LINEAR SYSTEMS AND DIGITAL SIGNAL PROCESSING

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PRENTICE-HALL, INC., Englewood Cliffs, NJ 07632

PREFACE

Courses in Digital Signal Processing (DSP) have been offered at the graduate and senior level by Electrical Engineering departments for several years. As they require a course in linear systems as a prerequisite, many Electrical and Computer Technology students are precluded from taking DSP as linear systems courses are not usually offered in their programs.

This text is based upon the lectures that presented the concepts of DSP to the Bachelor of Engineering Technology students at Rochester Institute of Technology, and it reflects the concept that DSP should be presented as a regular part of a technology program. It begins with a chapter that serves to introduce the reader to linear systems and to provide the background for the DSP material of the later chapters.

This is followed by chapters that introduce sampling concepts, the representation of a discrete signal using the z transform, digital systems, and the Fast Fourier Transform (FFT). The material on analog processing and interfacing the analog and digital worlds was kept brief as there are many excellent texts and articles on these topics. This allowed more space to be dedicated to the main ideas of linear systems and DSP. An appendix on analog filters is included to serve as a refresher or as a brief introduction. This provides the reader with some background when analog filters are encountered in the text.

Due to the rapid development in the hardware used to process the digital signals, it was decided to concentrate on some of the ICs that are available and to present only a brief look at the devices presently in use. The large number of journals and magazines along with the literature provided by the manufacturers should be consulted to keep abreast of the introduction of more powerful and faster devices.

In its present form, the text is suitable for people in Electrical Engineering who would like an introduction to DSP. It can also be used as a primer before reading the classical texts on the topic. In addition, the large number of people in other areas of Engineering, Computer Science, and Computer Technology will also find it valuable as an introductory source of information.

I would like to take this opportunity to thank the manufacturers who allowed me to use their data sheets, the students who provided feedback on the material of the text, and the people at Prentice-Hall whose work helped bring this text into the world.

THOMAS YOUNG

CONTENTS

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1 LINEAR SYSTEMS

The main purpose of this text is to introduce the reader to *digital signal processing* (DSP); however, it is advisable to present the concepts and terminology of linear systems first, as many of the techniques and applications of DSP require an understanding of them. In this chapter we consider the linear system in sufficient detail to provide the background required for later chapters.

When applied to a system, the concept of linearity allows one to use relatively simple analytical techniques to determine the input and output (I/O) relations of the system or its transfer function. These techniques will be examined from the time- and frequency-domain points of view. This beginning will lead us to the concepts of convolution (filtering) and correlation, preparing us for their use in DSP.

An *electrical system* is defined as any combination of electrical components. The system can be as simple as an *RC* circuit or as complex as a computer-controlled space shuttle. Regardless of a system's actual composition, we will assume that the relationship between its output and input signal is linear to simplify our analysis. Figure 1–1 will be used to represent an arbitrary system.

As the first step in establishing the concept of linearity, the definition of a function is presented. This will be useful as the values assumed by the input

Figure 1-1 Block representation of an arbitrary system.

and output signals of the system are determined by their functional relationships with time and frequency.

1.1 DEFINITION OF A FUNCTION

A function is defined when the following three items are provided: (1) a collection (set) of numbers, real or complex, called the *domain*; (2) a second set of numbers, also real or complex, called the *range*; and (3) a rule that relates the numbers in the range to numbers in the domain.

1.1.1 Domain

In electronics, the domains of interest usually consist of time or frequency values. Once a domain is decided upon (e.g., time), we can arbitrarily select any one of its members. As the values in the domain are chosen at our convenience, the domain is termed the *independent variable*.

In specifying a domain, maximum and minimum values are indicated which form the upper and lower limits of the domain. If all numerical values between the limits, possibly including them, are elements of the domain, it is termed *continuous*; if only a finite number of values are allowed, the domain is *discrete*. As an example, if all values of time between, and including, the limits of 0 and 10 seconds form the domain, it is continuous; if only integer values of time (e.g., 0s, 1s, 2s, . . . , 10s) form the domain, it is discrete.

If either or both of the limits of the domain are infinity, the domain, whether discrete or continuous, is said to be *infinite*. As an example, it is possible to consider all values of frequency from $-\infty$ to $+\infty$ as the domain, making it continuous and infinite in extent. Had we chosen a discrete set of frequency values as our domain, it would be discrete and infinite in extent. When representing functions in two- or three-dimensional graphs, the domain is the *abscissa*.

1.1.2 Range

The range usually consists of a collection of real or complex numbers that represent either voltage, current, power, the amplitude of a frequency component, or a phase-shift angle. As with the domain, the range can be continuous (all values allowed) or discrete (a finite amount of values allowed), finite or infinite in extent. As the numbers that form the range are determined by the choice of elements in the domain, the former are termed *dependent variables*. Even though a dependency exists between the values of the elements of a domain and its range, the range can be continuous or discrete, independent of the domain: a continuous domain, time, may give rise to either a continuous or a discrete range. Examples of this are the voltage output of a sine wave generator (continuous domain-continuous range) or the voltage of a digital signal (continuous domain-discrete range). A discrete domain may also give rise to either a continuous or a discrete range, as with the amplitudes of frequency components obtained by a Fourier analysis (discrete domain-continuous range) or the output of an analog-to-digital converter (ADC) (discrete domain-discrete range) in which the output levels can assume only discrete values at discrete sampling times. These relationships are displayed in Figures 1–2 and 1–3. In a graphical representation of the range, it is termed the *ordinate*.

1.1.3 Rule

The rule that states how to relate numbers in the domain (e.g., time, t, or frequency, f) with numbers in the range (e.g., voltage, v, power, p, or spectral component amplitude) completes the definition of a function. As an example, consider the voltage function, $v = v(\cdot)$, where $\cdot$ represents an arbitrary set of numbers forming the domain of interest, v represents the set of numbers forming the range, and $v(\cdot)$ represents the rule that specifies the way to relate values in the domain (t or f) to those of the range (e.g., v). Another name for the term inside the parentheses is the *argument*.

Choosing the time domain and selecting an element, t_1, the rule will identify a particular value of the range, v_1, associated with t_1. This is shown in Equation 1–1.

$$v(\cdot) = 10 \sin 2\pi f \cdot \quad (1\text{–}1a)$$

$$v = v(t) = 10 \sin 2\pi ft \quad (1\text{–}1b)$$

$$v_1 = v(t_1) = 10 \sin 2\pi ft_1 \quad (1\text{–}1c)$$

where Equation 1–1a specifies the rule, Equation 1–1b indicates the range and domain, and Equation 1–1c identifies the two numbers, v_1 and t_1, that are related by the rule. For any value of t (continuous and infinite domain), the values the voltage can assume are limited to all values between and including +10 and −10 V (continuous and finite range). What mathematicians call a

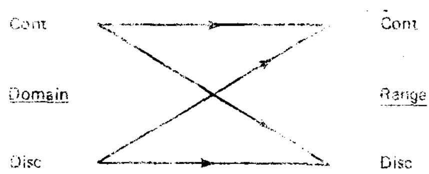

Figure 1–2 Representation of the domain/range relationship.

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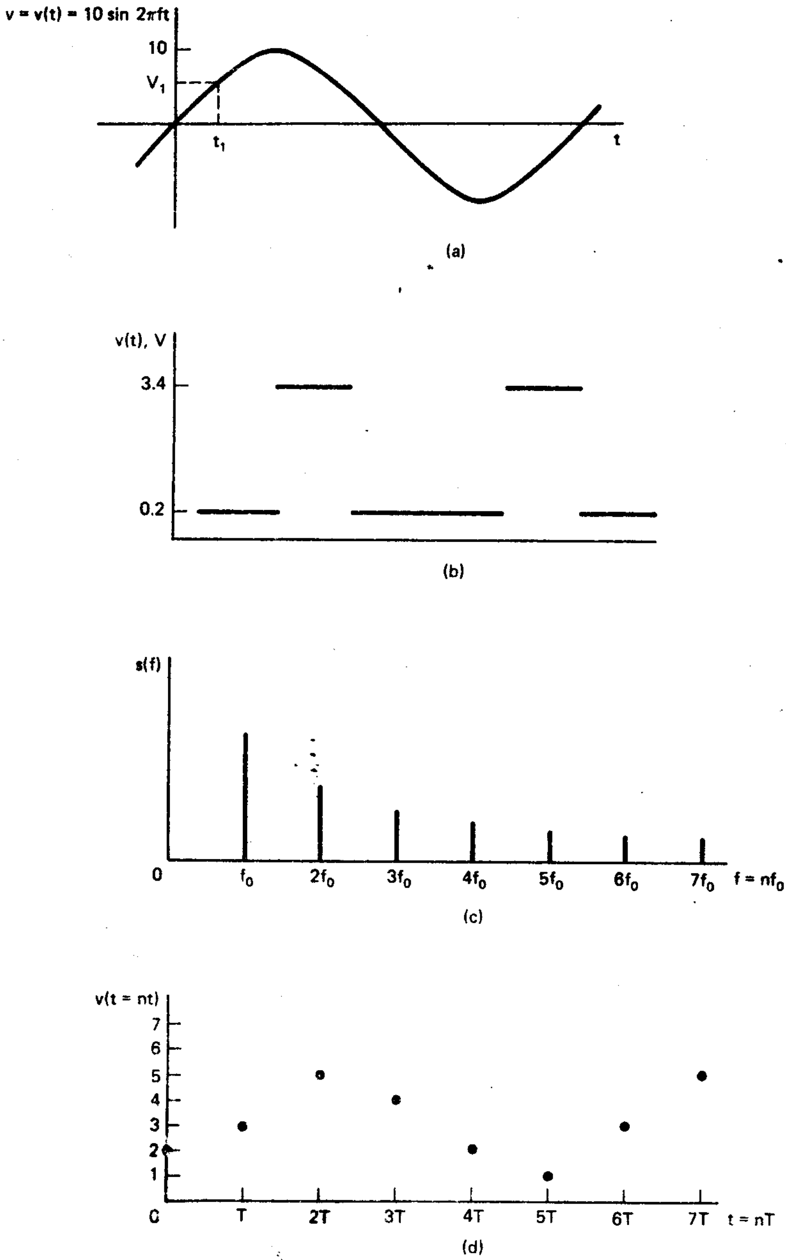

Figure 1-3 Graphical representation of a function; (a) continuous domain, continuous range; (b) continuous domain, discrete range; (c) discrete domain, continuous range; (d) discrete domain, discrete range.

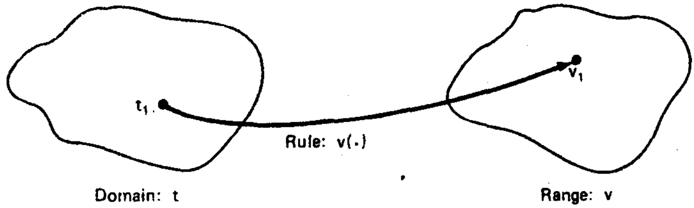

Figure 1–4 Pictorial representation of a time function.

function, we in the area of electronics call a signal: a voltage or current whose value (the range) can be measured or observed at different times (the domain). We will use the terms signal and function interchangably. A signal whose domain and range are continuous is termed *analog*.

In order to have our results relate to practical experience, the functions we will use in this text are absolutely integrable [i.e., the integral of the function's magnitude over all time must be finite]. This is expressed in Equation 1–2.

$$\int_{-\infty}^{\infty} |f(t)|\, dt \leq M < \infty \tag{1–2}$$

where | | indicates magnitude and M is a finite constant. If the function represents voltage or current, this is another way of stating that the signal contains a finite amount of energy. Figures 1–3, 1–4, and 1–5 are graphical and pictorial representations of a function.

1.1.4 Implicit Function

If the domain of a given function is itself the range of another function, an implicit relationship exists between the functions. This occurs, for example, when we specify the domain in radians, ω, in the function $v = v(\omega)$. ω is also the range for the function $\omega = \omega(f) = 2\pi f$. This yields the implicit function $v = v(\omega(f)) = v'(f)$. Either by specifying $f = f_1$ for $v'(f)$ or $\omega = \omega_1$ for $v(\omega)$, we will obtain $v = v_1$. This is shown pictorially in Figure 1–6.

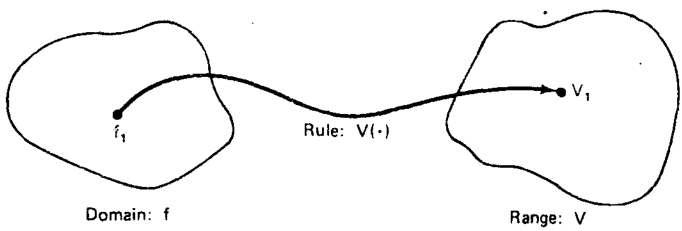

Figure 1–5 Pictorial representation of a frequency function.

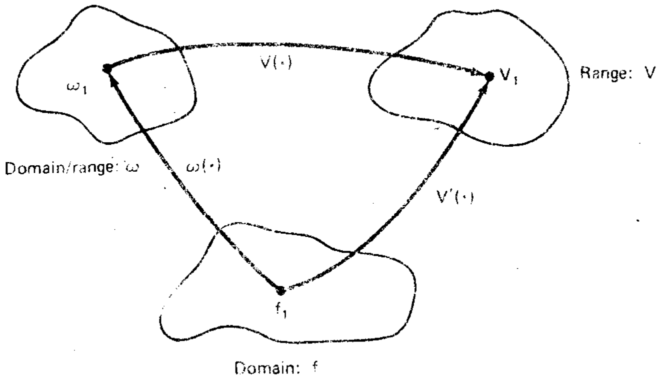

Figure 1–6 Pictorial representation of an implicit function.

By the foregoing definition of a function, $v(\omega) \neq v'(f)$; however, as the arguments are functionally related [$\omega = \omega(f)$ in this situation] and they have the same range, we will consider $v(\omega)$ and $v'(f)$ as being equivalent: $v(\omega) \Longleftrightarrow v'(f)$. From a graphical point of view, this implies that we are using a different abscissa to display the function.

Example 1–1

Represent the magnitude of the frequency components of a periodic signal (e.g., a square wave) using radians (ω) and hertz (f).

Solution This is given in Figure 1–7, where $|C_n|$ has been plotted against both domains, ω and $f = \omega/2\pi$.

In Example 1–1, both domains were discrete, while the range was continuous.

When dealing with implicit functions, we will make use of their equivalency in choosing a domain that will provide a different quantity of information.

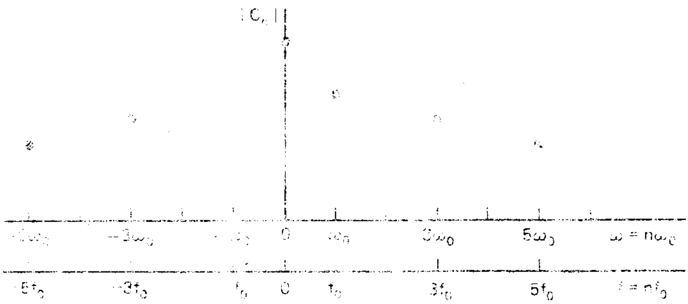

Figure 1–7 Solution to Example 1–1.

1.2 LINEARITY

Linearity can be defined using the arithmetical concepts of addition and multiplication as applied to functions.

1.2.1 Addition

Given a system with an input that consists of a sum of signals, if the system is linear, its output is obtained by summing the individual outputs for each of the input signals. This can be shown by considering the following. Given two system inputs, $x_1(t)$, and $x_2(t)$, that produce the outputs $y_1(t)$ and $y_2(t)$, respectively, $(x_i(t) \longrightarrow y_i(t))$, an input $x(t) = x_1(t) + x_2(t)$ will produce an output $y(t) = y_1(t) + y_2(t)$. This is shown in Figure 1–8. For N inputs, we can generalize the definition of linearity as in Equation 1–3.

$$\sum_{i=1}^{N} x_i(t) = x(t) \quad \longrightarrow \quad y(t) = \sum_{i=1}^{N} y_i(t) \tag{1–3}$$

where $y_i(t)$ is the output due to the input $x_i(t)$.

Example 1–2

$x_1(t) = 3t + 2$ and $x_2(t) = 2 - t/2$. If a linear system produces $y_1(t) = (t/4) - 1$ and $y_2(t) = t$ for the inputs above, what is the system output if its input is $x(t) = x_1(t) + x_2(t) = (3t + 2) + (2 - t/2)$?

Solution Using Equation 1–3, we have

$$\begin{aligned} y(t) &= y_1(t) + y_2(t) \qquad (1–3) \\ &= \left(\frac{t}{4} - 1\right) + t \\ &= \frac{5t}{4} - 1 \end{aligned}$$

1.2.2 Multiplication

If a system is linear, multiplying its input by a (complex) constant will cause the output to be multiplied by the same constant. If an input signal

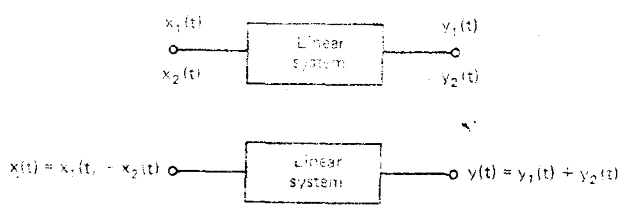

Figure 1–8 Representation of a linear system: addition.

$x(t)$ produces an output $y(t)$, then $cx(t)$ applied at the input will produce the output $cy(t)$, where c can be real or complex. This is given in Equation 1–4 for N inputs, each multiplied by a (possibly) different constant, c_j.

$$\sum_{j=1}^{N} c_j x_j(t) = x(t) \longrightarrow y(t) = \sum_{j=1}^{N} c_j y_j(t) \tag{1–4}$$

where $y_i(t)$ is the output due to $x_i(t)$ and c_i may be real or complex. In this text we assume that all systems discussed are linear.

1.3 TIME INVARIANCE

If the components that form the system are constant in value (i.e., they do not change with time) the system is said to be *time invariant*. To define time invariance mathematically, we introduce the time-shifted, or time-delayed function. If $s(t)$ is a given function, then a time-delayed version of this function is $s(t - \tau)$, where τ is the amount of the delay. This is shown in Figure 1–9.

The effect of changing the domain (argument) from t to $t - \tau$ is to *shift the graphical representation of the function to the right on the time axis* for values of $\tau > 0$. This can be stated as follows: The time-shifted function will provide the same set of values (range) for $s(t - \tau)$ as $s(t)$; that is, their ranges are identical when their arguments assume the same value. This is shown in Equation 1–5, which is the functional form of the waveform in Figure 1–9.

$$s(\cdot) = \begin{cases} \dfrac{2\text{ V}}{T_1}(\cdot), & 0 \le \cdot \le \dfrac{T_1}{2} \\ \dfrac{-2\text{ V}}{T_1}(\cdot) + 2\text{ V}, & \dfrac{T_1}{2} \le \cdot \le T_1 \\ 0, & \text{otherwise} \end{cases} \tag{1–5a}$$

$$s(t) = \begin{cases} \dfrac{2\text{ V}}{T_1} t, & 0 \le t \le \dfrac{T_1}{2} \\ \dfrac{-2\text{ V}}{T_1} t + 2\text{ V}, & \dfrac{T_1}{2} \le t \le T_1 \\ 0, & \text{otherwise} \end{cases} \tag{1–5b}$$

$$s(t - \tau) = \begin{cases} \dfrac{2\text{ V}}{T_1}(t - \tau), & 0 \le t - \tau \le \dfrac{T_1}{2} \\ \dfrac{-2\text{ V}}{T_1}(t - \tau) + 2\text{ V}, & \dfrac{T_1}{2} \le t - \tau \le T_1 \\ 0, & \text{otherwise} \end{cases} \tag{1–5c}$$

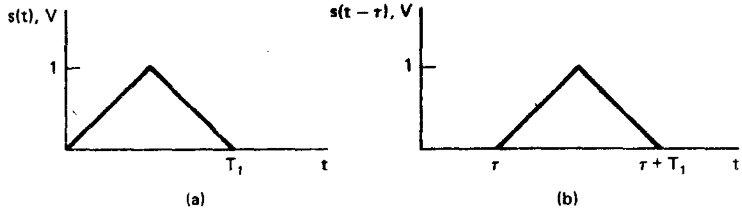

Figure 1-9 A signal and its time delayed form.

Equation 1-5b is the nonshifted waveform (Figure 1-9a) and Equation 1-5c is the time-shifted waveform (Figure 1-9b). Equation 1-5c is obtained from Equation 1-5a by replacing $(\cdot)$ with $(t - \tau)$, where τ is a parameter [i.e., a "constant" variable], that indicates the amount of the shift or delay. For negative values of τ, the shift would be to the left, representing an advanced signal. To observe the values of t for which $s(t - \tau)$ has a nonzero value, we add τ to both sides of the inequality in Equation 1-5c to obtain Equation 1-5d.

$$s(t-\tau) = \begin{cases} \dfrac{2\text{ V}}{T_1}(t-\tau), & \tau \le t \le \dfrac{T_1}{2} + \tau \\ \dfrac{-2\text{ V}}{T_1}(t-\tau), & \tau + \dfrac{T_1}{2} \le t \le T_1 + \tau \\ 0, & \text{otherwise} \end{cases} \quad (1\text{-}5\text{d})$$

The effect of time invariance on a linear system can be stated as follows: If an input signal $s(t)$ produces an output $y(t)$, a time-invariant (TI) system will produce $y(t - \tau)$ if the input is $s(t - \tau)$. This is shown in Figure 1-10. For a time-invariant system, there is no change in the shape of the output

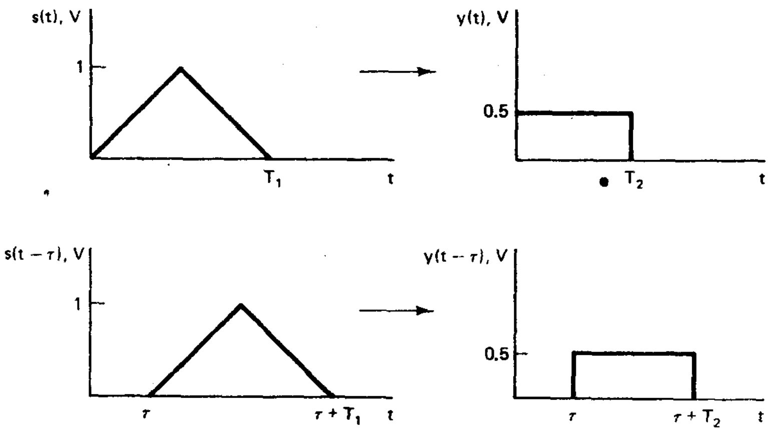

Figure 1-10 Effect of time invariance on system output.

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