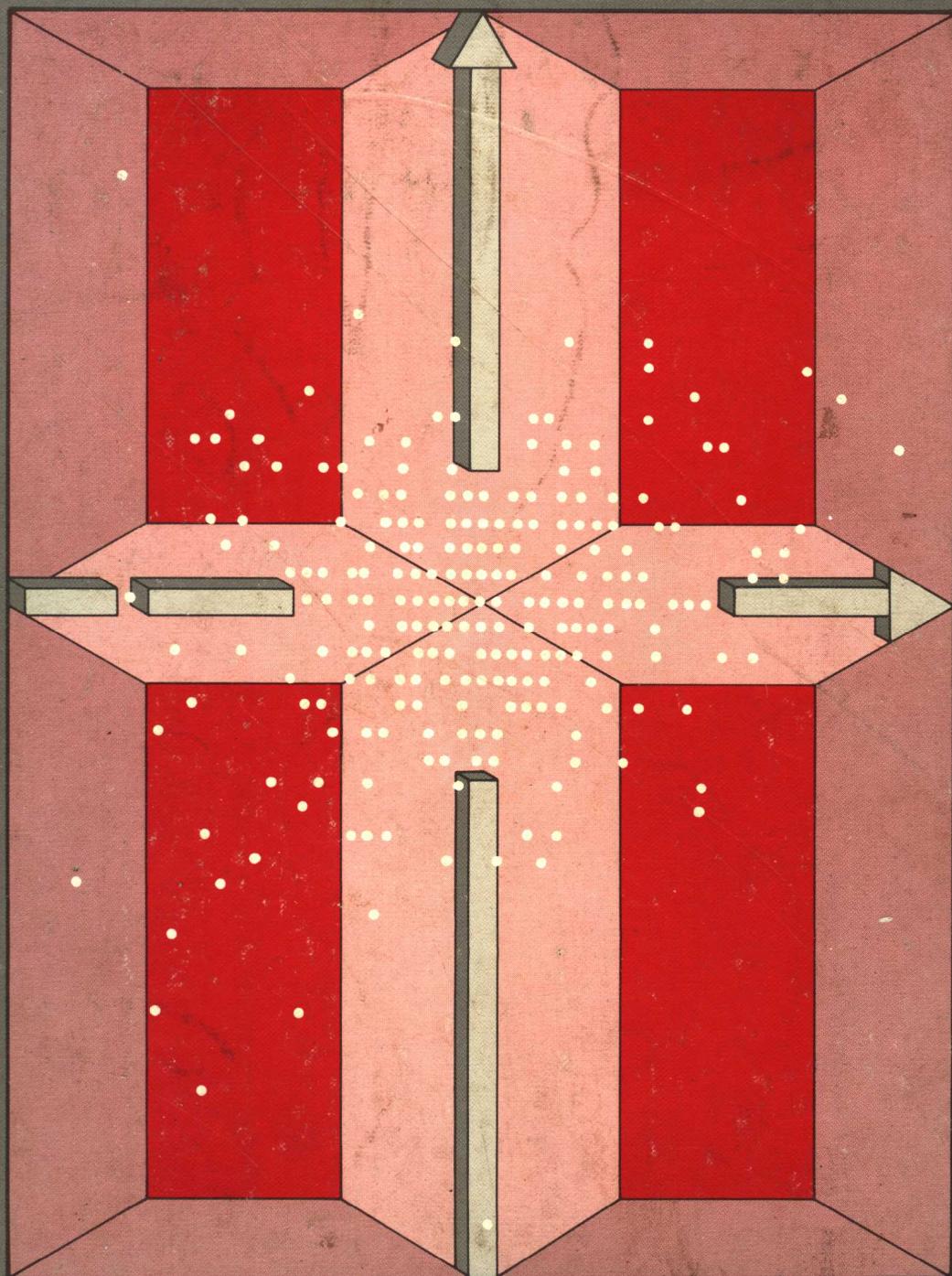


TIME SERIES AND SYSTEM ANALYSIS WITH APPLICATIONS



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PREFACE

Engineers and scientists in system analysis use mathematical models, usually differential equations, developed from a conjectured physical mechanism. For complicated systems, an empirical approach, such as frequency analysis, is employed. On the other hand, statisticians and economists approximate their models, using difference equations, from plots of empirical autocorrelations and spectra. If time series and system analysis are brought together, it should be possible to avoid the considerable trial and error presently needed in both fields and vastly improve their applications. In the 1973 Ph.D. dissertation of S. M. Pandit, with S. M. Wu as advisor, a new philosophy of system analysis that bypasses this element of trial and error and provides models in the form of difference/differential equations directly from the observed data was outlined with the requisite mathematical foundation. The enthusiastic response of faculty and students to this philosophy and its extensive applications in diverse research investigations led to the need for this book.

An application of this new philosophy to time series modeling yields a sequential strategy, as in regression analysis. Once the time series is considered as the response of a system, it can be modeled with increasing degrees of freedom justified by the data. Successively higher order models are fitted by least squares until the improvement in the fit is statistically insignificant. The sequential modeling strategy can be conveniently carried out as a result of the ever-increasing capabilities of computers. The new modeling strategy can greatly reduce the tedious chore of searching for an appropriate model. We hope that this book will bring together time series and system analysis to provide system analysis specialists with a new tool and to make time series analysis useful to engineers and scientists.

The book is application-oriented. The new tool has been used for system identification, signature analysis, physical characterization, control, and even engineering design. Obviously, it is most useful for forecasting, which was the original purpose of the time series development. To enhance the ability for long-term forecasting, both stochastic and deterministic approaches are presented.

The first draft of the book grew out of lecture notes prepared for a one-semester course taught in 1973 at the University of Wisconsin, Mad-

ison, to graduate and senior undergraduate students. Revised drafts have been used in subsequent years for this course, as well as for an undergraduate-graduate two-course sequence at other schools. Students from disciplines such as civil and environmental, electrical, industrial, mechanical, mining, metallurgical, and nuclear engineering, as well as from economics and business have enrolled. Many of the examples in this book are based on the class projects undertaken by these students. We are grateful to the students for their gratifying response.

We especially thank Dr. Shiv. G. Kapoor and William Wittig for their contribution toward developing computer programs to fit models and for their help, along with Drs. T. Ungpiyakul and W. T. Tsai, in improving the final manuscript. We are indebted to Professor R. E. DeVor, University of Illinois, Urbana, and Professor W. R. DeVries, Rensselaer Polytechnic Institute, for their constructive comments.

Sudhakar M. Pandit

Shien-Ming Wu

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