DIGITAL TECHNIQUES

in Simulation, Communication, and Control

Spyros G. Tzafestas Editor

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PREFACE

This work represents the proceedings of the 1984 IMACS European Meeting on "Digital Techniques in Simulation, Communication, and Control (DIGITECH '84)" held at Patras University, Greece (July 9-12, 1984). This Meeting belongs to a series of IMACS meetings in European countries with objective the exchange of the latest research and practical developments in the field of "System Simulation" and closely related areas. DIGITECH '84, which took place in parallel with the "First European Workshop on Real-Time Control of Large Scale Systems" has really provided a unique opportunity to our colleagues from seventeen countries for crossfertilizing interactions in the digital system engineering field.

The book involves 90 papers which are classified in the following five parts:

1. Modelling and simulation,
2. Digital signal processing and 2-D system design
3. Information and communication systems,
4. Control systems, and
5. Applications (robotics, industrial and miscellaneous applications).

The volume contains sufficient amount of information which reflects very well the state-of-art of the field of digital techniques.

I am grateful to the members of the scientific committee for their help in selecting the papers, the session chairmen for their assistance in running the meeting, and the authors of the papers for their high-level presentations.

Especially, I would like to thank Professor Robert Vichnevetsky, the President of IMACS, for his coming at the Meeting. His presence, together with the presence of Professor Manfred Thoma, the President of IFAC, who came for the Workshop, gave a special emphasis on the importance of the coupling between the IMACS Meeting and the EEC Workshop. Many thanks are also due to our distinguished colleagues who presented their exciting invited plenary papers.

Finally, a special word of thank should be addressed to the University of Patras for its hospitality and generous support.

In recent years, Greece has become the heart of a conference activity on systems, control and information sciences. It is hoped that this activity will steadily continue for the benefit of the whole Eastern Mediterranean and Middle East regions.

Patras, July 1984

Spyros G. Tzafestas

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1. MODELLING AND SIMULATION

DIGITAL TECHNIQUES in Simulation, Communication and Control
Spyros G. Tzafestas (editor)
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MODEL REDUCTION BY WALSH FUNCTION TECHNIQUES

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This paper discusses the application of Walsh functions expansion to reduce the order of a linear time-invariant system. First, model reduction of linear continuous systems by matching the Walsh spectra of output responses of the original and reduced models, subject to the specific inputs, is discussed. Secondly, model reduction under linear constraints on the structure of the reduced model is discussed. The latter has the advantage that the reduced model is stable and/or cause no steady-state error.

1. INTRODUCTION

Because of its importance in systems analysis and in the design of controllers, model reduction methods have received considerable attention over the past two decades [1]. The object of model reduction is to find a lower order model which preserves the dynamics of more complex, higher order system in both time and frequency domains. From this point of view, the model reduction in some aspects can be considered as a data matching process. The reduced order model may be determined by applying an identification procedure to input-output data obtained by driving the original system with a special input.

Recently, the Walsh functions have been used by many workers to analyse a wide range of systems [2]-[4]. The Walsh functions appear to be suited for digital processing of continuous time signals, and Walsh spectra characterisation of signals reduces the calculus of dynamic systems to an algebra in the approximate sense of least squares, through the so-called operational matrices.

In this paper, a new method via the Walsh function techniques is proposed for obtaining a reduced model for high order systems. First, the output data of the original and reduced models with respect to polynomial inputs are transfered into the Walsh spectra. Then by matching the two spectra, the parameters of the reduced model can thus be determined. Secondly, in order to preserve the stability requirement and/or to achieve steady state agreement between the original and reduced models, model reduction under linear constraints on the structure of the reduced model is discussed. Example for illustrative purpose is given with satisfactory result.

2. WALSH FUNCTIONS

The Walsh functions are a set of square waves and the system of Walsh functions is orthonormal and complete [5]. Fig. 1 shows the functions from ϕ_0 to ϕ_7 in the dyadic order.

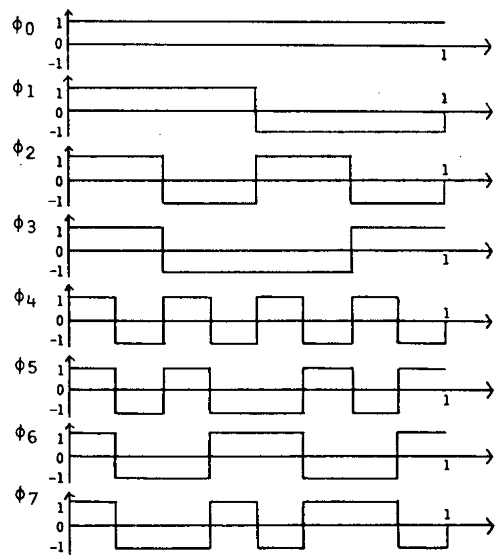

Fig. 1 Walsh functions

It is well known [6] that a square-integrable function f(t) on the interval [0,1) may be approximated in terms of the Walsh functions as

$$f(t) \simeq \sum_{i=0}^{N-1} f_i \phi_i(t) \qquad (1)$$

where $N = 2^k$, k an intger. $\phi_i(t)$ is i-th Walsh function defined in [0,1), and f_i the corresponding coefficient. Eqn. (1) can be concisely written as

$$f(t) \simeq F\Phi_N(t) \qquad (2)$$

where

$$F = [f_0, f_1, \cdots, f_{N-1}] \qquad (3)$$

and

$$\Phi_N(t) = [\ \phi_0(t),\ \phi_1(t),\ \cdots\ ,\ \phi_{N-1}(t)]^T \qquad (4)$$

The coefficient f_i are chosen to minimize

$$\varepsilon = \int_0^1 [f(t)-F\Phi_N(t)]^2\,dt \qquad (5)$$

and it is uniquely given by

$$f_i = \int_0^1 f(t)\phi_i(t)\,dt \qquad (6)$$

$\{f_i\}$ is also referred to as the spectrum of f(t).

The integration of Walsh function vector is related approximately to the Walsh function vector itself. That is,

$$\int_0^t \Phi_N(t)\,dt \approx P_N\Phi_N(t) \qquad (7)$$

where

$$P_N = \begin{bmatrix} P_{\frac{N}{2}} & -\frac{1}{2N} I_{\frac{N}{2}} \\ \frac{1}{2N} I_{\frac{N}{2}} & 0_{\frac{N}{2}} \end{bmatrix} \qquad (8)$$

$$P_2 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & 0 \end{bmatrix}$$

is called the Walsh operational matrix for integration.

Repeated application of P_N for the repeated integration implies that

$$\underbrace{\int_0^t\int_0^t \cdots \int_0^t}_{j\text{-times}} \Phi_N(t)\,dt^j \approx P_N^j\Phi_N(t) \qquad (9)$$

Thus the integration is approximately achieved by premultiplying the spectral vector with the operational matrix. The result is of considerable importance to us increasing the calculus of continuous dynamical systems to an approximate (in the sense of least squares) matrix algebra.

3. MODEL REDUCTION

Consider a linear time-invariant continuous system whose transfer function is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_1s^{n-1} + b_2s^{n-2} + \cdots + b_n}{s^n + a_1s^{n-1} + \cdots + a_n} \qquad (10)$$

where Y(s) and U(s) are respectively the Laplace transforms of the input y(t) and the output u(t). Eqn.(10) can also be represented by a differential equation

$$y^{(n)}(t) + a_1y^{(n-1)}(t) + \cdots + a_ny(t)$$

$$= b_1u^{(n-1)}(t) + b_2u^{(n-2)}(t) + \cdots + b_nu(t) \qquad (11)$$

with zero initial conditions. Integrating both side of (11) n times, we have

$$y(t) + a_1\int_0^t y(t)dt + \cdots + a_n\int_0^t\int_0^t\cdots\int_0^t y(t)dt^n$$

$$= b_1\int_0^t u(t)dt + \cdots + b_n\int_0^t\int_0^t\cdots\int_0^t u(t)dt^n \qquad (12)$$

Both y(t) and u(t) may be approximately expressed respectively by Walsh functions of size N as

$$y(t) \approx Y\Phi_N(t) \qquad (13)$$

$$u(t) \approx U\Phi_N(t) \qquad (14)$$

We now deal with the case of input functions of the form

$$u(t) = \beta_0 + \beta_1t + \beta_2t^2 + \cdots + \beta_l t^l \qquad (15)$$

where l is some intger. Note that for $l = 0$ we include step function, and for $l = 1$ the ramp function. The Walsh coefficient vector of u(t) in (15) is

$$U = \sum_{i=0}^{l} (i!)\beta_i \underline{e}P_N^i \qquad (16)$$

where

$$\underline{e} = [1,\ 0,\ \cdots\ ,\ 0] \qquad (17)$$

is the Walsh coefficient vector of the unit-step function and is derived from (6).

Substituting (13), (14) into (12) and application of (7) yields

$$Y[I + a_1P_N + \cdots + a_nP_N^n]\cdot\Phi_N(t)$$

$$= U[b_1P_N + b_2P_N^2 + \cdots + b_nP_N^n]\cdot\Phi_N(t) \qquad (18)$$

Since eqn.(18) must be satisfied for any value of t, equating of coefficients of $\Phi_N(t)$ gives

$$Y[I + a_1P_N + \cdots + a_nP_N^n]$$

$$= U[b_1P_N + b_2P_N^2 + \cdots + b_nP_N^n] \qquad (19)$$

For given values of a_i and b_i, the Walsh coefficient vector of the output is calculated as

$$Y = U\{\sum_{i=0}^{n} b_iP_N^i\}\{\sum_{i=0}^{n} a_iP_N^i\}^{-1} \qquad (20)$$

$$\because\ a_0 = 1,\ \ b_0 = 0$$

Assume that the transfer function of the reduced model is of order m with m < n. Then

$$\hat{G}(s) = \frac{\hat{Y}(s)}{U(s)} = \frac{\hat{b}_1s^{m-1} + \hat{b}_2s^{m-2} + \cdots + \hat{b}_m}{s^m + \hat{a}_1s^{m-1} + \cdots + \hat{a}_m} \qquad (21)$$

where $\hat{Y}(s)$ is the Laplace transform of output $\hat{y}(t)$ of the reduced model. The coefficient $\hat{a}_i$

and $\hat{b}_i$ are to be determined so that $\hat{G}(s)$ may be an approximate model for $G(s)$.

Letting the Walsh functions expansion of $\hat{y}(t)$ be

$$\hat{y}(t) \approx \hat{Y}\Phi_N(t) \quad (22)$$

where

$$\hat{Y} = [\hat{y}_0, \hat{y}_1, \cdots, \hat{y}_{N-1}] \quad (23)$$

Similar to the case (10), we get

$$\hat{Y}(\sum_{i=0}^{m} \hat{a}_i P_N^i) = U(\sum_{i=0}^{m} \hat{b}_i P_N^i) \quad (24)$$

$$: \hat{a}_0 = 1, \quad \hat{b}_0 = 0$$

The Walsh spectra matching means that letting $\hat{Y} = Y$ in (24). Thus

$$Y(\sum_{i=0}^{m} \hat{a}_i P_N^i) = U(\sum_{i=0}^{m} \hat{b}_i P_N^i) \quad (25)$$

Since Y is evaluated from eqn.(20) and U is given, (25) may be used to estimate the parameters of the reduced model. For $N > 2m$, the unknown parameters $\{\hat{a}_i\}$ and $\{\hat{b}_i\}$ can be obtained by the least square estimate.

Let the equation error be

$$e = Y \sum_{i=0}^{m} \hat{a}_i P_N^i - U \sum_{i=0}^{m} \hat{b}_i P_N^i$$

$$= Y - X\Theta \quad (26)$$

where

$$X = [-YP_N, -YP_N^2, \cdots, -YP_N^m, UP_N, \cdots, UP_N^m] \quad (27)$$

$$\Theta = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_m, \hat{b}_1, \cdots, \hat{b}_m]^T \quad (28)$$

It is desired to obtain the best estimate of the coefficient vector such that the cost function

$$J = e^T We \quad (29)$$

is minimized, where W is a weighting matrix. The least square estimate of Θ is

$$\hat{\Theta} = (X^T WX)^{-1} WY \quad (30)$$

Once $\hat{\Theta}$ is obtained, the reduced model of (21) is established.

REMARK : Recall that the Walsh functions are defined on the interval [0,1). Hence, if we evaluate the responses of the original and reduced models on the interval [0, T), we may change the time scaling for normalizing, by letting $\bar{t} = t/T$. Then, the Walsh operational matrix should be

$$\bar{P}_N = T P_N \quad (31)$$

Further, in order to maintain accuracy, computations have to be made with increased N, the size of Walsh functions.

4. MODEL REDUCTION UNDER LINEAR CONSTRAINTS

The above model reduction method cannot guarantee to obtain a stable reduced model if the original model is stable one, and to cause no steady state response error between the original and reduced models.

For the stability requirement, combined methods may be used. That is, the conventional stable methods such as dominant pole retention, Routh approximation, Hurwitz polynomial approximation, etc. are used to determine the coefficients $\{\hat{a}_i\}$ of the denominator of transfer function of the reduced model. Then, Walsh spectra matching is used to determine the coefficients $\{\hat{b}_i\}$ of the numerator of the reduced model.

Also, the condition that the reduced model does not produce steady state error to step-input is

$$y(\infty) = \hat{y}(\infty) \quad (32)$$

which implies that

$$b_n/a_n = \hat{b}_m/\hat{a}_m \quad (33)$$

It follows that under the constraint (33) the Walsh spectra matching must be applied.

These constraints on the structure of the reduced model can be, in general, expressed as

$$R\Theta = \Gamma \quad (34)$$

Therefore the cost function (29) is to be minimized under the linear constraint (34). Let $R^{\#}$ be any matrix which renders

$$R_* = \begin{bmatrix} R^{\#} \\ R \end{bmatrix}$$

nonsingular, and define as

$$\bar{\Theta} = R^{\#}\Theta \quad (35)$$

$$XR_*^{-1} = [X_1 \quad X_2] \quad (36)$$

then, equation error (26) can be rewritten as

$$e = (Y - X_2\Gamma) - X_1\bar{\Theta} \quad (37)$$

Hence, the least squre estimate of $\bar{\Theta}$ is

$$\hat{\bar{\Theta}} = (X_1 W X_1)^{-1} W(Y - X_2\Gamma) \quad (38)$$

and $\hat{\Theta}$ is given by

$$\hat{\Theta} = R_*^{-1} \begin{bmatrix} \hat{\bar{\Theta}} \\ \Gamma \end{bmatrix} \quad (39)$$

Thus the optimal coefficients of the reduced transfer function are completely determined.

5. ILLUSTRATIVE EXAMPLE

To illustrate the method, a model representing the pich rate control system of a supersonic aircraft [8] is considered. This is one of models considered by many workers. The transfer function is given by

$$G(s) = \frac{375000(s+0.08333)}{s^7+83.64s^6+4097s^5+70342s^4+853703s^3+2814271s^2+3310875s+281250} \quad (40)$$

The input is a unit step. Then the output's Walsh coefficient vector can be obtained from eqn.(20), and this is used for the determination of systems of order two or three.

With N = 16, T = 10 sec, and weighting matrix W = I, the following reduced models are obtained.

$$\hat{G}_1(s) = \frac{0.0212s+0.2999}{s^2+2.213s+2.545} \quad (41)$$

$$\hat{G}_2(s) = \frac{-0.0474s^2+0.5887s+0.03592}{s^3+3.988s^2+5.107s+0.3302} \quad (42)$$

In Fig. 2, a comparison is made between the step responses of the original and reduced models. As can be seen from the figure, the response of $\hat{G}_2(s)$ is a good approximation to reponse of the original system over the interval [0, 10) that was considered in the derivation of $\hat{G}_2(s)$. Since true response does not reach steady state in 10 sec, some steady state error has to be expected, and amounts to 2.1%.

In the next place, the poles and zero of the original system are

zero : -0.08333

poles : -0.09193, -2.02439 ± 0.96465,
-7.67437 ± 13.4461, -32.0752 ± 38.849

For the application of the second method, let the denominator polynomial of the reduced model be

$$s^2+4.04879s+5.02873$$

by retaining only the dominant complex pair. Using Walsh spectra matching to determine the coefficients of the numerator, for N = 16, T = 10, gives

$$\hat{G}_3(s) = \frac{-0.02874s+0.59797}{s^2+4.04879s+5.02873} \quad (43)$$

The comparison of the unit step responses of the original and reduced models are shown in Fig.3. The reduced systems show slight deterioration in the steady state responses, but this is overcome by applying spectra matching under constraint (33).

It is noted that the results could be improved by increasing the size of the Walsh spectra and/ or the time interval. Also only step response matching was examined, similar analysis could be used to match responses to other kinds of input.

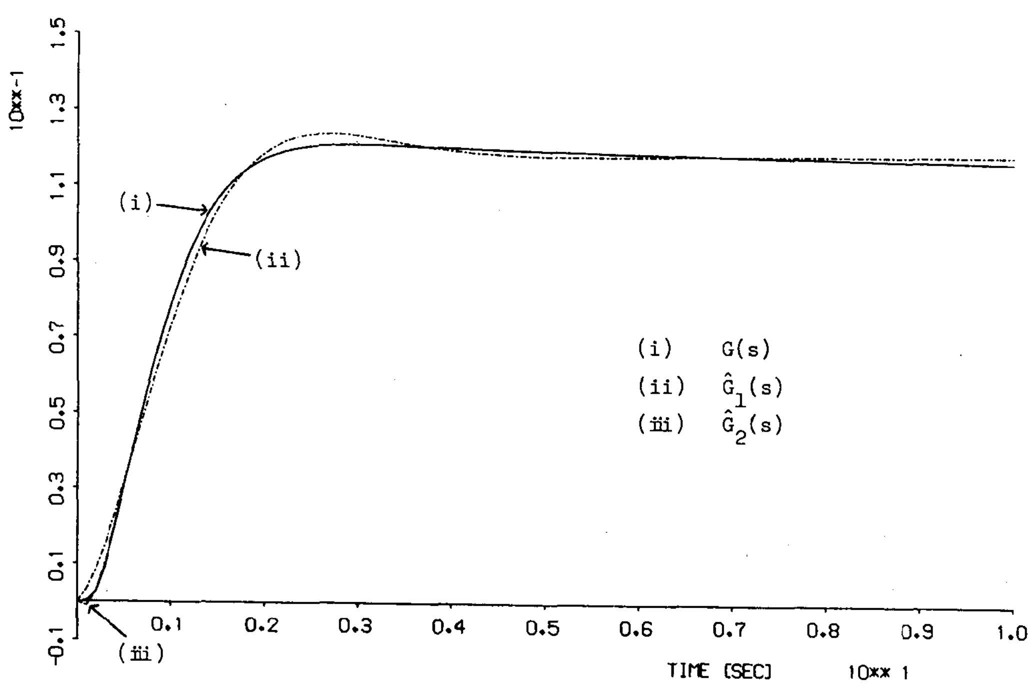

Fig.2 Step responses of the original and reduced models

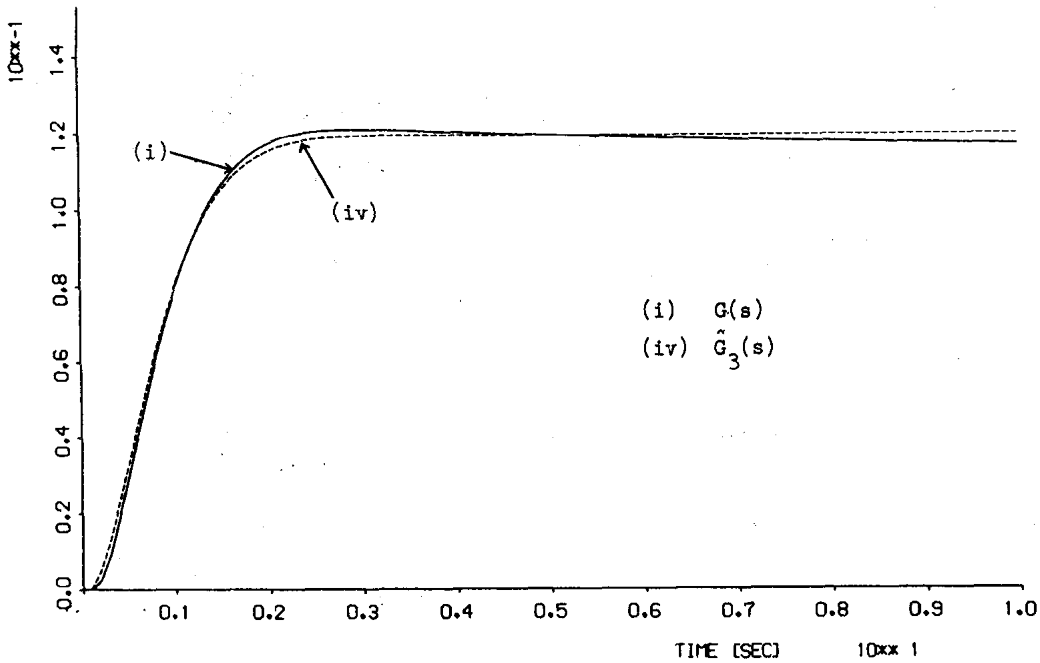

Fig.3 Step responses of the original and reduced models

6. CONCLUSION

Since information can be well kept under the Walsh transformation, the new method of using Walsh spectra matching can reserve the time-domain characteristics of the original systems satisfactory, and can be easily programmed on a digital computer. Further, by using reduction method under linear constraints, the reduced model is stable provided the original model is stable, and does not cause steady-state response error.

Other basis functions, particularly the block-pulse functions can be also used. There is no difference in the philosophy, and the format of the algorithm is the same as Walsh functions except the operational matrix for integration. Finally, it should be mentioned that the basic idea can be applied to discrete systems.

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