GENERALIZED MOMENT METHODS IN ELECTROMAGNETICS

Formulation and Computer Solution of Integral Equations

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PREFACE

Since the late 1950s, high-speed digital computers have become increasingly available, and their use to solve electromagnetic problems not amenable to classical analytic methods has proliferated. In 1968, R. F. Harrington unified the majority of these methods by identifying the underlying principle for them, which he called the *method of moments* (MM) in his now well-known book published that year. The MM of Harrington is a method which transforms a continuous operator-equation describing the physical problem into a set of matrix equations by first discretizing the operator equation and then performing a scalar (or symmetric) product on it with selected weighting functions. Today, the MM has become a widely used computational method in electromagnetics, and is being taught as a graduate-level course, in one form or another, at universities with a graduate curriculum in electromagnetics.

However, there seems to be an ever-increasing number of new computational methods in electromagnetics that may bewilder even the well-read scholars of the field. Thus, it is desirable to unify and consolidate these methods, whenever possible, into an organized body of knowledge. The author observed that the direct MM of Harrington, iterative methods, the reaction integral equation method, and mode-matching, etc. can all be presented within the framework of the *generalized method of moments* (generalized MM) as defined in the first chapter. This effort of unification and consolidation is the first goal of this book.

The second goal of this book is to present an updated and fairly complete coverage of the subject so that this book may serve both as a textbook and as a reference book. I also recognize that there are many more users than developers of MM computer codes. For the code users, the book must be highly tutorial and easy to understand. With these code users also in mind, I have chosen a direct and reiterative style in exposing the basic principles, technical issues, and analytic and programming methods, particularly in their introductory phase. There are three levels of discussions in this book. Chapter 1 provides a bird's-eye view of the subject. Chapters 2 and 3 present basic principles and techniques. Chapters 5 through 12 are devoted to a thorough and detailed presentation of the subject after a review of the theorems and techniques in electromagnetics in Chapter 4.

Electromagnetic theory is an integral part of the generalized MM. The electromagnetic theory in this book reflects new concepts and insights gained in the last three decades, which have not appeared in textbooks. For example, the fields in the source region, traditionally an obscure and overlooked topic, are a central issue in the MM and are carefully and extensively dealt with. As another example, the magnetic current and magnetic medium (both real and equivalent) are treated as basic parameters, as the electric current and conductor have been, in recognition of their rapidly ascending role in modern concepts and applications. As a result, this book can also be used as a textbook in advanced electromagnetic theory.

This book is written with a balanced amount of mathematical rigor and generality so that it will be sufficiently rigorous and encompassing, yet readable and not too pedantic for engineers and scientists. Occasionally, a more expository approach at the expense of rigor is taken if the theory or finding (even though being widely used and accepted) is a recent one still being disputed. This compromise can be justified by recalling the appearance and acceptance of the Dirac delta function in the physics and engineering community amidst criticisms by mathematicians about half a century ago.

In writing this book, I had in mind a reader with a minimum of one year of graduate-level courses in electromagnetic theory and some training and experience in Fortran programming. For a reader with such a background, the present book should help him to comprehend journal papers; to use, decipher, and modify existing MM programs; and, in some cases, to write computer programs without further literature study.

This book is intended to be fairly complete. After a general introduction in Chapter 1, the iterative MM is discussed in Chapter 2; the direct MM is much simpler and has been clearly introduced in Chapter 1. Chapter 3 addresses the formulation of the integral equation as a first step in the generalized MM, together with various basic issues associated with the integral equations. A number of fundamental concepts, theorems, and techniques in electromagnetic theory relevant to the generalized MM are reviewed in Chapter 4.

The generalized MM problems are then divided into seven major classes based on their physical and mathematical commonalities, each of which is discussed in a separate chapter with some overlaps in other chapters. One of the simplest of these is the wire problem, which is discussed first in Chapter 5. The surface integral equation approach and the volume integral equation approach are two fundamental branches of the generalized MM and are applicable to many types of geometries; they are presented in Chapters 6 and 7 respectively.

Chapter 8 addresses a class of problems involving structures with boundaries suitable for formulation by eigenfunction expansions; they can be considered as generalized cylindrical waveguides. Chapter 9 deals with planar structures including apertures and microstrip antennas.

Chapter 10 deals with infinite planar phased arrays and periodic structures. The basic technique is to exploit the periodicity of the geometric structure so that the effective domain of the integral equation can be reduced to a small unit cell. For planar structures, the Floquet-mode expansion makes such a unit-cell approach feasible in the exterior radiation region.

The generalized MM discussed in Chapters 1 through 10 is for problems in the frequency domain. The extension of the generalized MM to the time domain is discussed in Chapter 11. Chapter 12 addresses computational techniques that deal with the difficulties and limitations in the software and hardware of modern digital computers employed in generalized MM solutions.

Since a major purpose of this book is to serve as a graduate-level text, initially at the School of Electrical Engineering of the Georgia Institute of Technology, exercises are provided at the end of most chapters to help the students gain insight and appreciation of the fine points, the difficulties, subtleties, and usefulness of the principles and techniques discussed in the text. Several sample computer programs are also listed in the appendices, with explanations to reveal the inner workings and building blocks of the generalized MM programming.

When using this book as a text for a short one-semester or one-quarter course in numerical methods, the author suggests that the WIRE89 code be given to the students at the beginning of the course to solve a few simple problems. Simultaneously the instructor can select materials for lectures leading toward, for example, the TM scattering of a conducting infinite cylinder in Section 6-4. The students are then led to develop a computer code for this problem (Exercises 6-3 and 6-4).

The author has made an effort to present the electromagnetic theory in a fairly rigorous and updated manner so that this book can also be useful as a companion textbook in advanced electromagnetic theory, theoretical physics, and applied mathematics, especially for students whose ultimate goal is the numerical solution of problems.

It is my pleasure to acknowledge the secretarial and artwork support provided by the Georgia Tech Research Institute of the Georgia Institute of Technology. The word processing skill of Ms. Lois Randolph-Savwoir is particularly appreciated. In the course of preparing the manuscript, a great deal of personal time was devoted to this effort—this would not have been possible without the support and encouragement of my wife, Lillian.

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Atlanta, Georgia
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CONTENTS

CHAPTER 1

INTRODUCTION

1-1. The Generalized Method of Moments

Since the late 1950s, high-speed digital computers have become increasingly available to engineers and scientists. As a result, computer-based techniques for solving physical problems unamenable to classical methods have proliferated. In electromagnetics the *moment method* plays a predominant role. In other disciplines, the *finite element method* and *finite difference method* are most widely used.

The terminology "*moment method*" or "*method of moments*" (to be denoted by MM throughout this book) was introduced to the electromagnetics community by R. F. Harrington [1] in 1968 in a timely effort to unify the rapidly growing yet poorly organized numerical methods. At that time, the MM merely represented a basic approach to transform an operator equation into a finite matrix equation, which could then be solved by either a direct or an iterative matrix-solution method.

Today, more than two decades after Harrington's unifying effort, the ranks of numerical methods in electromagnetics appear again teeming with many seemingly unrelated varieties. This obviously poses as a stumbling block to new students trying to gain an elementary, yet basic and general, understanding of numerical methods. This is also frustrating to mature engineers and scientists; even they may be overwhelmed by the many seemingly new methods appearing in the latest journals.

The main purpose of this book is to unify and organize a collection of closely identifiable numerical methods and present them in a succinct and efficient way under the *generalized method of moments* (generalized MM). In the process, the electromagnetic theory and the integral equation methods, which are integral parts of the numerical methods and have undergone fundamental improvements in the last four decades, are also updated.

The generalized MM is, as the subtitle of this book suggests, *the* *complete process of formulation and computer solution of integral* (or *integro-differential*) *equations*. Before entering a more detailed discussion on the generalized MM, it is desirable to distinguish it from the finite element method and the finite difference method by their historical background and prevailing usages in the literature.

The *finite element method* was first introduced in structural mechanics in the 1950s as an approximation technique to discretize the geometric area or volume of a physical problem into small "elements", thereby reducing the operator equation to a finite matrix equation, which is then solved numerically. During the last thirty years, the meaning of the finite element method has been expanded to very broad ones, often encompassing the finite difference method. In fact, a book on the finite element method even states that the moment method is essentially a finite element method.

The *finite difference method* is the oldest among the three methods, being originally a method for approximating differential equations by discretization. It has been substantially blended into the literature of the finite element method.

The terminology *method of moments* or *moment method* (MM) has been used in several ways to refer to various techniques for solving linear operator equations. It was probably first discussed in applied mathematics as a direct method [2] and as an iterative approach [3] for solving linear operator equations. It has a different meaning in nuclear physics, being a technique for solving problems in many fermion systems. In electromagnetics, this terminology was first used by R. F. Harrington [1] in 1968 to specify a certain general method for reducing linear operator equations to finite matrix equations. Today, the method of moments in electromagnetics is used to mean sometimes the narrowly defined matrix-generation technique of Harrington, and sometimes broader and more extended methods [4, 5].

All these three methods are *discretization methods*, and they have been developed fairly independently. In the course of their expansions, some overlaps, ambiguities, and inconsistencies arise, and it has become unclear as to what each method represents. Therefore, at the beginning of this book, we will try to distinguish these subjects and to draw a dividing line between the generalized moment method and the finite element and finite difference methods.

It is fairly easy to draw a line between the generalized moment method and the other two methods based on current usages. Computations employing the generalized MM invariably reduce the physical problem, specified by the Maxwell equations and the boundary conditions, into *integral equations* (by which we mean integro-differential equations throughout this book) which have finite, and preferably *small*, domains. In doing so, the problem is cast concretely in a small domain.

And it is in this small domain that discretization, the expansion of the unknowns as a series of basis functions, is carried out. (*Note that the small size of the domain is essential in accommodating the limited memory capacity of the digital computer.*) On the other hand, the finite element and finite difference methods generally deal with the differential equations directly.

In bounded problems such as those in structural mechanics or the infinitely long uniform waveguides, the unknowns are limited by their physical nature to a finite region, which can be directly discretized within the memory-storage capacity of the computer. These methods of solution have been called the finite element or finite difference methods, depending often on whether one chooses to emphasize the differential equation or the discretization process. A survey of the over one-hundred books under the title catalogue of the finite element method, as well as investigation in the general practice in electromagnetics, supports this observation.

Thus *the formulation of a problem by integral (or integro-differential) equations with a finite and usually small domain signifies the first step of a generalized moment-method solution.* That the generalized MM has been playing a predominant role in electromagnetics is due to the generally unconfined nature of the electromagnetic wave, such as radiation in an open region, which can be reduced to a finite domain by integral equations. (In some cases, such as the periodic structures or phased arrays, the domain of the integral equation may appear infinite, but is actually finite or can be considered to be finite.) On the other hand, problems in mechanics and simple waveguide problems, etc., are physically limited to a small finite region, and therefore can be directly discretized by the finite element or finite difference method on the differential equations without consideration of the boundary and radiation conditions.

We have now established a clear exterior boundary for the generalized MM. Next we will examine what is inside it. At present, the generalized MM encompasses the direct matrix method of Harrington, the iterative methods, the mode-matching methods, the reaction integral equation method [6], and a large body of time-domain techniques, etc. The "spheres of influence" of these numerical methods and their mutual relationships are illustrated in Fig. 1-1 in a qualitative yet lucid manner.

Note that we have incorporated the entire numerical process, including the formulation of the operator equation, into the context of the generalized MM. We will call the MM of Harrington the *direct* MM; the MM initiated by Vorobyev, the *indirect* or *iterative* MM; and we will show that the reaction integral equation method is a direct MM. *We have defined the generalized moment method as a class of numerical solution techniques in which a physical problem is formulated into integral*

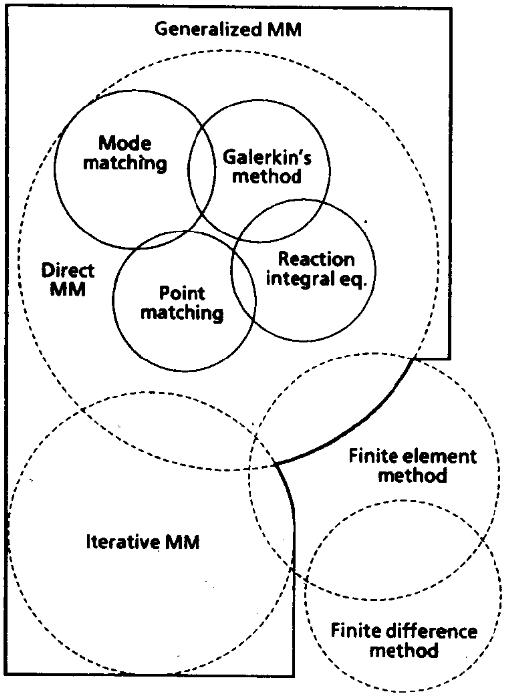

Figure 1-1. Major numerical methods for solving linear operator equation.

equations in a small finite domain, on which chosen unknowns are discretized and solved on a digital computer by a direct or iterative method.

Before entering detailed discussions, we would like to point out that the direct and iterative MM are not the *direct (exact) and indirect (iterative) matrix solution methods* generally referred to in the solution of matrix equations, even though they are related. In the generalized MM, an integral equation (sometimes an integro-differential equation) is numerically solved. Although in an N-dimensional space there is a one-to-one correspondence between an operator and a matrix, different matrix operators can be used in direct MM to approximate the integral operator. Thus the use of a matrix to represent a linear operator as often seen in the literature, though enlightening in many instances, may sometimes lead to a narrow and limited view on the subject.

The *direct MM* is a generalized MM that formulates a problem into a specific *matrix*, which is then solved by an exact or iterative matrix method. Because it terminates in an exact, predetermined number of arithmetic steps, it is called a *direct* method. The *indirect* or *iterative* MM is, in general, not explicitly associated with a particular matrix, and is an iterative process that terminates after an indeterminable number of steps.

The generalized MM as a numerical procedure is outlined in Fig. 1-2. The major steps consist of the formulation of an appropriate integral equation (IE) for the problem, the discretization (approximation) process, and the solution of the discretized integral equation by either a direct or an iterative method. These steps will be discussed in details in the following sections.

In this book, the generalized method of moments will be systematically introduced so that the reader can see how an electromagnetic problem, described fully by the Maxwell equations and the boundary conditions, is reduced to integral equations in a finite domain; how the integral equations are approximated by discretization methods; and how the discretized linear operator equations are numerically solved by a direct or indirect (iterative) method. The general approach is outlined in this chapter and discussed more thoroughly in Chapters 2 and 3. After a review of some basic theorems and techniques in electromagnetics in Chapter 4, the specific and detailed techniques are discussed later.

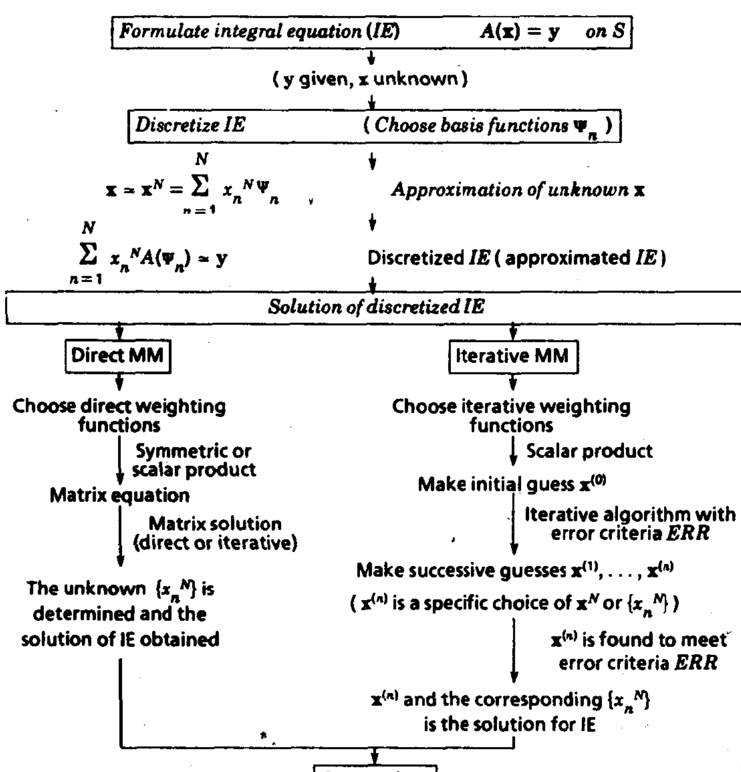

Figure 1-2. Outline of the generalized MM.

1-2. The Physical Problem

A macroscopic electromagnetic problem in a *linear, isotropic* medium is fully described by the Maxwell equations, the constitutive relationships, and the boundary conditions. Spurious solutions are eliminated by the use of the edge condition and the radiation condition. Since most practical problems are time-harmonic, and since the time-harmonic formulation has one fewer parameter, the time t, and therefore easier for computation, we will limit the physical problems under consideration to be time-harmonic, except in Chapter 11, which will deal with problems in the time domain. For time-harmonic fields, the Maxwell equations with $e^{j\omega t}$ dependence are

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B} - \mathbf{M} \tag{1-1}$$
$$\nabla \times \mathbf{H} = j\omega\mathbf{D} + \mathbf{J} \tag{1-2}$$
$$\nabla \cdot \mathbf{B} = m \tag{1-3}$$
$$\nabla \cdot \mathbf{D} = \rho \tag{1-4}$$

where $j = \sqrt{-1}$, $\omega = 2\pi f$, etc., are standard notations in the rationalized mksc system [1, 4, 5]. (If the $e^{-j\omega t}$ dependence is chosen, the j's in all the equations in this book should be changed to $-j$.) The constitutive relationships are

$$\mathbf{D} = \varepsilon\mathbf{E} \qquad \mathbf{B} = \mu\mathbf{H} \tag{1-5}$$

and the equations of continuity are

$$\nabla \cdot \mathbf{J} = -j\omega\rho \qquad \nabla \cdot \mathbf{M} = -j\omega m \tag{1-6}$$

In the above equations, $\varepsilon = \varepsilon' - j\varepsilon''$, $\mu = \mu' - j\mu''$, and ε and μ are the complex permittivity and permeability respectively. In the eight equations above, only six of them are independent (appropriate for the six unknowns). For example, Eqs. (1-3) and (1-4) can be deduced from Eqs. (1-1), (1-2), and (1-6).

The differential form of the Maxwell equations is predicated on the existence of the first derivative and hence continuity of the fields and media. However, we can apply the Maxwell equations to locations with medium discontinuity by assuming that the change in medium takes place gradually and continuously in a finite, small interval. By doing so, we can derive a set of boundary conditions at medium discontinuities. Let a medium discontinuity exist on surface S between regions 1 and 2, as shown in Fig. 1-3, four boundary conditions can be derived from the