

Fundamental Electromagnetic Theory and Applications

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130
Prentice-Hall, Inc., Englewood Cliffs, N.J. 07632

8850130

Library of Congress Cataloging in Publication Data

KING, RONOLD W. P., (date)

Fundamental electromagnetic theory and applications.

Bibliography

Includes indexes.

1. Electromagnetic theory. I. Prasad, Sheila.

II. Title.

QC670.K48 1986 530.1'41 85-9360

ISBN 0-13-336959-5

Portions of this book have been previously published under the title
of *Fundamental Electromagnetic Theory*.

Editorial/production and supervision
and interior design: *Theresa A. Soler*
Cover design: *20/20 Services, Inc.*
Manufacturing buyer: *Rhett Conklin*

DR52/21

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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-336959-5 01

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall of Southeast Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Whitehall Books Limited, *Wellington, New Zealand*

010028

Preface

Electromagnetism is one of the cornerstones of classical physics. For the student it is usually the first introduction to a fully developed field theory. The concept of a vector field (the electromagnetic field) with its sources (positive and negative electric charges) and their mutual interactions is new and difficult. The fact that such interactions require a finite time (i.e., propagate with a finite velocity) is a philosophically and mathematically complicating aspect.

Electromagnetism is also one of the cornerstones of modern technology—indeed, of modern life. Electromagnetic waves through space or along wires, optical fibers and waveguides provide our principal means of communication; electric currents activate our lights, motors, computers, television and radio transmitters and receivers. Antennas on roofs, on automobiles and ships, space shuttles and satellites expand the range of our eyes and ears.

How should the physics and mathematics of electromagnetic field theory be presented to the modern student who would rather program a computer than acquire the mental and technical expertise needed to understand the fascinating complexities of the physical world? The explicit introduction of computer-based methods into a course on electromagnetic theory is not the answer. Numerical methods and computer software are invaluable tools for the modern scientist and engineer which he must certainly learn to use. But this does not mean that skill in the use of techniques like the method of moments is an acceptable alternative to physical understanding or mathematical analysis. Accordingly, this introduction to electromagnetism seeks to develop an understanding of electromagnetic phenom-

ena, their mathematical representation, and their manifold applications without the explicit added distractions and complications of computer-related methods.

In the first section of his distinguished book, *Electrodynamics*, (volume III in his *Lectures on Theoretical Physics*) Arnold Sommerfeld describes the surprising new insights which he gained as a student from the great work of Heinrich Hertz, "On the Basic Equations of Electrodynamics for Bodies at Rest." In particular, he points out that in Hertz's presentation—in contrast with the "old-style lectures" that begin with Coulomb's law and electrostatics—the equations of Maxwell are introduced initially and axiomatically as the basis of electromagnetism. He emphasizes that from them "the entirety of electromagnetic phenomena can be deduced logically and systematically. Coulomb's law, that used to provide the initial foundation, now appears as a necessary consequence of the comprehensive theory."

Fundamental Electromagnetic Theory and Applications departs from the traditional "old style" to follow the lead of Hertz in the spirit of Maxwell. It provides a moderately comprehensive and logically coordinated introduction to electromagnetism based directly on Maxwell's equations. The basic theory in the first six chapters borrows heavily from the senior author's earlier work, *Fundamental Electromagnetic Theory*, but with appropriate rearrangement and a completely modernized symbolism and standardized notation. Since the book begins with first principles, it does not depend directly on previous work in electricity and magnetism, but does presuppose an intermediate level of physical and mathematical maturity.

Chapter 1 serves a double purpose. Although directed primarily toward defining the density functions which subsequently appear in Maxwell's equations and formulating the principle of conservation of electricity, it also introduces the vector operators in terms of fundamental physical concepts, rather than merely as a mathematical symbolism. Instead of summarizing vector analysis in a separate introductory chapter or in the appendix, it is made an integral part of the logical formulation of electromagnetic principles. In this manner, it becomes associated with tangible pictures that are basic to the very subject of study. The outline of classical electromagnetism begins in Chapter 2 with the definition of the electromagnetic vectors and continues in the succeeding three chapters with the introduction of potential and energy functions. Chapter 6 is concerned with the formulation of general theorems and their applications. Beginning with Chapter 7, the theory is applied to the scattering and diffraction of plane waves, linear antennas and arrays, the foundations of electric circuit theory and the loop antenna, transmission-line theory, the insulated antenna, the theories of metal and dielectric waveguides, and to waves and antennas near and across the boundary between electrically different half-spaces.

Because investigators concerned with outward traveling waves are accustomed to the time dependence $\exp(-i\omega t)$ while those dealing with electric circuits use $\exp(j\omega t)$, both forms appear in this book. The choice is made dependent on the particular subject being discussed. The conversion from the one to the other involves the simple relation $i = -j$ except in Hankel functions, where $H_n^{(1)}(x)$ goes

over into $H_n^{(2)}(x)$ when i is changed to $-j$. Note that in the exponential the letter j is used consistently and exclusively with the positive sign, the letter i with the negative sign.

Boldface type is used for space vectors whether real or complex. Real and complex scalars are in lightface type. Readers are reminded that the so-called rotating "vectors" used to represent periodic phenomena are actually complex numbers. The "rotating vector" is a rotating pointer in the complex plane. A complex vector in boldface is a shorthand for a real space vector in boldface (often a unit vector) multiplied by a complex number in lightface.

All of the material in this book except that which appears in the last chapter has been tested in the classroom.

Acknowledgments

The first draft of the manuscript was typed in part by Barbara Cottam and Ravindran. The final draft was typed by Pat Kent and Karen Kent. Margaret Owens corrected the entire manuscript and typed the last chapter. The authors wish to acknowledge the help and encouragement given by Bernard Goodwin, Senior Editor, Prentice-Hall, Inc., throughout the preparation of the manuscript.

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