COLD CATHODE DISCHARGE TUBES

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PREFACE

THE aim of this book is to present a treatment of the factors involved in the design of cold cathode discharge tubes which is as comprehensive as a single volume will allow. It has been found convenient to divide the book into two parts. Part I is concerned with the theory of the fundamental processes of electrical discharges in gases. These theories lay the foundations for Part II, which deals with the application of these ideas to the design of actual tubes.

The book is largely based on a course of post-graduate lectures given in the Department of Physics at the Northern Polytechnic, London. The authors are grateful to Ericsson Telephones Ltd for allowing the use of a great deal of unpublished data and experience obtained in the laboratory of their Tube Division.

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CONTENTS

PART II. PRACTICE AND DESIGN

PART I: GENERAL THEORY

INTRODUCTION

THE aim of Part I of this book has been to present a reasonably comprehensive survey of the subject of conduction of electricity in gases. Although those aspects of the subject which are directly relevant to the design of cold cathode discharge tubes have naturally been emphasized, a number of topics not having any direct application have been included. This has been found necessary for the logical development of the subject.

Part I is based on a post-graduate lecture course given by one of the authors (J. D. Swift) at the Northern Polytechnic Physics Department over the last few years. The intention throughout has been to concentrate on the physical principles involved, while mathematical complexity has been avoided as far as possible.

Fundamental collision processes in gases are briefly discussed in an introductory chapter. A full treatment of this subject is beyond the scope of this book, however, and the interested reader is recommended to consult a work such as *Electronic and Ionic Impact Phenomena* by H. S. W. Massey and E. H. S. Burhop. Chapter 2 deals with the motion of slow electrons in gases. Although this is concerned largely with the calculation of electron energy distribution functions, a brief survey of recent microwave studies of slow electrons is also included.

The next few chapters deal with the mobility and diffusion coefficients of positive ions and electron–ion recombination. An account of recent afterglow investigations is given here. Ionization by electron collision in gases is considered in Chapter 5, while a brief discussion of electron attachment and negative ion formation follows in Chapter 6.

Secondary ionization processes leading to breakdown are dealt with in Chapter 7. The spark breakdown process itself is discussed at some length in Chapter 8. The emphasis here is on breakdown phenomena at low gas pressures ($<$ 500 mm Hg), and little has been said regarding sparking at pressures exceeding atmospheric.

The last two chapters of Part I are devoted to low current self-maintained discharges and glow discharges. The cathode region of the glow discharge has naturally been the main concern, but some account of the positive column is also included. High frequency and arc discharges have not been discussed, these topics being outside the scope of this book.

Although a number of references are included at the end of each chapter of Part I, no attempt has been made to give a comprehensive bibliography, and the author wishes to apologize for any important omissions.

1

FUNDAMENTAL COLLISION PROCESSES

1.1. COLLISION CROSS-SECTIONS

In discussing electrical discharges in gases we shall be very frequently concerned with problems involving collisions between electrons, ions, and gas molecules. It is convenient, therefore, to begin by defining a suitable coefficient which will be a measure of the probability of a particular collision process.

Consider a beam of I particles of homogeneous velocity c moving through a gas containing N molecules per cubic centimetre, the particle concentration being much less than N and the molecular velocity much less than c. Then the number of particles suffering collisions per second $\mathrm{d}I$ is proportional to N, c, and I. Hence

$$\frac{\mathrm{d}I}{I} = Nq_c c \qquad \ldots (1.1)$$

where the constant of proportionality q_c, which has dimensions $[L]^2$, is the total collision cross-section. The above definition may be extended to include the various types of collision that may occur.

It is sometimes more convenient to use the mean free path λ_c for the type of collision under consideration. Since $\mathrm{d}I = (c/\lambda_c)I$ we have

$$\lambda_c = \frac{1}{Nq_c} \qquad \ldots (1.2)$$

1.2. ELASTIC COLLISIONS

If c is sufficiently small the only type of collision that need be considered is an elastic collision; this involves a redistribution of the kinetic energy but no changes in the internal energy of the interacting particles.

We will consider the simple case of an elastic collision between two spherical particles of masses m and M (see Figure 1.1). Suppose the velocity of m before the collision is c, while M is initially at rest. If m rebounds with velocity c_1 at an angle θ to its initial direction of motion we have, from considerations of conservation of momentum and energy

$$mc - mc_1 \cos\theta = Mw \cos\phi \qquad \ldots (1.3)$$

$$mc_1 \sin\theta = Mw \sin\phi \qquad \ldots (1.4)$$

$$\tfrac{1}{2}mc^2 - \tfrac{1}{2}mc_1^2 = \tfrac{1}{2}Mw^2 \qquad \ldots (1.5)$$

w being the velocity of M after the collision. From these equations we obtain

$$w = \frac{2m}{M+m} \cdot c \cos \phi \qquad \ldots (1.6)$$

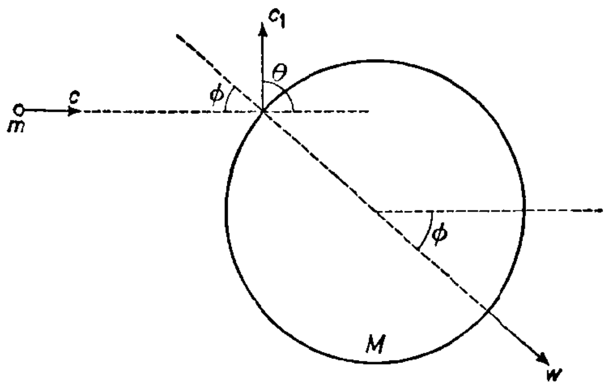


Figure 1.1. Elastic collision of two particles

If $f(\theta)$ is the fraction of its energy that the particle m loses in the collision then

$$f(\theta) = \frac{c^2 - c_1^2}{c^2} = \frac{M}{m} \cdot \frac{w^2}{c^2}$$

Hence
$$f(\theta) = \frac{4Mm}{(M+m)^2} \cdot \cos^2 \phi \qquad \ldots (1.7)$$

1.3. ELECTRON–ATOM ELASTIC COLLISIONS

In this case $m \ll M$ and $\theta \simeq \pi - 2\phi$. Hence

$$f(\theta) \simeq \frac{2m}{M}(1 - \cos \theta) \qquad \ldots (1.8)$$

Let q_c be the total elastic collision cross-section for electrons of velocity c. Thus $Nq_c c$ is the fraction of the electrons suffering collisions per second. Consider those collisions in which the electron is deflected through an angle between θ and $\theta + d\theta$ into a solid angle $d\Omega = 2\pi \sin \theta \, d\theta$ (Figure 1.2). Then

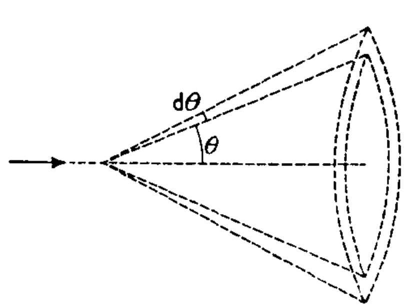


Figure 1.2. Differential collision cross-section

$\sigma(c, \theta)\, d\Omega$ is the differential cross-section for elastic scattering through an angle θ into the solid angle $d\Omega$. Hence

$$q_c = 2\pi \int_0^\pi \sigma(c, \theta) \sin\theta \, d\theta \qquad \ldots (1.9)$$

Since $[\sigma(c, \theta)\, d\Omega]/q_c$ is the fraction of the collisions that result in a deflection through θ into $d\Omega$ and $f(\theta)$ is the fractional energy loss of the electron in each such collision, we have for the mean fractional energy loss per collision

$$\frac{2\pi}{q_c}\int_0^\pi \sigma(c,\theta) f(\theta) \sin\theta\, d\theta = \frac{2m}{M}\cdot\frac{2\pi}{q_c}\int_0^\pi \sigma(c,\theta)(1-\cos\theta)\sin\theta\, d\theta$$
$$= \frac{2m}{M}\cdot\frac{Q_c}{q_c}$$

where

$$Q_c = 2\pi \int_0^\pi \sigma(c,\theta)(1-\cos\theta)\sin\theta\, d\theta \qquad \ldots (1.10)$$

The mean fractional energy loss per electron per second is then

$$\frac{2m}{M}\cdot\frac{Q_c}{q_c}\cdot N q_c c = \frac{2m}{M}\cdot N Q_c c \qquad \ldots (1.11)$$

It is seen that if the total elastic cross-section q_c is replaced by Q_c then the average fractional energy loss per elastic collision f_{el} may be taken to be $(2m/M)$. The quantity Q_c is frequently referred to as the momentum transfer cross-section.

In the case of ideal elastic spheres the scattering is spherically symmetrical, all directions of motion after a collision being equally probable. The differential collision cross-section is then independent of θ giving

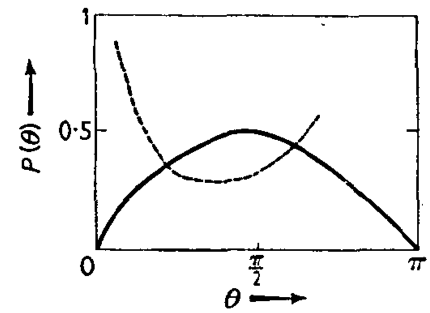
$$\sigma_c = \frac{q_c}{4\pi} \text{ and } Q_c = q_c \qquad \ldots (1.12)$$

Then $P(\theta)\, d\theta$, the fraction of the collisions resulting in deflections between θ and $\theta + d\theta$, which in general is

$$\frac{\sigma(c,\theta)}{q_c}\cdot 2\pi \sin\theta . d\theta$$

becomes $\frac{1}{2}\sin\theta\, d\theta$. The average value of θ is then $\pi/2$.

For sufficiently slow electrons the scattering is spherically symmetrical and the two cross-sections Q_c, q_c are therefore equal. At higher energies there are serious divergences from the ideal case[1] (see Figure 1.3). Q_c and q_c only differ appreciably when there is a pronounced concentration of scattering in either the backward or forward directions. In Figure 1.4, which compares the values of Q_c and q_c for helium, neon, and argon, the values of Q_c are obtained from the observed angular distributions. (Cross-sections are



——— Ideal elastic spheres - - - - - 10 eV electrons in He

Figure 1.3. Elastic scattering of electrons

normally measured in units πa_0^2, where $a_0 = 0{\cdot}53 \times 10^{-8}$ cm = the radius of the first Bohr orbit of the hydrogen atom.) It will be seen that the assumption that $f_{el} = 2m/M$ can frequently be made without serious error. If the mean energy $\bar{\epsilon}_g$ of the gas molecules cannot be ignored compared to the energy $\epsilon = \frac{1}{2}mc^2$ of the electron the expression for f_{el} is modified as follows

$$f_{el} = \frac{2m}{M}\left(1 - \frac{4}{3}\frac{\bar{\epsilon}_g}{\epsilon}\right) \qquad \text{. . . (1.13)}$$

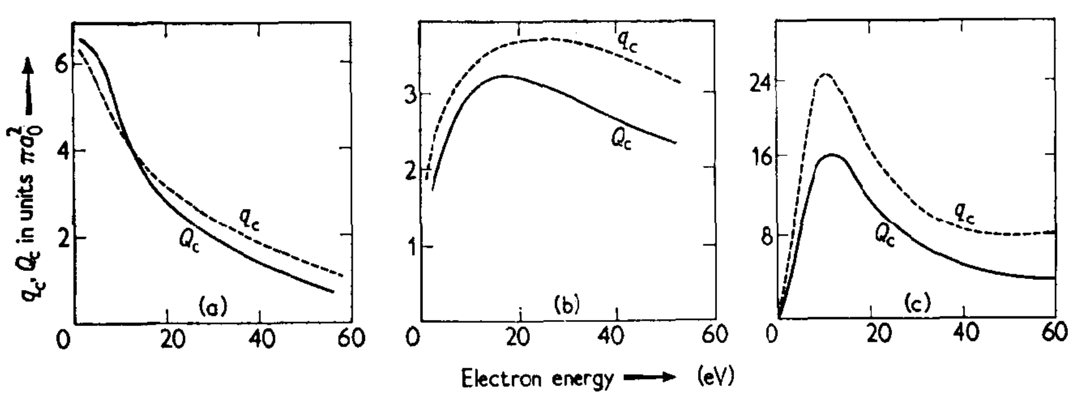


Figure 1.4. Comparison of momentum transfer and total elastic cross-sections for (a) helium, (b) neon, and (c) argon. After Massey and Burhop, reference 1, p. 15

If the electrons also have an energy distribution with a mean value $\bar{\epsilon}$, then

$$f_{el} \simeq \frac{2m}{M}(1 - \bar{\epsilon}_g/\bar{\epsilon}) \qquad \text{. . . (1.14)}$$

The exact value of the numerical factor does, however, depend on the form of the distribution function.

The marked dependence of q_c on the electron velocity c was discovered independently by Ramsauer[2] using electron beams of homogeneous velocity, and by Townsend and Bailey using a more indirect method involving electron swarms (see sub-section 2.3.4). The notable transparency of the heavier rare gas atoms towards electrons of energy ~ 1 eV is frequently referred to as the Ramsauer–Townsend effect (see Figure 1.5).

It must be emphasized that the observed variation of $P(\theta)$ with θ and q_c

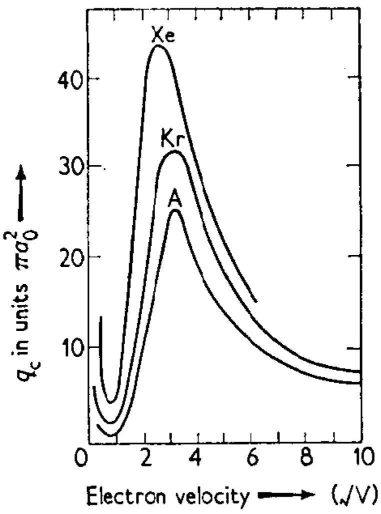


Figure 1.5. Total cross-section curves for argon, krypton, and xenon

with c cannot be explained on the basis of a classical description of the scattering of an electron by gas atoms. If a quantal description of the process is used, the electron is regarded as being represented by a wave packet and the results can be accounted for on the basis of the diffraction of the electron wave by the field of the atom.

Actually, on classical ideas the total collision cross-section q_c approaches infinity as the angular resolving power of the apparatus is increased since some deviation will occur as long as some field exists between an electron and an atom. This difficulty disappears when quantum uncertainty effects are allowed for,[3] and a finite value of q_c is predicted provided the force between an electron and an atom falls off at large values of the separation r faster than $1/r^3$.

1.4. POSITIVE ION–ATOM ELASTIC COLLISIONS

We must now return to the general equation for $f(\theta)$ the fractional energy loss of a particle m in a collision with M involving deflection θ (equation (1.7))

$$f(\theta) = \frac{4Mm}{(M+m)^2} \cdot \cos^2 \phi$$

and consider the case where m and M are of the same order of magnitude. Let the radius of M be r.

The probability of a collision taking place with the angle of impact ϕ in the range $d\phi$ is equal to the ratio of the projected area $2\pi r \sin \phi . \cos \phi . r\, d\phi$ to the whole area presented for collision πr^2, namely $2 \sin \phi . \cos \phi . d\phi$. We thus obtain for f_{el}, the mean fractional energy loss per collision

$$f_{el} = \frac{8Mm}{(M+m)^2} \cdot \int_0^{\pi/2} \cos^3 \phi . \sin \phi \, d\phi = \frac{2Mm}{(M+m)^2} \quad \ldots (1.15)$$

If $m = M$, this gives $f_{el} = 0{\cdot}5$. This shows that positive ions which are receiving energy from an electric field will, in general, lose a large fraction of this energy in elastic collisions with gas atoms (see Section 3.1).

Equations (1.3) to (1.5) give $\theta = (\pi/2) - \phi$ if $m = M$. Thus $P(\theta)\,d\theta$, the fraction of the collisions that result in deflections between θ and $\theta + d\theta$, is given by (see Figure 1.6)

$$P(\theta) = \sin 2\theta \qquad \ldots (1.16)$$

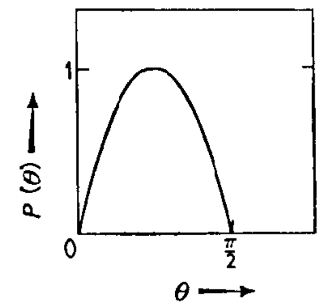


Figure 1.6. Elastic scattering of ions and atoms of equal mass

Since $P(\theta)$ is only finite for $0 < \theta < \pi/2$ this indicates that only forward scattering occurs. In view of this result, persistence of velocities is important in the theory of the motion of positive ions in gases (see Section 3.1).

1.5. INELASTIC COLLISIONS BETWEEN ELECTRONS AND GAS ATOMS

Provided the electron energy $\epsilon = \frac{1}{2}mc^2$ is less than the difference in energy between the lowest excited state and the ground state of the atom only elastic collisions can occur. When ϵ exceeds this critical value, however, excitation of the gas atoms can take place. We may define a cross-section q_n for the excitation of the nth state of the atom, i.e. Nq_nc is the fraction of the electrons of velocity c undergoing collisions of the given type per second. If $\epsilon > \epsilon_i$, the minimum ionizing energy of the gas atoms, a similar cross-section q_i for ionization collisions must be included.

Thus, in general the total collision cross-section q_c may be written

$$q_c = q_{0c} + \sum_n q_n + q_i \qquad \ldots (1.17)$$

where q_{0c} now represents the total elastic collision cross-section and the summation includes the various possible excited states.

It is sometimes convenient to introduce the probability of a particular type of collision taking place. Thus the ionization probability K_i is given by

$$K_i = q_i/q_c \qquad \ldots (1.18)$$

and varies with electron energy as shown in Figure 1.7.[4] Thus for argon K_i is a maximum when the electrons have an energy $\sim$ 125 eV and even then less than half of the collisions result in ionization. For electron energies less than $3\epsilon_i$, K_i is given approximately by

$$K_i \simeq a(\epsilon - \epsilon_i) \qquad \ldots (1.19)$$

$K_{hn} = q_n/q_c$, the excitation probability for state n, as a function of ϵ has different forms according to the excited state involved. The curve for the excitation of a singlet state has a rather broad maximum at an energy value several times the minimum excitation energy ϵ_{hn}, decreasing slowly at higher energies. On the other hand, K_{hn} for a triplet state rises to a very sharp maximum just above ϵ_{hn} after which it falls away rapidly (Figure 1.8).[5]

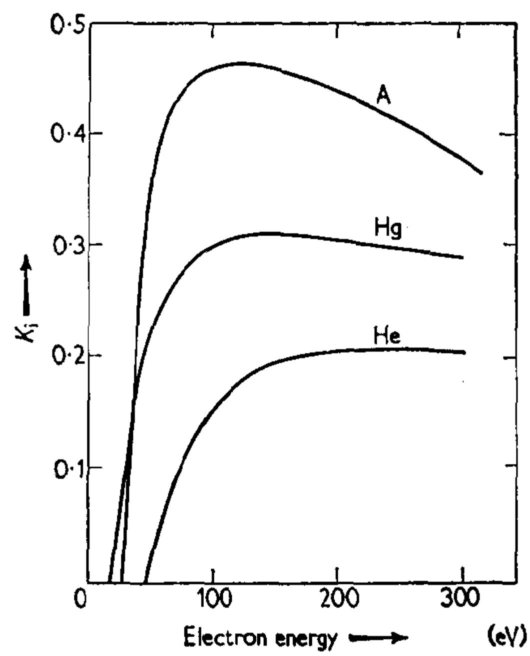


Figure 1.7. Ionization probability of electrons in argon, mercury, and helium. After Arnot. Collision Processes in Gases, p. 39 (Methuen, London, 1950)

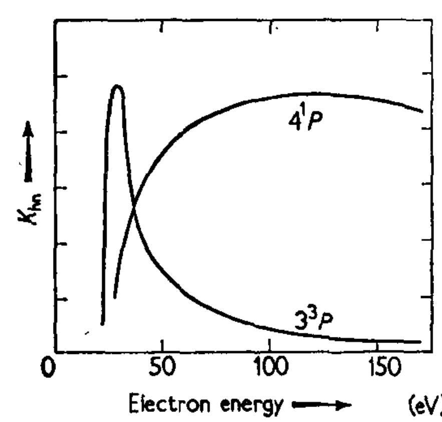


Figure 1.8. Excitation probability curves for singlet and triplet levels of helium. After Arnot. Collision Processes in Gases, p. 33 (Methuen, London, 1950)

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2. Ramsauer, C. *Ann. Phys., Lpz.* **64**, 513 (1921); **66**, 546 (1921)
3. Reference 1, p. 3
4. Reference 1, p. 37
5. Reference 1, p. 55

2

MOTION OF SLOW ELECTRONS IN GASES

2.1. CALCULATION OF ELECTRON ENERGY DISTRIBUTION FUNCTION

2.1.1. *Introduction*

Let us suppose that an electric current traverses a gas in which a uniform electric field X exists. The electron current is supposed to be sufficiently small for interaction between electrons to be unimportant compared with collisions between electrons and gas molecules.

In general, the energy distribution of the electrons will depend on the distance x which the electrons have travelled under the action of the field X. However, provided the distance x is sufficiently large and the electron current density is constant, the energy distribution attains a steady value independent of x. We shall be concerned at present with this case where the average rate of supply of energy to an electron from the field is equal to the average rate of loss of energy in collisions with gas molecules.

The detailed calculation of electron energy distributions is, in general, highly complex and only certain simple cases will be considered here. However, it may be noted that for a given gas the distribution depends only on $X\lambda$, the potential difference per mean free path of an electron. This quantity is generally expressed in the form X/p_0 where p_0 is the gas pressure reduced to 0° C. This can be done because the mean free path λ is inversely proportional to the gas density.

Now provided X/p_0 is not too large, two simplifying assumptions may be made: (1) The energy gained by an electron from the field in one free path is, in general, small compared with the energy of the electron, ϵ; (2) A negligible number of electrons acquire sufficient energy to enable them to ionize gas molecules on collision. It is an obvious consequence of (1) that the electron energy distribution is very nearly isotropic.

Since the mass of an electron m is very small compared with the mass of a gas molecule M, it is clear that the average fraction of its energy that an electron loses in a collision with a gas molecule f will be much less than 1 provided the collisions are mainly elastic. (It was shown in Section 1.3 that $f_{el} \simeq 2m/M$.) It obviously follows from this that only in the limiting case of infinitesimally small X/p_0 values will the electron swarm be in thermal equilibrium with the gas. In general, for finite X/p_0 the mean energy $\bar{\epsilon}$ of an electron in the swarm is much greater than $\frac{3}{2}\kappa T$.

If the electric field X and hence the drift motion of the electrons are confined to the x direction, we can assume that the electron energy distribution function is homogeneous and isotropic in any yz plane. The function should then depend only on the actual speed of an electron c and on the velocity component in the x direction ξ. Let $F(c, \xi)\, d\gamma$ be the number of electrons per unit volume whose velocity components lie in the range $d\gamma = d\xi . d\eta . d\zeta$ (Figure 2.1). F can be expanded in a series of Legendre functions of ξ/c

$$F(c, \xi) = F_0(c) + P_1\left(\frac{\xi}{c}\right).F_1(c) + P_2\left(\frac{\xi}{c}\right).F_2(c) + \ldots$$

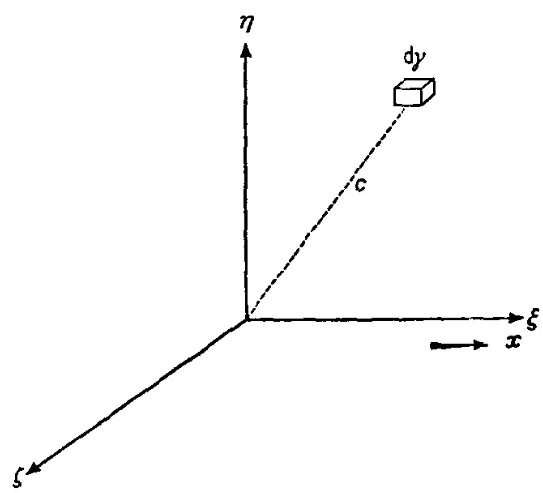

Figure 2.1. Electron velocity distribution function

The spherically symmetric term F_0 is much larger than the higher order terms because of the disordering effect of collisions. This expansion is rapidly convergent for the cases to be considered here. We can therefore write

$$F(c, \xi) = F_0(c) + \frac{\xi}{c} \cdot F_1(c) \qquad \ldots (2.1)$$

where $F_1(c)$ is normally a small term determining the drift motion due to the applied field X.

F is determined by considering the balance between loss and gain of electrons in the velocity element $d\gamma$. We shall calculate first of all the net number of electrons leaving $d\gamma$ per second due to the field $a\, d\gamma$.

2.1.2. *Calculation of 'a'*

The acceleration of an electron due to X is eX/m. This will be, therefore, the velocity with which the point representing the electron in velocity space is displaced in the x direction due to the field. Hence, the number of electrons entering $d\gamma$ per second due to X is

$$\frac{eX}{m} \cdot F . d\eta\, d\zeta$$