ANALYSIS OF ELECTRICAL NETWORKS

SHLOMO KARNI

ANALYSIS OF ELECTRICAL NETWORKS

SHLOMO KARNI
University of New Mexico

JOHN WILEY & SONS
New York Chichester Brisbane Toronto Singapore

Library of Congress Cataloging in Publication Data:

Karni, Shlomo, 1932–
Analysis of electrical networks.

Includes indexes
1. Electric network analysis I Title
TK454.2.K27 1986 621 319'2 85-12206
ISBN 0-471-80268-9

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ABOUT THE AUTHOR

Shlomo Karni was born in Poland in 1932. He received his BSEE degree (cum laude) from the Technion, Israel Institute of Technology, in 1956; his M.E. from Yale University in 1957, and a Ph.D. in electrical engineering from the University of Illinois, Urbana, in 1960.

Since 1961 he has been with the Department of Electrical and Computer Engineering at the University of New Mexico in Albuquerque, New Mexico, where he is a Professor and the Director of Graduate Studies. During 1954–1956, Professor Karni was employed by the Palestine Power Company, Haifa, Israel; 1956–1957 was spent as a teaching assistant at Yale University. From 1957 to 1961, he was Instructor, then Assistant Professor, at the University of Illinois. He was Visiting Professor, Department of Electrical Engineering, University of Hawaii, during 1968–1969; a Visiting Professor and the first Acting Dean at the School of Engineering, Tel Aviv University, 1970–1971; and a Visiting Professor at the Technion, Israel Institute of Technology, during the summer of 1977.

Professor Karni has served as a consultant to Westinghouse Learning Corporation, Gulton Electronics Industries, the Air Force Weapons Labs, and the Los Alamos ERDA (now DOE) Labs.

Professor Karni was on the editorial board of the IEEE *Spectrum* (1969–1972) as Associate Editor and is currently Editor of the IEEE *Magazine on Circuits and Systems*. He has also served as a consultant to major publishing houses (John Wiley & Sons, McGraw-Hill, Prentice-Hall, Houghton Mifflin, Addison-Wesley, and others).

He is the author of three books: *Network Theory: Analysis and Synthesis*, (Boston: Allyn & Bacon, 1966; Russian translation: Moscow: Svyaz Publishing Co., 1973); *Intermediate Network Analysis*, (Boston: Allyn & Bacon, 1971); and *Analysis of Continuous and Discrete Systems*, with W. Byatt (Holt, Rinehart & Winston, 1981). In addition, he has written over 60 technical articles.

Dr. Karni is a Fellow of the IEEE, and a member of Tau Beta Pi (board of directors, 1972–1976); the New York Academy of Science (1970); Eta Kappa Nu; and The Academy of Hebrew Languages, Jerusalem, (correspondent, 1970).

PREFACE

This book is intended for a one-semester (or two-quarter) second course in circuit analysis. The student is expected to have had the following topics as prerequisites: R, L, M, and C as circuit elements, analysis of resistive networks via node and loop equations, dc Thévenin and Norton equivalent circuits, power and energy, superposition, classical solutions of first-order (RL, RC) circuits and of second-order (RLC) circuits, simple operational amplifier circuits, and sinusoidal steady-state (phasor) analysis.*

To help the student and the instructor in the review of some of this material, Chapter 1 provides a summary leading to the general time-domain formulation of dynamic equations for RLC networks. Appendix A contains the essentials of linear algebraic equations, matrices, and determinants. In Appendix B, a brief survey is given of the op-amp, together with some of its common circuits. Students and instructors should use their discretion in studying this review material and referring to it as needed. Appendix C, on scaling, may be conveniently studied or reviewed at this early stage.

The rest of the book is devoted to more advanced topics in circuit analysis. Interconnections of networks, topology, and signal flow graphs (Appendix D) stress the uniformity and the organized formulation of network equations. Convolution reiterates the principle of superposition. The Laplace transform and the Fourier transform serve as powerful tools in the solution of network equations; in addition, they provide the necessary tools for many related subjects, such as stability and frequency response. State variable analysis and solution give us, in addition to mathematical elegance, a deep insight into the physical behavior of networks. The last chapter gives a brief introduction to linear, time-varying networks.

Throughout the text, the following features are used:

1. Problems relevant to a particular topic are listed by number next to the discussion of that topic. It is hoped that such an arrangement will make the learning of the material more systematic and more helpful.

2. Topics which are more advanced are marked with an asterisk (*) at the beginning of the appropriate section. They may be skipped during the first reading without loss of continuity. Later, the student and the instructor are encouraged to return and integrate them into the study.

3. Examples are marked clearly with a title ("Example 3") and with a square mark (□) at the end, all between two thick lines.

4. In Appendix F, selected solutions and hints to problems are given. The student should use these judiciously as a helpful tool, not as a substitute for thinking and

* See, for example, R. E. Thomas and A. J. Rosa, *Circuits and Signals*. New York: John Wiley & Sons, 1984, and D. F. Mix and N. M. Schmitt, *Circuit Analysis for Engineers: Continuous and Discrete-Time Systems*. New York: John Wiley & Sons, 1985.

working! A more complete Solutions Manual is available to instructors upon adoption.

5. References and bibliographical listings at the end of each chapter are kept to a minimum. Too many references may look impressive, but they tend to discourage, by their sheer numbers, even the most well-intended student. This is not to say, however, that a resourceful instructor should not encourage the better students to read additional material. The given lists are a good start in that direction.

I gratefully acknowledge the help of Robin Morel, Ireena Erteza, and Sabina Erteza, who helped me in the preparation of Appendix E. My secretary, Mrs. Joan Lillie, typed the final manuscript and managed the administrative aspects of my work. My editors at John Wiley & Sons, along with the staff, provided prompt, courteous, and efficient assistance throughout the stages of this project. My sincere thanks go to them.

Albuquerque, New Mexico
August 1985

Shlomo Karni

CONTENTS

1

INTRODUCTION

In this chapter, we review the basic properties of network elements and the formulation of loop equations and node equations.

1-1 SYSTEMS AND NETWORKS

In its broadest sense, a *system* can be defined as a group, or collection, of components, each with its specific characteristics, interacting in some prescribed manner. So, an automobile with all its parts, a generating station supplying electric power to customers through transmission lines, the nerves of a living organism, a group of people with certain mutual interests—all are examples of systems. In fact, any conceivable group of entities, interrelated in some fashion can be called a system.

In particular, an electrical network is a system, consisting of *elements* (components) such as resistors, capacitors, voltage and current sources, transistors, diodes, etc. Our aim in this book is the analysis of electrical networks, that is the development of the mathematical relations among the variables that describe the behavior of the network.

1-2 SOME CHARACTERISTICS OF ELECTRICAL NETWORKS

In certain cases, it will be useful to represent a network by the classical "black box." See Fig. 1-1. It has, in general, m inputs (excitations), $x_1(t)$, $x_2(t), \ldots, x_m(t)$ and n outputs (responses), $y_1(t), y_2(t), \ldots, y_n(t)$.[†] Note that, in general, m is not equal to n. We use the notation

$$S\{x_1, x_2, \ldots, x_m\} = \{y_1, y_2, y_3, \ldots, y_n\} \tag{1-1a}$$

[†] These are general symbols; others are $e(t)$ for excitation and $r(t)$ for response. Specific excitations and responses will be denoted by appropriate symbols such as $i(t)$ for a current, $v(t)$ for a voltage, etc.

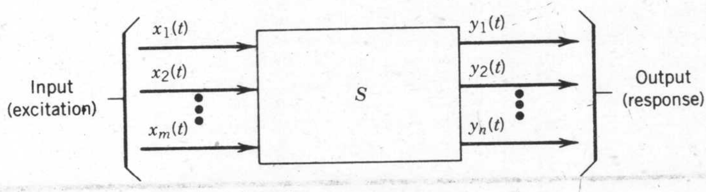

FIGURE 1-1. A "black box."

or

$$\{x_1, x_2, \ldots, x_m\} \xrightarrow{S} \{y_1, y_2, y_3, \ldots, y_n\} \tag{1-1b}$$

to mean, "the inputs $x_1, x_2, \ldots, x_m$ to network S produce the outputs $y_1, y_2, \cdots, y_n$." In network analysis, we deal in general with the following problems.

1. Given the network S, obtain the input-output relations in the mathematical form. In other words, find the explicit form of Eq. (1-1) for a given network.
2. Having obtained the equations for the given network, solve them to obtain the outputs corresponding to given inputs.
3. Discuss the mathematical properties of the equations of a network, and, hence, certain properties of the network.

A *continuous-time* network is characterized by inputs and outputs which are functions of the *continuous variable t*. In a *discrete-time* network, they vary only at discrete values of time. In the former case we designate these inputs and outputs as $x(t)$ and $y(t)$, whereas in the latter case they are denoted by $x(n)$ and $y(n)$, where $n = 1, 2, 3, \ldots,$

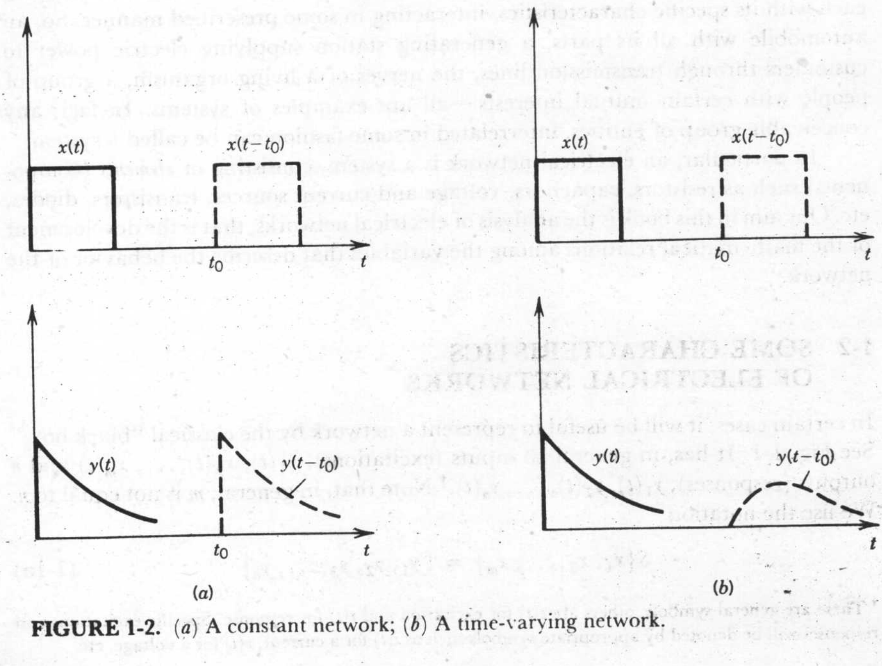

FIGURE 1-2. (*a*) A constant network; (*b*) A time-varying network.

are the discrete points on the time axis. Continuous-time networks can be described, in general, by *differential equations*, and discrete-time networks by *difference equations*.

A *constant* (*time-invariant*) network is one whose response has a *shape* depending only on the *shape* of the excitation and not on the time of application of the excitation (see Fig. 1-2). Mathematically, this can be expressed as follows: with all initial conditions zero, if

$$S\{x(t)\} = y(t) \tag{1-2a}$$

then

$$S\{x(t - t_0)\} = y(t - t_0) \tag{1-2b}$$

where the notation of Eq. (1-1) is used. In other words: a time-shift t_0 in the input produces an identical time-shift in the output.

In a *time-varying* network, Eq. (1-2b) does not hold. Typically, the parameters of a time-invariant network will be constants, and those of a time-varying network will be functions of time.

Example 1

A (hypothetical) network is described by the equation $v(t) = i(t/2)$, where v is the output voltage and i is the input current. Applying Eq. (1-2), we obtain

$$S\{i(t - t_0)\} = i[\tfrac{1}{2}(t - t_0)]$$

but on the other hand,

$$v(t - t_0) = i(\tfrac{1}{2}t - t_0)$$

obviously, $i(t/2 - t_0) \neq i[1/2(t - t_0)]$, and, therefore, the network is time-varying. □

Example 2

The output current of a certain network is given by $i(t) = [v(t)]^2$, where v is the input voltage. With Eq. (1-2), we obtain

$$S\{v(t - t_0)\} = [v(t - t_0)]^2$$

on one hand, and

$$i(t - t_0) = [v(t - t_0)]^2$$

on the other. This network, then, is constant. □

A *lumped* element has physical dimensions which do not affect its describing equation. More precisely, if d is the largest dimension of the element, and λ is the wavelength of the signal, then a lumped element satisfies

$$d \ll \lambda \tag{1-3a}$$

The wavelength λ is given by

$$\lambda = \frac{c}{f} \tag{1-3b}$$

where $c = 3 \times 10^8$ m/s = velocity of electromagnetic waves (velocity of light), and f, in Hertz, is the frequency of the signal. *Total* differential equations describe a network made up of lumped elements. In a *distributed network*, variations with respect to, say, the length of the components *are* important; such a network is described by *partial* differential equations, involving space variation as well as time variation.

Example 3

At audio frequencies, with f = 1,000 Hz, $\lambda = 3 \times 10^5$ meters ≈ 186 miles. At typical microwave frequencies, $f = 10^{10}$ Hz, then λ = 3 cm ≈ 1.2 inches. Thus, for audio circuits, all elements are considered lumped. In microwave circuits, elements such as waveguides are distributed and their describing equations will involve space variations as well as time variations.

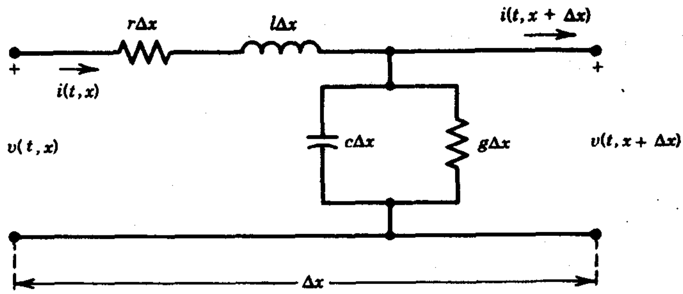

FIGURE 1-3. A model of a distributed network.

The model of the transmission line, Fig. 1-3, is that of a distributed network, since its equations are[1]

$$-\frac{\partial v(t,x)}{\partial x} = ri(t,x) + l\frac{\partial i(t,x)}{\partial t}$$

$$-\frac{\partial i(t,x)}{\partial x} = gv(t,x) + c\frac{\partial v(t,x)}{\partial t}$$

showing dependence on time and position; voltage and current at a given time will vary along the length of transmission line. □

1-3 ELEMENTS AND SOURCES

It is both an amazing and a comforting fact that we can model, analyze, and design the most complicated electrical networks using only a few basic elements. Let us

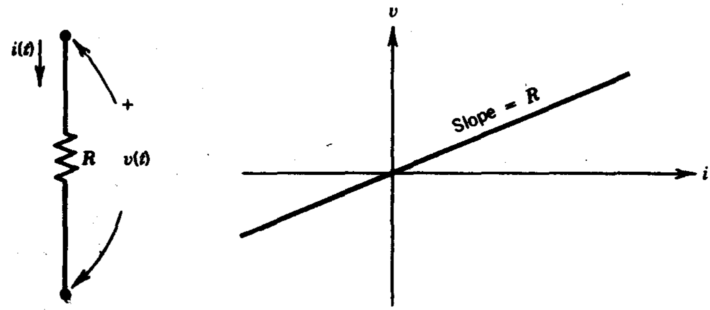

FIGURE 1-4. A constant resistor and its v–i characteristic.

review their properties and introduce certain additional concepts associated with them.

The Resistor

A lumped, time invariant, *linear* resistor is defined by its voltage-current relation

$$v(t) = Ri(t) \tag{1-4a}$$

as shown in Fig. 1-4. The voltage $v(t)$ and the current $i(t)$ have their reference signs as indicated.† The units of $v(t)$ are *volts* (V), of $i(t)$ are *amperes* (A), and the resistance R is in *ohms* (Ω).

The inverse relationship, i in terms of v, is, of course,

$$i(t) = \frac{1}{R}v(t) = Gv(t) \tag{1-4b}$$

where G, the *conductance*, is in *mhos* ($\mho$).

The relationships in Eq. (1-4) indicate that the output of a resistor, either v or i, depends, at any time $t = t_0$, only on the input, i or v, respectively, *at that time* $t = t_0$, and not on past values of the input. Such an element is called *instantaneous* (*memoriless*).

The Capacitor

A constant, lumped, linear capacitor is defined by its charge–voltage relation

$$q(t) = Cv(t) \tag{1-5}$$

as shown in Fig. 1-5. The charge $q(t)$ is measured in *coulombs* and the capacitance C is in *farads* (F). In network analysis, we are interested in the relations between currents and voltages; therefore, we recall the basic relation between current and charge

$$i(t) = \frac{dq(t)}{dt} \tag{1-6}$$

† Throughout this book, lower case letters will denote functions of time, for example, $v(t)$, $i(t)$. Sometimes the parenthetical t may be omitted for convenience.

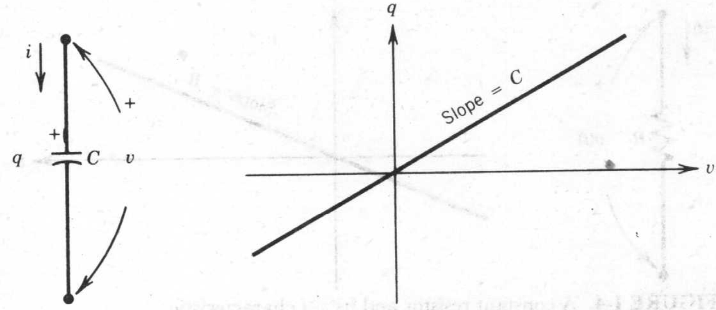

FIGURE 1-5. A constant capacitor and its q–v characteristic.

and differentiate Eq. (1-5). The result is

$$\frac{dq(t)}{dt} = i(t) = C\frac{dv(t)}{dt} \tag{1-7a}$$

and it provides the i–v relationship for the capacitor.

The inverse relationship, v in terms of i, is obtained from Eq. (1-5) and by integrating Eq. (1-7a):

$$\frac{q(t)}{C} = v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx \tag{1-7b}$$

The integral of i, from $-\infty$ to any time t, represents the total charge on the capacitor at time t. The dummy variable x is used in the integrand in order not to confuse it with t.

Unlike the resistor, the capacitor has a *memory*: past values of the input (i) also affect the output (v), as seen in Eq. (1-7b). Such an element, where the output at $t = t_0$ depends on present *and* on past values of the input, is called *dynamic*.

Finally, it will be convenient for us to rewrite Eq. (1-7b) by expressing the integal as a sum of two integrals:

$$\int_{-\infty}^{t} i(x)\,dx = \int_{-\infty}^{0} i(x)\,dx + \int_{0}^{t} i(x)\,dx = q(0) + \int_{0}^{t} i(x)\,dx \tag{1-8}$$

where $t = 0$ is some convenient *initial* time, and $q(0)$ is the initial charge (initial condition) on the capacitor. Then Eq. (1-7b) becomes

$$v(t) = \frac{q(0)}{C} + \frac{1}{C}\int_{0}^{t} i(x)\,dx \tag{1-9}$$

that is,

$$v(t) = v(0) + \frac{1}{C}\int_{0}^{t} i(x)\,dx \tag{1-10}$$

with $v(0)$ the initial voltage across the capacitor.

The Inductor

A constant, lumped, linear inductor is defined by its flux-current relation

$$\phi(t) = Li(t) \tag{1-11}$$

as shown in Fig. 1-6. The flux $\phi(t)$ is measured in *webers* and the inductance L in *henries* (H).

In order to obtain the v–i relations, we recall that, according to experimental observations (of Faraday, Lenz, and others)

$$v(t) = \frac{d\phi(t)}{dt} \tag{1-12}$$

and differentiate Eq. (1-11)

$$\frac{d\phi(t)}{dt} = v(t) = L\frac{di(t)}{dt} \tag{1-13a}$$

This is the desired v–i relationship for the inductor. It is interesting and instructive to recognize that Eqs. (1-7a) and (1-13a) are of identical form, but with voltage and current exchanging places; that is, for the capacitor

$$i_C(t) = C\frac{dv_C(t)}{dt} \tag{1-7a}$$

while for the inductor

$$v_L(t) = L\frac{di_L(t)}{dt} \tag{1-13a}$$

Here we have added the subscripts C and L for clarity. Such relationships, where elements obey the same equation in form but with v and i replaced, are called *duals*. The principle of duality will be very useful to us.

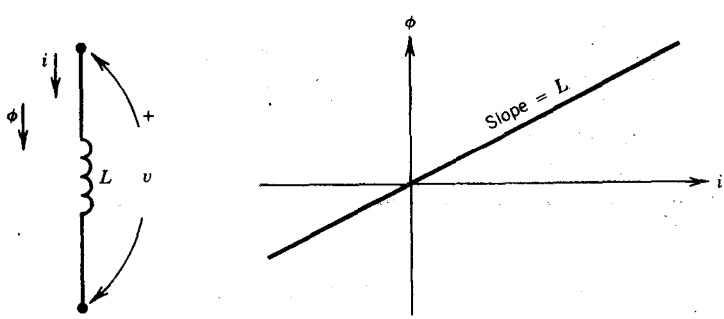

FIGURE 1-6. A constant inductor and its ϕ–i characteristic.