

# Game Theoretic Revenue Management Models for Hotel Room Inventory Control

## 基于博弈理论的 酒店收益管理

宋敬普 著

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# Chapter I

## Introduction

This chapter provides an introduction to the topic of revenue management(RM) and game theoretic applications. We start with an explanation of RM and its history. We then briefly describe the industry profiles of hotels which make the RM practices distinguish from the applications in other industries and the objective of hotel RM. Next, we highlight the importance of combining ideas from RM and game theory. Finally, we conclude by giving an outline of the remaining chapters in this thesis.

### **1.1 Revenue Management and Its History**

Revenue management(RM) first appeared in the airline industry in the late 1970s when the deregulated industry attempted to maximize profit by ensuring all seats were occupied before take-off and offering varied prices to the customers. RM marries operations research/management science, statistics, economics, and software development to manage demand for a firm's inventory with the goal of maximizing revenue. Practitioners usually find that it is easier to

define the objective of revenue management rather than explain what it actually is—the outcomes are easier to understand than the process. Revenue management is about marketing mix, cost/price relationships and product distribution, which allows business to “sell the right product, to the right customer, at the right time, at the right price” (Smith et al. [45]). It is a suite of components that, when working in harmony, will present the best opportunity to maximize revenue. In 1992, Weatherford and Bodily [50] proposed to replace the term revenue management with a new, more appropriate term, Perishable-Asset Revenue Management(PARM). The element that links the industries implementing revenue management is that all of their inventories are perishable. Once the plane takes off, there is nothing one can do about trying to sell any of the seats on the plane. Similarly, when a room is empty overnight, the opportunity for revenue is lost forever.

Today, revenue management also has spread out to other industries such as hotels, retailers, car rental agencies, Internet service providers (ISP), railways, cruise lines, electric power supply and restaurants. Basically, these industry sectors all share certain characteristics that make them particularly suited for revenue management. These characteristics have been identified by Kimes [26] as: relatively fixed capacity; ability to segment markets; perishable inventory; product sold in advance; fluctuating demand and low marginal sales cost and high marginal capacity change costs. Although similar in these respects, there are still some explicit differences when different industries are subject to different combinations of duration control and variable pricing. Kimes and Chase

[27] demonstrate such differences with a pricing and duration positioning table; see Table 1.1.

**Table 1.1 Pricing and duration positioning table for various industries**

		PRICE	
		Fixed	Variable
DURATION	Predictable	Quadrant 1:  Movies Stadiums/Arenas	Quadrant 2:  Hotel Rooms Airline Seats Rental Cars
	Unpredictable	Quadrant 3:  Restaurants Golf Courses Internet Service Providers	Quadrant 4:  Continuing Care Hospitals

Comparatively, airlines, hotels and car-rental firms are more able to apply variable pricing for a product which has a more predictable duration. They conclude that successful revenue management applications generally occur in these industries(Quadrant 2), because they can manage both price and capacity. But there are still a wide variety of complications when firms implement RM techniques because every firm possesses its own industry-specific characteristics such as technology standards, consumer behavior, pricing policies etc. Carrol and Grimes [10] summarize the impact of these factors on three industries: airline, hotels and car rental firms. Nair and Bapna [35] use Weatherford and Bodily's taxonomy [50] to compare the Internet Service Provider(ISP) problem with hotel and airline revenue management problems. Several review papers describing the theory and applications of revenue management in

the airline industry have been published in recent years(see Bitran and Caldentey [5], Kevin and Piersma [25], McGill and van Ryzin [33], and Weatherford and Bodily [50]).

## 1.2 Hotel Revenue Management

The hotel industry began to apply the concept of revenue management in late 1980s when the industry faced excess capacities, competitive markets, liquidity problems and recession; all of which affected operations and resulted in lower revenue(See Hansen and Eringa [21]). Hotels can be classified as business, resorts, extended-stay, or a mix of business and leisure and also by size and location. Some hotels manage only individual properties, while large hotel chains can own hundreds of properties. A hotel, typically, offers rooms for many day-to-day lodgings of various types of customers. Despite some of the similarity with the airline customer types, the segmentation used in hotel RM are different. For example, advance-purchase discounts, a prominent segmentation mechanism of airlines, are not commonly used by hotels. Since hotels also generate significant revenues from other sources such as food, entertainment, and function space, the value of a customer is hard to determine exactly. However, these additional sources of revenue are usually not considered in hotel RM applications.

There are many different room types, such as standard rooms, deluxe rooms, executive rooms, rooms with a view, single or double bed rooms, smoking and nonsmoking rooms, etc. They can be

grouped together into three or four categories for capacity control purposes. Hotels typically aggregate both the room rate and the customer types, leading to about 3 to 10 rate bands for RM purposes. The room rates are usually adjusted only once or twice a year. Normally, a hotel room booking is made directly with the hotel (walk-in, through Internet, or by call). However, in a large hotel, approximately 20 to 40 percent of bookings come from Global Distribution Systems (GDS). The cancellation of a booking happens not only when the customer cancel the booking before the date for accommodation, but also when the customer decides to check out early. Therefore, the future capacity of the hotel is often uncertain and overbooking is widely practiced in the hotel industry.

Hotel RM mainly focuses on selling rooms in a way that maximizes total room revenue, rather than trying to sell all available rooms. For example, hotels sometimes make the customer “walk” (i.e., send elsewhere) a less valuable customer even when a room is available, to avoid walking a more valuable customer who is arriving later. This strategy may be risky since the arrivals of high-revenue customers in the future are not guaranteed. However, it is a systemized occupancy-price strategy for controlling the room rates and occupancies to maximize the total revenue. Some recent studies perceive revenue management as a managerial tool for maximizing profits, rather than revenue. For example, Donaghy et al. [13] and Griffin [20] point out that the total income calculations should include cost considerations and revenue management should move from a revenue-to a profit-generating tool. However, due to the high capital investments but low variable costs of hotel operations, increasing revenue essentially results in an

increase in operating profits.

### 1.3 Game Theory and Revenue Management

Game theory concerns itself with the analysis of competition and cooperation situations. It has found applications in diverse areas such as anthropology, auctions, biology, business, economics, management-labour arbitration, philosophy, politics, sports and warfare. During the 1960s and the 1970s, academic researchers began to apply game theory in operations research/management science area. Several reviews focussing on the application of game theory in economics or management science have appeared in the last five decades. An early survey of game theoretic applications in management science was given by Shubik [43]. Feichtinger and Jørgensen [14] published a review that was restricted to differential game applications in management science and operations research. A review of applications of differential games in advertising was given by Jørgensen [22]. Wang and Parlar [49] presented a survey of the static game theory applications in management science problems. In addition, several books (e. g. , Chatterjee and Samuelson [11], Gautschi [17], and Sheth et al. [42]) partially reviewed some specific game-related topics in management science. More recently, Leng and Parlar [29] present a review of the existing supply chain game models, under a topic classification of five areas: (i) Inventory control, (ii) production and pricing competition, (iii) service and product quality competition, (iv) sharing issues in sup-

ply chain management, and (v) strategic competition in marketing.

To the best of our knowledge, there are currently no detailed survey papers on game theoretic models in RM problem and there are very few published works directly concerning such problems. Most studies assume that the company handling perishable products (such as airline, hotel, restaurant, etc.) exists as a distinct entity. In reality, there are usually more than one company dealing with “substitutable” products in a specific geographical market. In this situation, one company’s decisions on inventory rationing, pricing, or both might be affected by the decisions of other companies. Therefore, more significant and interesting topics arising from revenue management allow us to address the following questions: How do they set the booking limits or protection levels of multiple classes products? Is there an equilibrium in inventory allocations? Is it more beneficial to be the “leader” in a Stackelberg game? How to find the optimal rationing policies when one firm has an incomplete information of the others? How much can RM increase the overall revenue if the firms cooperate? As a result, a prime methodological tool for dealing with these problems is game theory that focuses on the simultaneous or sequential decision-making of multi-players under complete or incomplete information in a competitive or cooperative context.

## **1.4 Organization of the Book**

The rest of this book is organized as follows.

Chapter 2 presents a comprehensive discussion on the existing

mathematical models which can be applied to hotel room inventory control problems. We then look at several game models which can be applied to hotel revenue management.

Chapter 3 addresses a single-period two-player two-fare-class hotel room rationing game. First, we investigate the best response functions of both players and corresponding properties. A unique Nash equilibrium of booking limit decisions is found in the competitive situation. Next, we assume that one hotel acts as the “leader” and the other as the “follower”; under this scenario we examine the Stackelberg equilibrium. For this case, we identify a situation in which the Stackelberg game is equivalent to the Nash game. This result shows that if one player’s booking limit is reached, i. e., if he always rejects low-fare customers, neither of the two players prefers to be the “leader” in the game. Finally, we examine the cooperative case where the hotels “cooperate” to maximize a system-wide objective function and find that the profit loss is substantial if there is a lack of cooperation between two players.

In Chapter 3 and 4, we assume that the games are played under complete information, i. e., each player knows the booking arrival patterns, transfer rates and rejection costs of both players. In Chapter 5, we relax this assumption and examine the static game problem under incomplete information. More specifically, we assume one player’s rejection cost and transfer rate of low-fare class customer as the incomplete information. By employing the different types of information (secret, private and public information), we discuss the game theoretic solution for the incomplete information game, which is known as Bayesian Nash equilibrium. Another

goal of the study in this chapter is to evaluate the different information types. Accordingly, we first analyze the conditions in which the information is positive(or negative). Then, we compare different information values to see which type is most valuable, and under what conditions the information benefits the player most and in what content.

Chapter 6 summarizes the concluding remarks of this book and provides some potential research directions for the future studies.