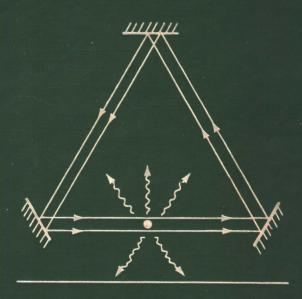
# ATOM-PHOTON INTERACTIONS

Basic Processes and Applications



Claude Cohen-Tannoudji Jacques Dupont-Roc Gilbert Grynberg

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#### **Preface**

The spectacular development of new sources of electromagnetic radiation, covering a range of frequencies from radio waves to far ultraviolet (lasers, masers, synchrotron radiation, microwave sources), has resulted in considerably renewed interest in photon-atom interactions. New methods have appeared for obtaining more precise information about the structure and dynamics of atoms and molecules, for controlling their internal and external degrees of freedom, and for generating new types of radiation. These developments have caused a growing number of physicists, chemists, researchers, and engineers to become interested in interactions occurring between matter and low-energy radiation. With these two books on photons and atoms, our aim is to provide the theoretical bases necessary for undertaking the study of these processes beginning at a level of quantum mechanics and classical electromagnetism corresponding to that of first-year graduate course.

Such a program is naturally composed of two parts. First, one must introduce a theoretical framework that can be used to describe the quantum dynamics of the global system "electromagnetic field + nonrelativistic charged particles" and discuss the physical content of the theory, as well as its different possible formulations. These problems have been studied in a previous volume entitled *Photons and Atoms—Introduction to Quantum Electrodynamics*. Second, one must show how such a theoretical framework can be used to analyze the interactions between photons and atoms as they appear in atomic and molecular physics, quantum optics, and laser physics. This is the goal of the present volume entitled *Atom-Photon Interactions: Basic Processes and Applications*. The goals of these two volumes are thus clearly distinct and, depending on the concerns and needs of the reader, one or the other or both volumes of this work may be used.

It is, of course, impossible to present in a single volume an exhaustive study of the interaction between matter and radiation and of all the related physical phenomena. We have thus emphasized the aspects that we consider to be essential. First we will analyze in detail the elementary processes in which photons are emitted, absorbed, scattered, emitted and reabsorbed, or exchanged between atoms. Extensive use of diagrammatic representations will allow us to visualize the processes being described. A knowledge of these elementary processes is, nevertheless, not always sufficient for analyzing in simple terms the extremely large variety of

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phenomena which may result from the interplay of these processes. Thus we thought it important to bring together in this book different theoretical approaches, which are usually dispersed in more specialized works, and which are more particularly adapted to one aspect or another of the phenomena being discussed (perturbative methods, resolvent method, master equation, Langevin equation, optical Bloch equations, dressed atom method, etc.). Finally, we have decided to illustrate each of these methods in simple systems, so as to be able to show as clearly as possible their significance and their limitations. Our hope is to have integrated in this volume the basic elements allowing the physics of the matter-radiation interaction be mastered in all its different aspects.

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We are also especially grateful to Martine Guillaume, Patricia Bouniol, and Catherine Emo who were in charge of preparing the manuscript, and to Christophe Salomon, who checked the translation.

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