

INTRODUCTION TO STRUCTURES

W. R. SPILLERS. B.Sc., M.Sc., Ph.D.



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W. R. SPILLERS, B.Sc., M.Sc., Ph.D.

Professor of Civil Engineering
Rensselaer Polytechnic Institute, Troy, New York



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Foreword

Over the past 20 years massive changes have occurred in the practice of structural analysis. Those of us who graduated from an engineering school in the 1950s were taught relatively simple skills and hoped that we would never be asked to analyse structures with more than three redundants. Today a structure with 300 redundants is not considered large or thought to present any particular problem for analysis. In fact, a highly reliable solution for such a structure is usually available at a modest price. We have thus gone from a situation in which the analysis of a highly indeterminate structure posed considerable difficulty to one in which analysis capabilities are readily available and cheap.

Educational institutions have for the most part been slow to respond to these changes. While we did institute computer programming courses quickly and in some cases graduate courses in computer-aided structural analysis, surveys have shown that the undergraduate curriculum – the backbone of professional engineering – is roughly the same as it was 20 to 30 years ago. To the extent that we teach engineering, not computer programming, this lack of response to the advent of the computer may have been appropriate. But in the long run the computer will surely have its impact upon the way we teach structures. While not presuming to know how matters will eventually turn out, it is the thesis of this text that because of the computer (if for no other reason) structural engineers must know more today.

The question is, of course, how to know more. As far as this text is concerned that question is answered in two ways. In terms of depth, an attempt has been made to discuss three-dimensional problems more than has been common in the past. In terms of scope, the text moves through statically indeterminate structures and on some plastic analysis. In order to do this it has been necessary to omit some (in this context) redundant topics such as the conjugate beam and the three-moment equations.

Otherwise, the outline of this text is straightforward. It moves logically from statically determinate structures to the computation of displacements to the analysis of statically indeterminate structures. Then follow four supplementary chapters dealing with plastic analysis, cables, moment distribution, and influence lines. In terms of style, there is a tendency to include more material than the reader might want on first reading. That

Foreword

is done in the hope that he or she will return for a second look and even try the references which are indicated.

There is a 200-year tradition in structures. As a result, those of us who call ourselves structural engineers spend much time learning – really taking – from others. In my own case this includes not only teachers and colleagues but also long-suffering students at both Columbia University and Rensselaer Polytechnic Institute to whom I am grateful. What we have shared is a common interest in how structures work.

Finally a word about Ellis Horwood and his publishing house. At a time of harsh economic realities, Ellis Horwood turns out to be a creative man of great energy and enthusiasm. He is not simply a man of his word; he returns publishing to what we commonly think of as better times.

William R. Spillers

CHAPTER 1

1. Introduction and Review

1.1 MODELING

Structural analysis, the subject of this text, is for the most part concerned with finding the structural response (the lateral deflection of a building under wind load, the reaction of a bridge to a moving train . . .) given the external loads. In all but the most trivial cases, real structures, that is structures without the simplifications commonly associated with analysis, turn out to be impossibly complex. And what is finally analyzed – the structural model – may appear at first glance to be quite different than the real structure.

Constructing a structural model of a given physical situation involves discarding certain features and emphasizing others in an attempt to develop a 'reasonable' representation. In doing so the engineer must exercise judgement in knowing what to discard and when he has reached a workable model. This brings up the difference between engineering and analysis.

This text is concerned with analysis, not engineering. Given the structural model and the type of analysis to be performed, actually performing the analysis should be a matter of routine and not involve engineering judgement. However, even with analysis, engineering judgement is required at two points. It is first of all necessary to use engineering judgement to construct the model, given the real structure. At some later point in time, given the analysis, the engineer must use judgement to decide – for whatever reasons – whether or not his results make sense.

It is not possible to over-emphasize the importance of these two steps. Eventually the engineer must 'accept' his analysis and move forward with the process of design and construction. If an error of analysis leads to a design failure, he cannot simply shrug his shoulders and walk off. He is legally and ethically responsible for producing a design which functions adequately. In practical terms the only way this can be done is through developing an 'understanding' of his structure to the extent that he knows

how the analysis will turn out before he actually does it. The curious part is that this understanding is developed through performing analyses and thus one of the facets of this text.

Modeling may proceed on many levels:

- (1) *Structural modeling.* Elementary structural analysis is concerned with *skeletal structures* or structures which can be represented by lines and properties associated with lines. For example, the primary analysis of the Rio-Niterói Bridge (see the frontispiece) was probably performed on a structural model which was a beam, represented by a single line.

In order to learn to model structures properly, it is important for the engineer to observe structures and try to understand how each structure functions. (This is equivalent to making a structural model in your mind as you pass a structure.) When the functioning of a structure is obvious, so is its structural model. The truss bridge schematic of Fig. 1.1 is a case in point. Here the primary structural elements are the

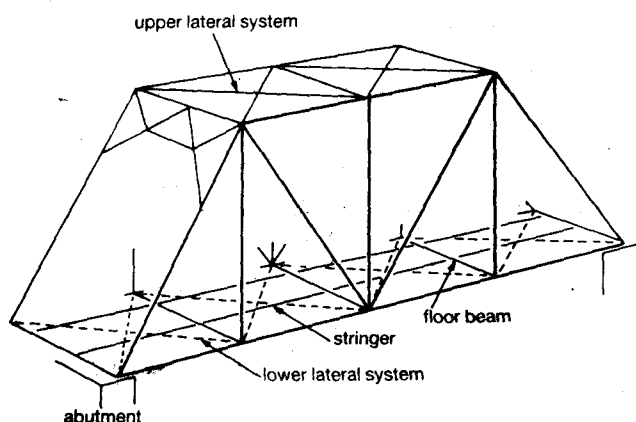


Fig. 1.1 - Schematic or 'model' of a truss bridge.

parallel trusses which transfer the loads from the bridge itself to the abutments. The typical load path involves a load on the bridge deck which is transferred to the stringers which are supported by the floor beams which frame into the truss joints. The upper and lower lateral systems are concerned with lateral load (e.g. wind) transfer and bracing against buckling.

A similar analysis can be made of the industrial building of Fig. 1.2. Schematically, the roof loads are transferred by the purlins to the

roof trusses which are supported by columns. The bracing systems again are primarily concerned with lateral load and buckling.

There is a full spectrum of structural complexity. While the two structures just mentioned function in rather obvious manners, a point

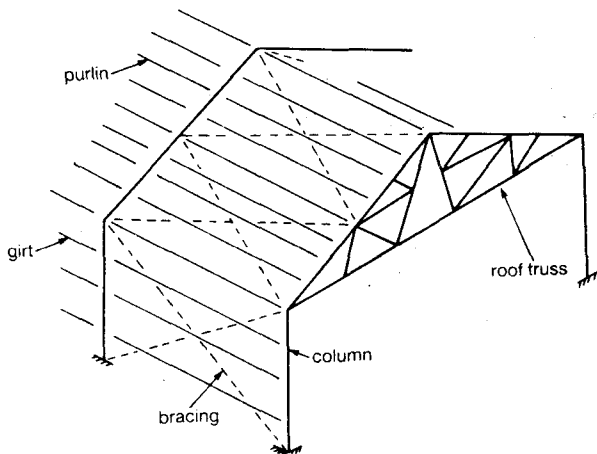


Fig. 1.2 – Schematic or 'model' of an industrial building.

load applied to a spherical shell or even a cable net (see Chapter 6) can produce a complex set of reactions which can be difficult to anticipate. In these latter cases it is even more important that the engineer develop some way in which he can 'understand' the structure for which he is responsible.

- (2) *Environmental modeling.* The term environmental modeling is first of all used here in connection with loads. While the dead load or weight of a structure should by definition be known accurately to its designer, most other cases turn out to be less clearly defined. The engineer frequently does not know precisely the uses to which his building will be put during its lifetime, he cannot anticipate all possible combinations of cars and trucks which will use his bridge, some loads such as wind, snow, and earthquakes possess a high degree of randomness...

From a practical point of view, many of these questions have been studied carefully over the years and in many cases the engineer *must* use loads specified in various codes of practice. For example, the Building Officials and Code Administrators (BOCA) code specifies various uniform live loads (loads associated with type of usage) for buildings as indicated in Table 1.1. Similarly, the American Association of State Highway and Transportation Officials

Table 1.1 Typical live loads for buildings

THE BOCA BASIC BUILDING CODE/1978

MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS

Occupancy or use	Live load (psf)
Apartments (see Residential)	
Armories and drill rooms	150
Assembly halls and other places of assembly:	
Fixed seats	60
Movable seats	100
Platforms (assembly)	100
Balcony (exterior)	100
One- and two-family dwellings only	60
Bowling alleys, poolrooms, and similar recreational areas	75
Cornices	75
Court rooms	100
Corridors	100
First floor	100
Other floors, same as occupancy served except as indicated	
Dance halls and ballrooms	100
Dining rooms and restaurants	100
Dwellings (see Residential)	100
Fire escapes	100
On multi- or single-family residential buildings only	40
Garages (passenger cars only)	50
For trucks and buses use AASHO ¹ lane loads (see Table 707 for concentrated load requirements)	
Grandstands (see Reviewing stands)	
Gymnasiums, main floors and balconies	100
Hospitals	
Operating rooms, laboratories	60
Private rooms	40
Wards	40
Corridors, above first floor	80
Hotels (see Residential)	
Libraries:	
Reading rooms	60
Stack rooms (boc) & shelving at 65 pcf) but not less than	150
Corridors, above first floor	80
Manufacturing:	
Light	125
Heavy	250
Marquees	75
Office buildings:	
Offices	50
Lobbies	100
Corridors, above first floor	80
File and computer rooms require heavier loads based upon anticipated occupancy	80

Open parking structures (passenger cars only)	50
Penal institutions:	
Cell blocks	40
Corridors	100
Residential:	
Multifamily houses	
Private apartments	40
Public rooms	100
Corridors	80
Dwellings:	
First floor	40
Second floor and habitable attics	30
Uninhabitable attics ²	20
Hotels:	
Guest rooms	40
Public rooms	100
Corridors serving public rooms	100
Corrisors	80
Reviewing stands and bleachers ³	100
Schools:	
Classrooms	40
Corridors	80
Sidewalks, vehicular driveways, and yards subject to trucking	250
Skating rinks	100
Stairs and exitways	100
Storage warehouse:	
Light	125
Heavy	250
Stores:	
Retail:	
First floor, rooms	100
Upper floors	75
Wholesale	125
Theatres:	
Aisles, corridors, and lobbies	100
Orchestra floors	60
Balconies	60
Stage floors	150
Yards and terraces, pedestrians	100

Note 1. American Association of State Highway Officials.

Note 2. Live load need be applied to joists or to bottom chords of trusses or trussed rafters only in those portions of attic space having a clear height of forty-two (42) inches or more between joist and rafters in conventional rafter construction, and between bottom chords and any other member in trusses or trussed rafters shall be designed to sustain the imposed dead load or ten pounds per square foot (10 psf) whichever be greater, uniformly distributed over the entire pan.

A further ceiling dead load reduction to a minimum of five pounds per square foot (5 psf) or the actual dead load, whichever is greater, may be applied to joists

in conventional rafter construction or to the bottom chords of trusses or trussed rafters under either or both of the following conditions:

- a. If the clear height is not over thirty (30) inches between joists and rafters in conventional construction and between the bottom chord and any other member for trusses or trussed rafter construction.
- b. If a clear height of greater than thirty (30) inches as defined in 'a' directly above, does not exist for a horizontal distance of more than twelve (12) inches along the member.

Note 3. For detailed recommendations see The Standard for Tents, Grandstands, and Air-Supported Structures Used for Places of Assembly listed in Appendix B.

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(AASHTO) code specifies standard trucks (see Fig. 1.3) to be used in the design of highway bridges, and the American Railway Engineering Association (AREA) specifies standard trains for railway bridges (see Fig. 1.4).

By definition, the design of a 'conventional' structure must follow some given building code. In the interesting case of a new type of structure or a structure with monumental proportions, the engineer will go to great lengths to ensure that his design is adequate. For example, wind loading on a fabric structure of unusual shape and certainly wind loading on a building of record height commonly require model studies in wind tunnels (at considerable expense). Finally, there are common problems such as foundation settlement which routinely require soil samples taken in the field to be tested in the laboratory. Less common are special problems of a corrosive environment (certain types of manufacturing, sanitary sewers . . .), dynamic effects (heavy manufacturing, crane loads . . .), wave action on ocean platforms . . . The list is endless but the point to be made is clear. The engineer must first understand the environment of his structure and then design for it.

- (3) *Material modeling.* The treatment of structural materials is another area in which it is common to make engineering approximations within a structural analysis. So far as this text is concerned, two types of material assumptions will be made. A material will be assumed to be