Applied Linear Programming
For the Socioeconomic and Environmental Sciences

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Preface

One of the shelves in my office is lined with books that have linear or nonlinear programming and optimization in the title. Why then write another linear programming book? Because I think it is useful to include the basic methods and a broad survey of applications under a single cover. Most of the dozen or so books I have used in my course since 1969 stress the mathematics of linear programming (e.g., Gass, Hadley, Simmons, Dantzig, Smythe and Johnson, and Spivey and Thrall). None emphasizes the range of linear programming applications that are relevant to students of socioeconomic and environmental management. Most books acknowledge the existence of only transportation applications. Those that do contain applications tend to present the proven applications, a few applications in depth, or linear programming and other models (e.g., Stark, Helly, Salkin and Saha, Driebeck, and DeNeufville and Marks). The major objective of this book is to stimulate new applications of linear and related programming by specialists and nonspecialists by providing many of the already reported uses of linear and related optimization techniques and the basic methods under a single cover.

This volume was molded on two other perspectives which, while not novel, are geared toward the nonspecialist student. Optimization models have become a tool that must be understood not only by the industrial engineer, operations research analyst, economist, and mathematician, but also by the planner, geographer, regional scientist, civil engineer, public servant, and other people without extensive mathematics backgrounds. Therefore, the second major goal of this book is to prevent the mathematics of linear programming from acting as a barrier to the potential user. While all users or readers of linear programming should understand the mathematics, they can understand it without corollaries and lemmas and mathematical presentations that assume backgrounds which many students do not possess.

A third goal of this book is to quickly move the student from the book to the computer. This goal is derived from years of teaching and talking with people who had not taken linear programming but were determined to look over the analyst's shoulder, people who had a course in linear programming but were never exposed to computer codes, and people who were unfamiliar with the plethora of information one can acquire beyond the optimum solution.

The three goals of this book are accomplished in eleven chapters. The first three chapters present the theory and mathematics of linear programming. Chapter 1 introduces the concept of optimization, a geometric interpretation, the mathematical formulation and solution, and a brief history of standard and new applications. Chapter 2 presents the essential matrix mathematics for solving linear programming problems. The simplex procedure is presented as the most useful technique for the potential user. Diagrams are used to demonstrate the feedback between the single- and two-phase solutions. The third chapter highlights special properties of linear programming (the dual and postoptimal analysis) and other optimizations methods (e.g., mixed integer, branch and bound, dynamic, and heuristic).

The fourth chapter presents MPS/360 and MPSX/370 computer codes for solving linear programming problems. Methods of preparing job control, system, and data cards are presented. System options are explained, including postoptimal analysis. Sample printouts are displayed and interpreted.

Chapters 5–11 provide applications of interest to planners, managers, economists, environmental and regional scientists, geographers, civil engineers, and public servants. Each chapter has three components: a broad overview of linear programming applications, one or more examples to be solved by the reader on the computer, and a bibliography. The author has chosen to emphasize breadth rather than depth in the application chapters. Five areas of application are presented: solid waste in Chapters 6 and 11; water resources in Chapter 7; health, education, and law enforcement in Chapter 8; intraregional land use and transportation in Chapter 9; and economic development and transportation in Chapter 10. Within each chapter, as much breadth in applications as possible is presented.

The breadth approach has three important advantages. First, the user should see the great extent to which applications vary. For example, in some applications optimizing behavior seems like a reasonable assumption;

in others this is dubious. Some applications require a readily available and a narrowly defined data set; others have data requirements that must be met with estimates. Numerous examples are necessary to highlight the subtle and sometimes not so subtle differences between problems. Second, with the exception of a few review articles, the vast majority of the applications in this book are available only in technical literature that may be difficult to locate and/or to readily comprehend for the nonspecialist. More than sixty percent of the applications in this book have been published during the 1970s. By bringing together many of the recent applications under a single cover, it is hoped that the inventory in and of itself will spawn new applications.

The breadth approach to applications has drawbacks. The most important drawback is overselling optimization. Optimization is only one of many mathematical and nonmathematical approaches used in the socioeconomic and environmental sciences. In many of the disciplines presented in the book, linear programming is not the most important mathematical tool. To mitigate against misrepresenting the role of linear programming, Chapter 5 compares optimization and other approaches and presents guidelines for mathematical programming applications. Chapter 11 expands on Chapter 5 through a case study.

Another important decision was to aggregate the applications on the basis of disciplines rather than by type of application. The latter includes such groupings as personnel allocation, transportation, siting, and capital replacement. A discipline classification appeals to the discipline specialist who may be primarily interested in solid-waste and water-resources applications, or land-use and transportation applications. This book gives the discipline specialist the opportunity to select one or two of the applications chapters to read in depth.

The discipline approach will not appeal to the reader who perceives applications as multidisciplinary. To this reader, sitings of a landfill, a hospital, a school, or other facilities are similar. The author decided on discipline aggregation because the vast majority of people with whom he spoke favored the discipline classification.

Recapitulating, the three major goals of this volume are to teach the mathematics of linear programming in as simple a manner as possible, to move the user to the computer as soon as possible, and especially to stimulate interest in new applications by presenting numerous examples of the use of linear and related programming. Overall, this book is intended to serve the large body of readers who are primarily interested in applications, not mathematics.

Acknowledgments

Many friends have contributed to this volume. Bill Spencer helped me learn goal programming. Tom Fitzgerald, Jim Gallagher, and John Caruana helped set up and run some of the problems at the ends of the chapters. Nancy Neuman made detailed comments on three chapters. Numerous colleagues contributed suggestions on specific chapters, in particular, George Carey and Don Krueckeberg of Rutgers University and Ethan Smith of the United States Geological Survey.

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Finally, I thank my many students at Rutgers, Columbia and Cornell Universities for motivating me to write this book and for reading it and offering suggestions that have greatly improved it.

Contents

Chapter 3 Beyond the Global Optimum

Chapter 4 Computer Algorithms

PART TWO APPLICATIONS

Chapter 5 Mathematical Models for Policy Analysis

Chapter 6 Solid Waste

Chapter 7 Water Resources Management

Chapter 8 Health, Education, and Law Enforcement

Chapter 9 Intraregional Land Use and Transportation

Chapter 10 Economic Development and Transportation

Chapter 11 A Linear Programming Case Study

PART ONE

THEORY AND METHODS

Chapter 1

Theory, Methods, and Applications

Linear programming is a specialized mathematical decisionmaking aid. In and of itself, it has no socioeconomic or environmental content. Like other widely used mathematical methods (e.g., regression, input–output analysis, and the calculus), it can only help us interpret data and to examine theories about the way things work or should work. If the data are spurious and incomplete, or if the match of model and theory is inappropriate, mathematical decisionmaking aids such as linear programming are more likely to confuse rather than clarify a decision. Given these important caveats, linear programming was developed for specific classes of applications which assume optimal behavior, linear relationships among different pieces of information, and partially restrictive specifications.

A. OPTIMAL BEHAVIOR

You must be able to assume optimal behavior if you hope to learn by applying linear programming to a problem. Optimal behavior implies that decisionmakers are aware of all the important information and will choose the most favorable decision, or at least would like to know the best decision. Not everyone can agree on the criteria for the best decision. Some people regard economic gain as something to be maximized; others prefer to minimize the use of resources. Still others seek to minimize political turmoil and others have additional "most favorable" criteria. Different most favorable criteria are based on the different philosophies and roles of the decisionmakers and the different information supplied to them.

Optimal behavior has been one of the cornerstones of economic theory and operations research.[1] Economic theorists, operations analysts, engineers, and others have assumed optimal behavior in modeling many private and sometimes public decisionmaking processes. However, it seems apparent that decisions are influenced by political, sociological, and psychological factors. While proponents of optimization assume that economic decisions are made on the basis of carefully reasoned debate, other theorists argue that decisions are more like psychological dramas in which the players, loaded with personal perspectives and axes to grind, arrive at clearly suboptimal decisions.

Most institutional decisions probably are made with explicit notions of optimal behavior and implicit psychological motivations. When the weight of the decisionmaking process is more on the side of information analysis than personal motivations, then optimal behavior is a possible behavioral theory from which to operate. However, if data are manipulated to support a pet intuition or past practice, or both, then optimal behavior cannot be assumed or may be assumed to occur only over a narrow range of decisions, most likely routinized operating decisions. On the whole, optimal behavior is most likely to be found when decisionmakers want to consider a relatively limited range of policies and choose from a broad range of specific alternatives drawn from each major policy. The limited range of major policies enables decisionmakers to grope toward specific tradeoff and contingency choices which can be modeled to seek optimality.

When optimal behavior is a reasonable-looking assumption, then the following mathematical conditions should exist if linear programming is to be used: constraints that partially restrict the range of the solution, nonnegative solutions, and linear relationships among the components of the problem.

B. MATHEMATICAL CONDITIONS FOR A SOLUTION

Linear programming may not be the appropriate decisionmaking aid, even when optimal behavior is assumed. Calculus is used to solve many optimization problems. Calculus methods cannot solve optimization problems with imprecise specifications and with nonnegativity requirements.

Imprecise specifications typify the socioeconomic and environmental planning sciences. For example, a water supply manager seeks to find a least-

[1] For a spectrum of viewpoints on optimality and decisionmaking, see Clarkson (1968), Ansoff (1969), Pugh (1971), and Starbuck (1971).

cost solution to meeting a public potable water demand of 10 million gallons per day (mgd). He may draw his supply from the local reservior or from a pipeline to an adjacent town. A solution that supplies 9.9 or less mgd or 10.1 mgd or more is not acceptable. The resource manager therefore seeks a least-cost solution while constrained by a specific demand requirement. If all the restrictions are as specific as the 10.0 mgd demand restriction, calculus can usually be used to solve the problem. Typically, however, other constraints are present but are not precise. The water supply manager's local reservoir and pipeline are likely to have imprecise constraints. The local reservoir has a daily yield of 5 mgd, which may not be exceeded. The manager can pump 5 mgd, but he may also pump less than 5 mgd. The reservoir restriction is an inequality rather than an equality. The pipeline also has two inequality restrictions. On the one hand, the community can only take 10 mgd because of the size of the pipeline, on the other, by contractual agreement it must take out at least 6 mgd. These typical inequality constraints may be treated with a linear programming formulation.

Another constraint is that the manager is not allowed to transfer negative amounts of water from the sources of supply to the areas of demand. This constraint is obvious to people but not to the computer. To the computer, it is cheaper to ship 5 mgd than 10 mgd, then why not ship -5 mgd? Linear programming problems are written with ≥ 0 constraints to prevent negative allocations.

Linear programming is one of a number of mathematical programming tools which seek a least-cost solution to meeting imprecise specifications. What distinguishes linear programming from the other mathematical programming techniques, which are briefly reviewed in the third chapter, is that linear programming requires linearly proportional relationships. The resources to be consumed by an activity must be linearly proportional to the level of the activity. For example, if the planner wants to double the number of garden apartments of a particular design in the community, then the amount of land and public services required for the previous garden apartments would have to double.

Similarly, the contribution of the activities to the goal that is to be optimized must be linearly proportional to the activity level. If 100 garden apartment units will cost the region $100,000 to service, then 200 garden apartment units will cost $200,000 to service. Economies and diseconomies of scale are not recognized. Finally, when added together, all activities must obey a materials balance. The sum of the resource inputs must equal the sum of the product outputs. Summarizing, linear programming is a mathematical tool for obtaining optimum solutions that do not violate imprecise constraints, that cannot have negative activities, that require linearly proportional relationships, and that account for all inputs and outputs within the system.

C. AN ILLUSTRATION OF DIFFERENT SOLUTIONS

At this point, it is appropriate to illustrate the theory and mathematical requirements of linear programming. Let us review the water manager's problem. The water department can utilize reservoir water which costs $300 for a million gallons or pipeline water which costs $500 for a million gallons. The reservoir water will be called activity X_1; the pipeline water will be called activity X_2. The community wishes to minimize the cost. The total cost and objective of the manager can be written as the following objective function:

Minimize cost: $$Z = 300X_1 + 500X_2 \tag{1}$$

We remember that the total demand for water is 10 mgd. Accordingly, the following equation states that activities X_1 (reservoir) and X_2 (pipeline) must supply 10 mgd:

$$X_1 + X_2 = 10 \tag{2}$$

Equation (2) is an example of a rigid specification, or equality constraint. Next, the water manager is permitted to take only 5 mgd from the reservoir:

$$X_1 \leq 5 \tag{3}$$

Equation (3) exemplifies imprecise specifications, or inequality constraints. Next, we need two constraints that specify that the pipeline can yield only 10 mgd, but that it must be used for more than 6 mgd:

$$X_2 \leq 10 \tag{4}$$

$$X_2 \geq 6 \tag{5}$$

Finally, we cannot supply negative amounts of water, so we need two non-negativity constraints:

$$X_1 \geq 0 \tag{6}$$

$$X_2 \geq 0 \tag{7}$$

The entire linear programming problem is summarized in the following:

Minimize: $$Z = 300X_1 + 500X_2 \tag{1}$$

subject to the constraints

$$X_1 + X_2 = 10 \tag{2}$$

$$X_1 \leq 5 \tag{3}$$

$$X_2 \leq 10 \tag{4}$$

$$X_2 \geq 6 \tag{5}$$

$$X_1 \geq 0 \tag{6}$$

$$X_2 \geq 0 \tag{7}$$

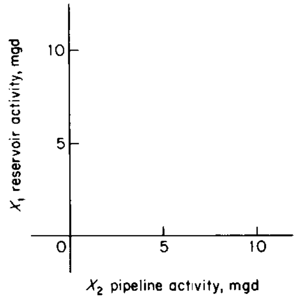

Fig. 1

This is the standard form for a linear programming problem. Let us briefly review the theory and mathematical conditions in this problem. The objective function assumes that the optimal least-cost choice is being sought. Equations (3)–(7) exemplify the imprecise inequality restrictions, and Eqs. (6) and (7) represent nonnegativity requirements. The proportional linear relationships are obvious in the objective function and the constraints (e.g., \$300 for 1 million gallon from X_1, \$600 for 2 million gallons, \$900 for 3 million gallons, etc.). Finally, the outputs of water from the two sources of water equal the inputs of water to the areas of demand.

1. A GRAPHICAL METHOD

In this section, we will review a graphical solution to a linear programming problem. Graphical solutions are impractical for real-world applications since most decisions involve many possible activities and many constraints. Nevertheless, the graphical solution is a device that may be used to explain how the more abstract algebraic methods are used to solve a linear programming problem and the possible solutions that can be obtained from a linear programming problem.

To picture the solution graphically (Fig. 1), we label one axis as activity X_1 (reservoir) and the other axis as activity X_2 (pipeline).

2. FEASIBLE SOLUTION

First, we seek a feasible solution. When a feasible solution has been found, then an optimal solution is sought. We find a feasible solution by drawing each constraint on the graph and locating the area of feasible solutions. The first constraint [Eq. (2)] is graphed in Fig. 2.