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几类动力学系统的对称性和守恒量研究

● 作者：傅景礼

● 专业：一般力学与力学基础

● 导师：陈立群

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答辩委员会对论文的评语

傅景礼同学的博士学位论文研究对称性和守恒量及其在机电系统分析力学和相对论性 Birkhoff 系统动力学中的应用,选题为分析力学的前沿课题,具有重要的理论意义.

主要创新工作包括:

1. 研究了非保守完整和非完整系统的非 Noether 对称性与非 Noether 守恒量;

2. 研究了有限自由度系统的定域 Lie 对称性并讨论了相应的守恒量;

3. 研究了机电耦合系统的 Noether 对称性、Lie 对称性、形式不变性和非 Noether 对称性,以及相应的 Noether 守恒量和非 Noether 守恒量理论;

4. 研究了相对论性 Birkhoff 系统的 Noether 对称性、Lie 对称性、对称性摄动与绝热不变量.

论文选题有相当难度,理论性强,工作量大,是一篇优秀博士论文.论文反映出作者较全面地掌握了与本课题相关的国内外发展动态,显示了作者具有坚实宽广的基础理论和系统深入的专门知识,具有很强的独立科研能力.

在答辩中论述清楚,回答问题正确.经答辩委员会投票表决全票(5 篇)通过博士学位论文答辩,并建议授予傅景礼同学博士学位.

答辩委员会表决结果

经答辩委员会表决，全票同意通过傅景礼同学的博士学位论文答辩，建议授予工学博士学位.

答辩委员会主席：**梅凤翔**

2004年6月25日

摘　要

本文研究约束力学系统、机电耦合系统和相对论性Birkhoff动力学系统的对称性和守恒量问题，包括Noether对称性、Lie对称性、形式不变性、非Noether对称性、速度依赖对称性、动量依赖对称性及对称性摄动和绝热不变量等. 第一章　前言. 综述Lie群分析、对称性和守恒量研究的意义、历史与现状；概述机电分析力学理论和相对论性Birkhoff系统动力学理论. 第二章　完整力学系统的几种对称性和守恒量. 研究有限自由度系统的定域Lie对称性，给出该对称性的确定方程、结构方程和守恒量的形式以及逆问题，给出与定域Noether对称性的关系. 研究保守和非保守Hamilton正则系统的形式不变性，给出形式不变性的定义、判据以及与Noether对称性、Lie对称性的关系等. 建立非保守系统的非Noether对称性和非Noether守恒量理论，包括系统的运动、非保守力和Lagrange函数之间的关系，直接导出非Noether守恒量，证明非Noether对称性导出Noether对称性的判据，指出非Noether守恒量是Noether守恒量的完全集. 建立保守和非保守Hamilton正则系统的动量依赖对称性理论，包括系统的动量依赖对称性的定义和结构方程，直接导出非Noether守恒量，并研究逆问题等. 第三章　非完整力学系统的几种对称性和守恒量. 研究非完整Hamilton正则系统的形式不变性，包括形式不变性的定义和判据，与Noether对称性、Lie对称性之间的关系，给出相应的守恒量. 引

入关于时间和准坐标的无限小变换，研究准坐标系下非完整系统的形式不变性、Noether 对称性和 Lie 对称性理论. 给出系统的形式不变性、Lie 对称性的定义和判据，给出系统的 Noether 定理，得到相应的守恒量，讨论三个对称性之间的关系. 研究非保守非完整系统的非 Noether 守恒量，包括系统的运动、非保守力、非完整约束力和 Lagrange 函数的关系，非保守力、非完整约束力满足的条件，系统的非 Noether 守恒量，非 Noether 对称性导出 Noether 对称性的判据等. 建立非完整系统的速度依赖对称性基本理论，包括正问题和逆问题. 给出非完整系统速度依赖对称性的定义、结构方程和确定方程，导出系统的非 Noether 守恒量等. 引入关于广义坐标和广义动量的无限小变换，建立非完整 Hamilton 正则系统的动量依赖对称性的基本理论，包括该对称性的定义、确定方程、结构方程和逆问题，给出系统的非 Noether 守恒量. 研究可控非线性非完整系统的 Lie 对称性和守恒量，给出包含控制参数的确定方程，存在 Noether 守恒量的条件和 Noether 守恒量的形式以及逆问题等. 第四章　机电系统的对称性和守恒量. 给出机电系统的 Noether 对称性的基本理论，包括系统的变分原理，Noether 对称性变换、Noether 准对称性变换、广义 Noether 准对称性变换、Killing 方程、Noether 定理和守恒量的形式，并给出应用例子；介绍机电系统 Lie 对称性的基本理论，包括确定方程、结构方程和守恒量的形式；研究机电系统的形式不变性的基本理论，包括形式不变性的定义、判据，形式不变性与 Noether 对称性、Lie 对称性的关系以及导出 Noether 守恒量的条件和形式等. 建立 Lagrange 机电系统的非 Noether 对称性理论，包括定义、判据、运动和 Lagrange 函数

的关系、非 Noether 守恒量、非 Noether 对称性和 Noether 对称性的关系等. 建立 Lagrange-Maxwell 机电系统的非 Noether 对称性理论,包括系统的运动、非势广义力和 Lagrange 函数的关系,非保守力和耗散力满足的条件,非 Noether 守恒量,非 Noether 对称性和 Noether 对称性的关系等. 第五章 相对论性 Birkhoff 系统的对称性和守恒量. 介绍相对论性 Birkhoff 系统的 Pfaff-Birkhoff 原理和 Birkhoff 方程. 基于相对论性 Birkhoff 系统的 Pfaff 作用量在无限小变换下的不变性,建立相对论性 Birkhoff 系统的 Noether 对称性理论,包括正问题和逆问题等. 基于相对论性 Birkhoff 方程在无限小变换下的不变性,建立相对论性 Birkhoff 系统的 Lie 对称性理论,包括正问题和逆问题等. 建立小扰动力作用下相对论性 Birkhoff 系统的对称性摄动理论,包括绝热不变量的定义和形式,对称性摄动的确定方程、结构方程,求解各阶摄动项等. 第六章 总结和展望. 给出本文的主要结果,提出未来研究的一些设想.

关键词 对称性和守恒量,约束力学系统,机电耦合系统,相对论性 Birkhoff 系统,对称性摄动

Abstract

This dissertation is devoted to the problems of symmetries and conserved quantities for constrained dynamical systems, mechanico-electrical coupling systems and relativistic Birkhoffian systems, including Noether symmetries, Lie symmetries, form invariance, non-Noether symmetries, velocity-dependent symmetries, momentum-dependent symmetries, perturbations of symmetry and adiabatic invariants of these systems. The first chapter reviews the significance, the history and the current state on studying Lie groups analysis, symmetries and conserved quantities, and summarizes the theory for the mechanico-electrical analysis mechanics and relativistic Birkhoffian dynamic system. The second chapter treats the symmetries and conserved quantities for holonomic mechanical systems. Firstly, the localized Lie symmetry for the finite degree freedom systems is investigated, including the determining equations, structural equation, conserved quantities and inverse problem, as well as the relationship between the localized Lie symmetry and localized Noether symmetry. Secondly, the form invariance for the conservative and the nonconservative Hamiltonian canonical systems are discussed, including the definitions and the criterions, the connections

among form invariance and Noether as well as Lie symmetries of the systems. Thirdly, the theory of the non-Noether symmetry and non-Noether conserved quantity for the nonconservative system is developed, comprising the relation among the motion, nonconservatiye forces and Lagrangian, obtaining the non-Noether conserved quantities directly, and proving the condition of that non-Noether symmetry educe Noether symmetry, as well as pointing out that the non-Noether conserved quantity being a set of the Noether conserved quantities for the system. At the end of the chapter, we found the theory of momentum-dependent symmetry for the conservative and nonconservative Hamilton canonical systems. The author gives the definitions and structure equations of momentum-dependent symmetry, and draw the non-Noether conserved quantity directly, as well as studies inverse problem of the systems. The third chapter deals with the symmetries and conserved quantities of nonholonomic systems. First, the theory of form invariance for nonholonomic canonical systems is established, including the definition and criterion of form invariance, the connections among form invariance and Lie symmetry as well as Noether symmetry, and the conserved quantity associated with nonholonomic canonical systems. Second, the infinitesimal transformations are introduced with respect to time and quasi-coordinates, and then the definitions and criterions of form invariance and Lie symmetry as well as

Noether' theorem, and the conserved quantities, the relationships among the three symmetries for nonholonomic systems etc, are presented. Third, the non-Noether conserved quantities of nonconservative nonholonomic dynamical systems are studied in the respects of the connection among the motion, the nonconservative forces, the nonholonomic-constrained forces, as well as Lagrangian of the systems, and the condition being satisfied by the nonconservative forces and the nonholonomic-constrained forces. Further, the non-Noether conserved quantities are derived, and the criterion of that non-Noether symmetry is proven to deduces Noether symmetry of the system. Fourthly, we present the foundation theory of velocity-dependent symmetry is presented for nonholonomic system, including direct and inverse problem, the author gives the definition, structure equation and determining equations of the symmetry, and the non-Noether conserved quantities of the system. Fifthly, the infinitesimal transformations are introduced as respect to generalized coordinates and generalized momentums so that the foundation theory of the momentum-dependent symmetry for nonholonomic systems is developed, including the definition, the determining equation, the structure equation, the conserved quantities and inverse problem of the system. At the end of the chapter, Lie symmetries and conserved quantities of controllable non-linear nonholonomic systems are addressed on the determining equations with controlled parameters, the conditions of possessing the conserved quantity, the form of conserved quantity, and the inverse problem of the systems. In the fourth chapter tackles the symmetries and conserved of the mechanico-electrical systems. Firstly, the foundation theory of Noether symmetry is presented for the mechanico-electrical systems, comprising variation principle, Noether symmetry transformation, Noether quasi-symmetry transformation and generalized Noether quasi-symmetry transformation, Killing equations, Noether theorem and conserved quantities, and an application example. Secondly, the basic theory of Lie symmetry is developed for the mechanico-electrical systems, including the determining equations, structure equation and conserved quantities. Thirdly, the foundation theory of the form invariance is given for the mechanico-electrical systems, including the definition and criterion of the form invariance, the connections among form invariance and Noether symmetry as well as Lie symmetry, the form of the conserved quantity, the condition that form invariance educes Noether conserved quantities for the systems. Fourthly, the theory of non-Noether symmetry is established for the Lagrange mechanico-electrical systems, including the definition and criterion of non-Noether symmetry, the relation between motion and Lagrangian, obtains the non-Noether conserved quantity directly, and proof of the connection between non-Noether symmetry and

Noether symmetry etc. At the end of the chapter, the theory of non-Noether symmetry is obtained for the Lagrange-Maxwell mechanico-electrical systems, including the connection the motion, generalized non-potential forces and Lagrangian, the condition satisfied by the nonconservative forces and the dissipative forces, direct approach to locate the non-Noether conserved quantities, and the relationship between non-Noether symmetry and Noether symmetry associated with the Lagrange-Maxwell mechanico-electrical systems. The fifths chapteris concerned with the symmetries and conserved quantities of relativistic Birkhoffian systems. Firstly, the Pfaff-Birkhoffian principle and Birkhoffian equations of relativistic Birkhoffian systems is introduced. Secondly, based on the invariance of the Pfaff action of the systems under the infinitesimal transformation with respect to time and generalized coordinates, Noether symmetric theory of relativistic Birkhoffian systems is developed, including direct and inverse etc. Third, based on the invariance of the Birkhoffian equation of relativistic Birkhoffian systems under the infinitesimal transformation, Lie symmetric theory of the systems is founded, including direct and inverse issue etc. At the end of the chapter, the theory of the symmetry perturbation is studied for relativistic Birkhoffian systems, including the definition and form of the adiabatic invariant, the determining equations, structure equation of the symmetry perturbation, and all-order terms of the symmetry perturbation associated with relativistic Birkhoffian systems. The sixth chapter ends the dissertation by summarizing the important results and proposing some ideas for the future research.

Key words Symmetry and conserved quantity, constrained mechanical system, mechanico-electrical system, relativistic Birkhoffian system, and symmetry perturbation

目　录