ and set in smaller print may be skipped without loss of continuity.*

The second edition has been written to make certain topics more accessible to beginning students and to update the coverage where a subject's fundamental ideas have changed. Specifically, the *major* revisions occur in Chaps. 6, 7, 10, 11, 12, and 16. The material on networks in Chaps. 6 and 7 is now organized so that each algorithm is presented in conjunction with the model's description and application. A slower-paced exposition of dynamic programming models is given in Chap. 10 the new material includes worked out numerical examples to clarify the approach. The explanation of models and methods for integer programming in Chap. 11 is extensively rewritten to reflect computational experience of recent years; approaches that have not worked well in practice are omitted. The material on stochastic programming in Chap. 12 is revised to be more suitable for assignment in an introductory course. Finally, the exposition of computer simulation in Chap. 16 is expanded and reorganized to give greater prominence to the model-building aspects of the approach.

The book contains more than 600 exercises, many of which have multiple parts. New exercises are added to most chapters in the second edition. The problems are grouped into three categories. The Review Exercises are keyed closely to the text and determine whether the student has understood the conceptual developments. The Formulation Exercises consist of "word problems" and test whether the student can translate a verbal statement into a precise mathematical model. The Computational Exercises provide practice problems for applying algorithms. There is "redundancy" in the multiple parts of many exercises, and consequently, you should examine the parts to ascertain which ones you want to assign.

This text does not treat the topics of deterministic and stochastic dynamic programming over an unbounded horizon, continuous nonlinear programming, transform methods and imbedded Markov chains for queuing models, and gives only a brief discussion of stochastic programming and network algorithms. An

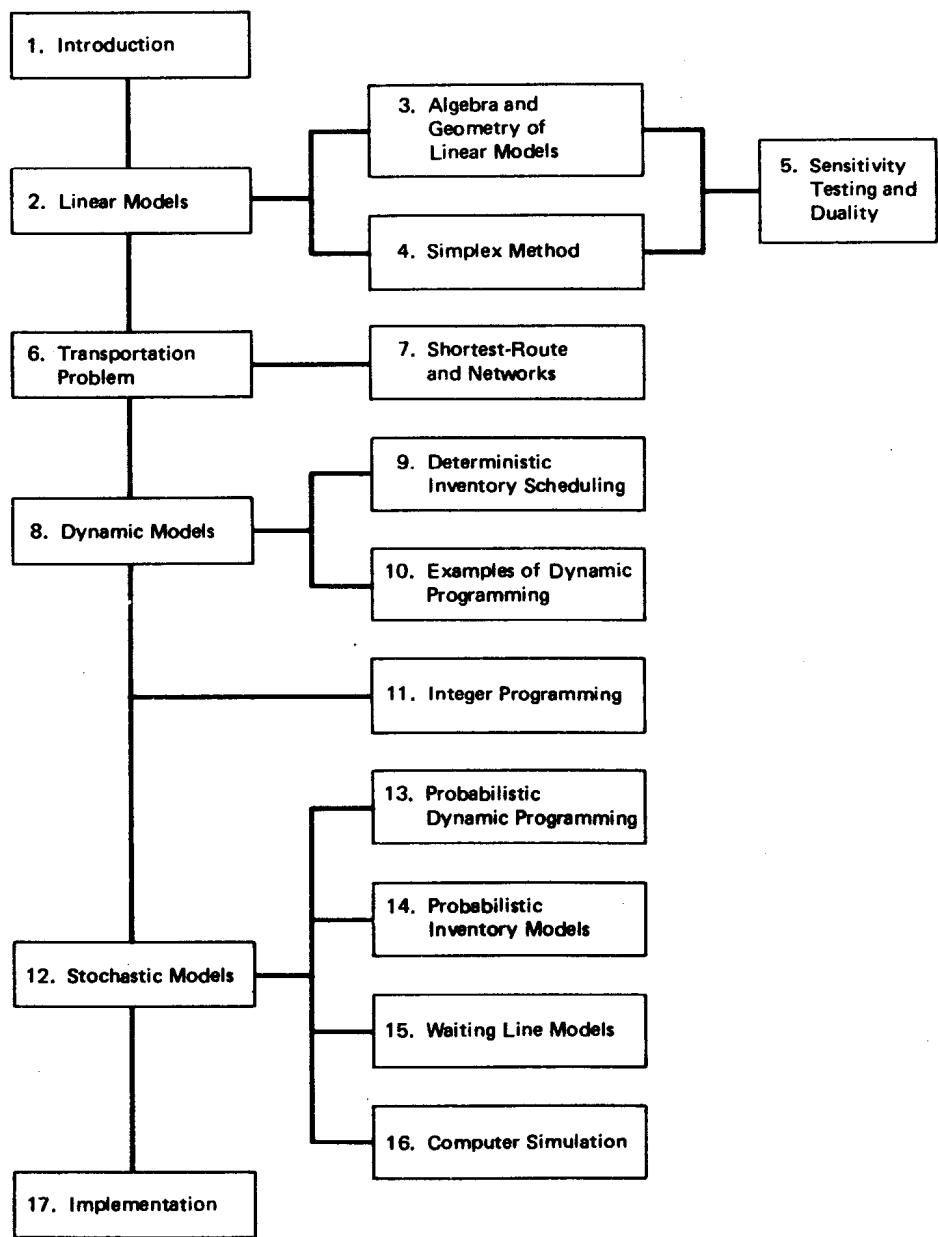


EXHIBIT A. Organization of Chapters.

CHAPTER	Immediate Predecessors
2 Linear Models	1
3 Algebra and Geometry of Linear Models	2
4 Simplex Method	2
5 Sensitivity Testing and Duality	3, 4
6 Transportation Problem	2, 5
7 Shortest-Route and Other Network Models	6
8 Dynamic Models	1, (7)
9 Deterministic Inventory Schedule	8
10 Examples of Dynamic Programming	8
11 Integer Programming	7
12 Stochastic Models	5
13 Probabilistic Dynamic Programming	10, 12
14 Probabilistic Inventory Models	(9), 12
15 Waiting Line Models	12
16 Computer Simulation	12
17 Implementation	1

*NOTE: Chaps. 12 through 16 require probability theory.
 Chap. 15 requires differential and integral calculus.
 () indicates desirable but not essential.*

EXHIBIT B

extensive coverage of these topics at an introductory level can be found in my *Principles of Operations Research*, Prentice-Hall, 1975. Important replacement models are included in the dynamic programming chapters. Game theory has been omitted. In practicing operations research, we have found that game theory does not contribute any *managerial insights* to real competitive and cooperative decision-making behavior that are not *already* familiar to church-going poker players who regularly read the Wall Street Journal. (This is not to say, however, that the intricacies of real competitive economic behavior, such as price wars, advertising campaigns, mergers, and acquisitions, have yet become phenomena fully understood by management scientists.)

ACKNOWLEDGMENTS

Over the years since the first draft of this text was written, I have received many helpful suggestions from students and their instructors who have used the book. The list of persons who have provided sage counsel is now far too long to record here, but I must offer special thanks at least to Richard W. Cottle (Stanford), Eric V. Denardo (Yale), Hamilton Emmons (Case Western Reserve), Donald Erlenkotter (University of California, Los Angeles), Charles H. Falkner (University of Wisconsin), Arthur M. Geoffrion (University of California, Los Angeles), Richard B. Hoffman (Arthur Young & Co., Washington, D.C.), Charles L. Hubbard (Georgia State), Charles H. Kriebel (Carnegie Mellon), Rudolph P. Lamone (University of Maryland), Richard P. O'Neill (Louisiana State), Alan J. Rolfe (IDA, Arlington, VA), and Matthew J. Sobel (Yale). I am deeply indebted to Professor Robert E. Machol (Northwestern) whose friendly advice prompted many of the revisions in this new edition. And I again want to acknowledge Professor Arthur F. Veinott, Jr. (Stanford) who has continued to deeply influence my own thinking about the principles of management science.

In great measure, the managerial relevance of the book has been enhanced through my association since 1960 with McKinsey & Co., as a Consultant to the Firm, and particularly through the personal guidance of Dr. David B. Hertz, Douglas Watson, Warren M. Cannon, and D. Ronald Daniel.

A fine Instructor's Manual for the second edition was patiently prepared by Arthur S. Estey and Richard A. Ehrhardt (both of Yale), and a debt of gratitude remains to John Chamberlin, John M. Harrison, and Michael Saunders who wrote the previous Manual.

The secretarial skill and encouraging dispositions of Ms. Sheila Hill (Stanford) and Ms. Marcia Wheeler (Yale) triumphed admirably over the seemingly never-ending strain of typing redrafts and meeting deadlines for the first edition. The limited pleasures of struggling over rewrites for the second edition were shared by Ms. Wilma Golden, Ms. Sara Martin, and Ms. Ellen Mester (all of Yale), at last count. And we all can be thankful for the unsparingly, but deftly, employed red pencils of editors Mr. Will Harriss (RAND) and Mr. Kenneth Cashman (Prentice-Hall).

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New Haven, Connecticut

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CHAPTER 1

The Art and Science of Executive Decisions

1.1 ONCE UPON A TIME...

The goddess Athena sprang forth full-grown and in full armor from the brow of Zeus. That is not quite the way *management science* made its debut on the contemporary scene, although there is a passing resemblance. Some of the subject's central ideas trace as far back as the 18th and 19th centuries; but at that time the prospects seemed dim that they would ever become practical tools for decision makers. Other important principles were discovered early in this century. The body of knowledge that is summarized in this text, however, by and large emerged during and since World War II.

It is not coincidental that computer technology developed in a parallel fashion. There would be negligible interest in management science beyond that generated within the fraternity of applied mathematicians were it not for the staggering analytic power made available by high-speed electronic computers. This augmentation of human intellect has turned theoretical findings into here-and-now results. To illustrate, experienced management scientists do not hesitate anymore to construct mathematical systems containing a 1000 simultaneous equations—only a few decades ago, thinking of solving such large-scale systems was as “blue-sky” as proposing to send men into outer space.

1.2 BY ANY OTHER NAME

Synonyms for the term *management science* are about as numerous and tenaciously adhered to as dialects of English. A frequent American substitute is *operations research*, and you will soon see that we use the two names interchangeably throughout this book. The British prefer *operational research*, and the subject used to be