Matrix Perturbation Theory in Structural Dynamic Design

结构动态设计的矩阵摄动理论

Chen Suhuan

Science Press
Beijing

(O-2702.0101)

A matrix perturbation theory in structural dynamic design is presented in this book. The theory covers a broad spectrum of subjects, the perturbation methods of the distinct eigenvalues and repeated / close eigenvalues, the perturbation methods of the complex modes of systems with real unsymmetric matrices, the perturbation methods of the defective / near defective systems, random eigenproblem and the interval eigenproblem for the uncertain structures. The contents synthesized the most recent research results in the structural dynamics. Numerical examples are provided to illustrate the applications of the theory in this book.

This book is recommended to graduates, engineers and scientist of mechanical, civil, aerospace, ocean and vehicle engineering.

ISBN 978-7-03-018698-0

9 787030 186980 >

定 价：68.00 元

Matrix Perturbation Theory in Structural Dynamic Design

结构动态设计的矩阵摄动理论

Chen Suhuan

Science Press
Beijing

Responsible Editors: Yan Deping　Yu Hongli

Published by Science Press
16 Donghuangchenggen North Street
Beijing 100717, China

Printed in Beijing

ISBN 978-7-03-018698-0
Science Press, Beijing

Preface

In modern engineering problem, the dynamic design of structures becomes more and more important. It is well known that in order to achieve an optimal design, we have to modify the structural parameters and resolve the generalized eigenvalue problem repeatedly. The iterative vibration analysis may be very tedious job for large and complex structures. Therefore it is necessary to seek a fast computation method for sensitivity analysis and reanalysis. The matrix perturbation method is an extremely useful tool for sensitivity analysis and reanalysis.

The matrix perturbation theory is devoted to the discussion how the natural frequencies and modal vectors change if small changes are imposed on the parameters of structures.In engineering problem, we shall encounter many small changes of the structural parameters, such as small structural modification,manufacture errors,iterative design of structural parameters, design sensitivity analysis, random eigenvalue analysis and robustness analysis of control system, etc. In developing this book, it is assumed that the reader has a university graduate level in mathematics, vibration theory and finite element method.

The contents of the book in general are as follows:

The first chapter is preliminaries to matrix perturbation theory and presents the basic conclusions of vibration theory and finite element method.

Chapter 2 contains the perturbation theory of the distinct eigenvalue. The methods for improving the first order perturbation of the modal vectors, such as high accurate modal superposition method, eigenvector derivatives of the free-free structures and etc., are discussed.

In Chapter 3, systems with repeated frequencies are considered. The matrix perturbation theory of vibration modes of such systems is developed, and the methods for computing the first order perturbation of the modal vectors are also presented.

Chapter 4 contains the theory of matrix perturbation of structures with close frequencies, the spectral decomposition of the stiffness and mass matrixes, and the derivatives of modes of close frequencies.

Chapter 5 presents the matrix perturbation theory of the complex modes of systems with real unsymmetrical matrices, and the discussion is limited to the nondefective systems. The contents include the matrix perturbation methods for distinct, multiple and close eigenvalues.

In Chapter 6, the defective systems are considered. The matrix perturbation theory for defective system is developed. The generalized modal theory and the method for computing generalized modal vectors are covered.

In Chapter 7, the matrix perturbation theory for near defective systems and a shift perturbation method for close eigenvalues are discussed.

Chapter 8 presents the random eigenvalue analysis of structures with random parameters. The contents include random finite element method, random perturbation for random eigenvalue analysis and statistical properties of random eigensolutions.

Chapter 9 presents the matrix perturbation theory for interval eigenproblems. The contents include an introduction to the interval mathematics, Deif's method for interval eigenvalue analysis, the generalization of Deif's method, the matrix perturbation based on Deif's method and interval perturbation method.

This book is recommended to graduates, engineers and scientists of mechanical, civil, aerospace, ocean, and vehicle engineering.

The author would like to express his gratitude to the National Natural Science Foundation of China and "985" Engineering of Jilin University for supporting during the author's research work. Thanks are also due to the author's graduate students of Department of Mechanics, Jilin University, for their assistance with the preparation of the computer routines and numerical examples.

Professor Chen Suhuan
Department of Mechanics
Jilin University
The People's Republic of China
May 2006

Contents

Chapter 1

Finite Element Method for Vibration Analysis of Structures

1.1 Introduction

Before presenting the matrix perturbation theory in this book, we will discuss some important basic considerations for the finite element method and the eigenproblem in structural vibration analysis in this chapter. Because in various sections of the following chapters, we will use the finite element method, and encounter vibration eigenproblem and the statement of their solutions. We shall not at that time discuss how to obtain the required eigenvalues and eigenvectors.

The finite element method is very important to obtain approximate solutions to problems in structural vibration analysis. After application the finite element method to the structure, a discrete analysis model to idealize the continuum can be obtained. The approximation achieved was shown to depend on the characteristics and the number of elements used. As we know that the finite element method is a form of Ritz analysis, so that the Ritz solutions are also applicable to finite element solutions.

In the following sections we present the finite element formulation of continuum mechanics problems and the finite element solutions as a Ritz analysis, such as finite element equations of structural vibration, matrices of element characteristics, vibration eigenproblem, statements of eigensolutions (natural frequencies and mode shape), Ritz analysis for eigenproblems, response analysis including modal superposition and direct integration methods, etc.

1.2 The Hamilton Variational Principle for Discrete Systems[4]

Considering the bending vibration of a beam in the $x - z$ plane, which is assumed to be a plane of symmetry for any cross section, the force vibration equation is

$$\rho A \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}(EI\frac{\partial^2 w}{\partial x^2}) = f(x,t), \quad 0 < x < L \tag{1.1}$$

where $w(x,t)$ is subject to the boundary conditions

$$\begin{aligned} w = 0 \quad \text{or} \quad & \frac{\partial}{\partial x}(EI\frac{\partial^2 w}{\partial x^2}) = 0 \\ \frac{\partial w}{\partial x} = 0 \quad \text{or} \quad & EI\frac{\partial^2 w}{\partial x^2} = 0 \end{aligned} \tag{1.2}$$

and initial conditions. In Eq.(1.1), $w(x,t)$is the transverse displacement, EI the flexural rigidity, A the cross section area, ρ the mass density of the material, $f(x,t)$ the excitation force per unit length of the beam.

The vibration problem expressed by Eqs.(1.1) and (1.2) can be also expressed by the Hamilton variational principle as follows

$$\delta \int_{t_1}^{t_2} (T - V)\mathrm{d}t + \int_{t_1}^{t_2} \delta W \mathrm{d}t = 0 \tag{1.3}$$

where T is the kinetic energy, V the potential energy, δW the virtual work of the external forces. It can be proved that the vibration problem expressed by Eqs.(1.1) and (1.2) is equivalent to that expressed by the variational equation (1.3).

For the complex structures in engineering, the method to form the vibration equation is the finite element method which can be understood as a specific form of the Ritz analysis. Therefore, it is necessary to give the Hamilton variational principle for the discrete systems based on the variational equation (1.3).

Let us now consider a system with n degrees of freedom, the generalized coordinates are $q_1, q_2, \cdots, q_n$, the kinetic energy of the system can be expressed by the generalized velocities $\dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n$ as follows

$$\begin{aligned} T &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{q}} \\ &= T(\dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n) \end{aligned} \tag{1.4}$$

where $\boldsymbol{M}$ is the mass matrix. The potential energy of the system can be expressed by the generalized coordinates as follows

$$\begin{aligned} V =& \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}q_iq_j = \frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{q} \\ =& V(q_1, q_2, \cdots, q_n) \end{aligned} \tag{1.5}$$

where $\boldsymbol{K}$ is the stiffness matrix. The virtual work of the external forces can be expressed by

$$\delta W = \sum_{i=1}^{n} Q_i \delta q_i \tag{1.6}$$

Substituting from Eqs.(1.4) to (1.6) into Eq.(1.3), We have

$$\int_{t_1}^{t_2} (\sum_{i=1}^{n} \frac{\partial T}{\partial \dot{q}_i} \delta \dot{q}_i - \sum_{i=1}^{n} \frac{\partial V}{\partial \dot{q}_i} \delta q_i + \sum_{i=1}^{n} Q_i \delta q_i) \mathrm{d}t = 0 \tag{1.7}$$

where

$$\begin{aligned} \int_{t_1}^{t_2} \sum_{i=1}^{n} \frac{\partial T}{\partial \dot{q}_i} \delta \dot{q}_i \mathrm{d}t &= \sum_{i=1}^{n} \frac{\partial T}{\partial \dot{q}_i} \delta q_i \mid_{t_1}^{t_2} - \int_{t_1}^{t_2} \sum_{i=1}^{n} \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial T}{\partial \dot{q}_i} \delta q_i \mathrm{d}t \\ &= - \int_{t_1}^{t_2} \sum_{i=1}^{n} \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial T}{\partial \dot{q}_i} \delta q_i \mathrm{d}t \end{aligned} \tag{1.8}$$

Substituting Eq.(1.8) into Eq.(1.7), one gets

$$\int_{t_1}^{t_2} [\sum_{i=1}^{n} (- \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial V}{\partial q_i} + Q_i)] \delta q_i \mathrm{d}t = 0 \tag{1.9}$$

Because the generalized coordinate variation δq_i is independent and arbitrary in Eq.(1.9), we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial T}{\partial \dot{q}_i} + \frac{\partial V}{\partial q_i} = Q_i, \quad i = 1, 2, \cdots, n \tag{1.10}$$

This is the Lagrange equation, i.e., the Lagrange equation is equivalent to the Hamilton variational principle for the discrete system. Substituting Eqs.(1.4) and (1.5) into Eq.(1.10), the vibration equation in matrix form for the discrete systems can be obtained

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{Q} \tag{1.11}$$

1.3 Finite Element Method for Structural Vibration Analysis[13]

The main idea of the finite element method is that the complex structure is reduced into an assembly of the discrete elements in which the displacements can be expressed properly by the discrete displacements; the kinetic and the potential energy of the element and the assembled structure can be computed; and then the vibration equations can be derived by the Lagrange equation.

As we know that the assumed displacement functions must be continuous and should preferably satisfy compatibility of deflections and slope on the boundaries.

In the Ritz analysis the displacements in the element can be expressed as the following series form

$$\{u(x, y, z, t)\} = \sum_{i=1}^{n} \phi_i \eta_i = \mathbf{\Phi} \eta \tag{1.12}$$

where $\mathbf{\Phi}$ are the function matrix which in general, consist of polynomial terms, η the Ritz coordinates to be determined. In the finite element method the node displacements of the element should be used as the Ritz coordinates. In order to replace the Ritz coordinates η by the node displacements of the element, from Eq.(1.12) we have

$$\mathbf{u}^e = \mathbf{A}\eta \tag{1.13}$$

where the elements of matrix $\mathbf{A}$ are the functions of the element node coordinates. Because the number of elements of η and $\mathbf{u}^e$ is identical, $\mathbf{A}^{-1}$ exists, from Eq.(1.13) we have

$$\eta = \mathbf{A}^{-1}\mathbf{u}^e \tag{1.14}$$

Substituting Eq.(1.14) into Eq.(1.12), one gets

$$\mathbf{u}(x, y, z, t) = \mathbf{\Phi}\mathbf{A}^{-1}\mathbf{u}^e = \mathbf{N}\mathbf{u}^e \tag{1.15}$$

where

$$\mathbf{N} = \mathbf{\Phi}\mathbf{A}^{-1} \tag{1.16}$$

$\mathbf{N}$ is the displacement shape function matrix.

The strain variable ϵ is expressed as a function of the nodal displacement variables

$$\epsilon = \mathbf{\Delta}\{u(x, y, z, t)\} = \mathbf{\Delta}\mathbf{N}\mathbf{u}^e = \mathbf{B}\mathbf{u}^e \tag{1.17}$$

where $\mathbf{\Delta}$ is the gradient operator matrix, and

$$\mathbf{B} = \mathbf{\Delta}\mathbf{N} \tag{1.18}$$

where $\mathbf{B}$ is the strain matrix of the element.

The stress vector of the element is

$$\sigma = \mathbf{D}\epsilon = \mathbf{D}\mathbf{B}\mathbf{u}^e = \mathbf{S}\mathbf{u}^e \tag{1.19}$$

where $\mathbf{D}$ is the elasticity constant matrix, $\mathbf{S}$ the stress matrix.

The element potential energy is

$$\begin{aligned} \mathbf{V}^e &= \frac{1}{2}\int_v \epsilon^{\mathrm{T}}\sigma \mathrm{d}v \\ &= \frac{1}{2}\int_v (\mathbf{u}^e)^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\mathbf{u}^e \mathrm{d}v \\ &= \frac{1}{2}(\mathbf{u}^e)^{\mathrm{T}}\left(\int_v \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\mathrm{d}v\right)\mathbf{u}^e \\ &= \frac{1}{2}(\mathbf{u}^e)^{\mathrm{T}}\mathbf{K}^e\mathbf{u}^e \end{aligned} \tag{1.20}$$

where

$$\mathbf{K}^e = \int_v \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\mathrm{d}v \tag{1.21}$$

$\mathbf{K}^e$ is the element stiffness matrix.

The velocity vector in the element is

$$\dot{\mathbf{u}}(x, y, z, t) = \mathbf{N}\dot{\mathbf{u}}^e \tag{1.22}$$

and the element kinetic energy is

$$\begin{aligned}\mathbf{T}^e &= \frac{1}{2}\int_V \rho\dot{\mathbf{u}}^{\mathrm{T}}\dot{\mathbf{u}}\mathrm{d}v \\ &= \frac{1}{2}\int_V \rho(\dot{\mathbf{u}}^e)^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\mathbf{N}\dot{\mathbf{u}}^e\mathrm{d}v \\ &= \frac{1}{2}(\dot{\mathbf{u}}^e)^{\mathrm{T}}(\int_V \rho\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}v)\dot{\mathbf{u}}^e \\ &= \frac{1}{2}(\dot{\mathbf{u}}^e)^{\mathrm{T}}\mathbf{m}^e\dot{\mathbf{u}}^e\end{aligned} \tag{1.23}$$

$$\mathbf{m}^e = \int_V \rho\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}v \tag{1.24}$$

$\mathbf{m}^e$ is the consistent mass matrix.

The equivalent nodal force vector $\mathbf{R}^e$ of the body forces $\mathbf{q}^e$ applied to the structure can be obtained by the virtual work of $\mathbf{R}^e$ equalling to that of $\mathbf{q}^e$, i.e.

$$\begin{aligned}\delta\mathbf{u}^{e^{\mathrm{T}}}\mathbf{R}^e &= \int_v (\delta\mathbf{u})^{\mathrm{T}}\mathbf{q}^e\mathrm{d}v \\ &= \int_v \delta(\mathbf{u}^e)^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\mathbf{q}^e\mathrm{d}v \\ &= \delta(\mathbf{u}^e)^{\mathrm{T}}(\int_{\mathbf{v}} \mathbf{N}^{\mathrm{T}}\mathbf{q}^e\mathrm{d}v)\end{aligned} \tag{1.25}$$

So we have the equivalent nodal force vector

$$\mathbf{R}^e = \int_v \mathbf{N}^{\mathrm{T}}\mathbf{q}^e\mathrm{d}v \tag{1.26}$$

The viscous damping force of the element can be expressed as

$$\mathbf{q}_f^e = -\gamma\dot{\mathbf{u}} = -\gamma\mathbf{N}\dot{\mathbf{u}}^e \tag{1.27}$$

Substituting Eq.(1.27) into Eq.(1.26), the equivalent nodal force vector of the viscous damping force can be expressed as

$$\begin{aligned}\mathbf{R}_s^e &= -\int_V \gamma\mathbf{N}^{\mathrm{T}}\mathbf{N}\dot{\mathbf{u}}^e\mathrm{d}v \\ &= -(\int_V \gamma\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}v)\dot{\mathbf{u}}^e \\ &= -\mathbf{C}^e\dot{\mathbf{u}}^e\end{aligned} \tag{1.28}$$

where

$$\mathbf{C}^e = \int_V \gamma\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}v \tag{1.29}$$

$\mathbf{C}^e$ is the viscous damping matrix of the element.

From Eqs.(1.24) to (1.29) it can be seen that the viscous damping matrix $\mathbf{C}^e$ is in proportion to the consistent mass matrix $\mathbf{m}^e$. The difference between $\mathbf{C}^e$ and $\mathbf{m}^e$ is only a proportionality constant .

Assume that the transformation matrix $\mathbf{L}$ related to the local and global coordinates systems, (x, y, z) and (x', y', z'), which is an orthogonal matrix, that is $\mathbf{L}^{-1} = \mathbf{L}^{\mathrm{T}}$. This leads to following relationships(Fig.1.1)

$$\mathbf{u}^e = \mathbf{L}(\mathbf{u}^e)' \tag{1.30}$$

$$\mathbf{R}^e = \mathbf{L}(\mathbf{R}^e)' \tag{1.31}$$

Substituting Eq.(1.30) into Eqs.(1.20) and (1.23), we have

$$(\mathbf{K}^e)' = \mathbf{L}^{\mathrm{T}}\mathbf{K}^e\mathbf{L} \tag{1.32}$$

$$(\mathbf{M}^e)' = \mathbf{L}^{\mathrm{T}}\mathbf{M}^e\mathbf{L} \tag{1.33}$$

Using Eqs.(1.30), (1.31) and (1.29), we have

$$(\mathbf{C}^e)' = \mathbf{L}^{\mathrm{T}}\mathbf{C}^e\mathbf{L} \tag{1.34}$$

and

$$(\mathbf{R}^e)' = \mathbf{L}^{\mathrm{T}}\mathbf{R}^e \tag{1.35}$$

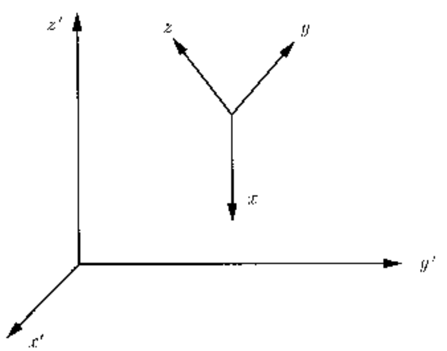

Fig. 1.1

For the sake of simplicity in the following discussion, $\mathbf{K}^e, \mathbf{m}^e, \mathbf{C}^e, \mathbf{R}^e$ and $\mathbf{u}^e$ still represent the corresponding variables in the global coordinate system.

The total potential energy of the structure is

$$\begin{aligned} V &= \sum_e V^e = \frac{1}{2}\sum_e (\mathbf{u}^e)^{\mathrm{T}}\mathbf{K}^e\mathbf{u}^e \\ &= \frac{1}{2}\mathbf{u}^{\mathrm{T}}\sum_e \mathbf{K}^e\mathbf{u} \\ &= \frac{1}{2}\mathbf{u}^{\mathrm{T}}\mathbf{K}\mathbf{u} \end{aligned} \tag{1.36}$$

where $\mathbf{u}$ is the displacement vector, $\mathbf{K}$ is the stiffness matrix of the structure,

$$\mathbf{K} = \sum_e \mathbf{K}^e \tag{1.37}$$

The total kinetic energy of the structure is

$$\begin{aligned} T &= \sum_e \mathbf{T}^e = \frac{1}{2}\sum_e (\dot{\mathbf{u}}^e)^{\mathrm{T}} \mathbf{m}^e \dot{\mathbf{u}}^e \\ &= \frac{1}{2}\dot{\mathbf{u}}^{\mathrm{T}}(\sum_e \mathbf{m}^e)\dot{\mathbf{u}} \\ &= \frac{1}{2}\dot{\mathbf{u}}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{u}} \end{aligned} \tag{1.38}$$

where $\mathbf{M}$ is the mass matrix of the structure,

$$\mathbf{M} = \sum_e \mathbf{m}^e \tag{1.39}$$

The total virtual work of the external forces and the damping forces is

$$\begin{aligned} \delta W &= \sum_e \delta W^e = \sum_e \delta(\mathbf{u}^e)^{\mathrm{T}}(\mathbf{R}^e + \mathbf{R}_f^e) \\ &= \delta\mathbf{u}^{\mathrm{T}} \sum_e (\mathbf{R}^e + \mathbf{R}_f^e) \\ &= \delta\mathbf{u}^{\mathrm{T}}(\sum_e \mathbf{R}^e + \sum_e \mathbf{R}_f^e) \\ &= \delta\mathbf{u}^{\mathrm{T}}(\mathbf{R} + \mathbf{R}_f) \end{aligned} \tag{1.40}$$

Using Eq.(1.28), we obtain

$$\begin{aligned} \mathbf{R}_f &= \sum_e \mathbf{R}_f^e = -\sum_e \mathbf{C}^e \dot{\mathbf{u}}^e \\ &= -(\sum_e \mathbf{C}^e)\dot{\mathbf{u}} = -\mathbf{C}\dot{\mathbf{u}} \end{aligned} \tag{1.41}$$

where $\mathbf{C}$ is the damping matrix of the structure

$$\mathbf{C} = \sum_e \mathbf{C}^e \tag{1.42}$$

Hence, Eq.(1.40) becomes

$$\delta W = \delta\mathbf{u}^{\mathrm{T}}\mathbf{R}' \tag{1.43}$$

where

$$\mathbf{R}' = \mathbf{R} + \mathbf{R}_f = \mathbf{R} - \mathbf{C}\dot{\mathbf{u}} \tag{1.44}$$

Substituting Eqs.(1.36), (1.38) and (1.44) into the Lagrange equation we have

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{R}' = \mathbf{R} - \mathbf{C}\dot{\mathbf{u}}$$