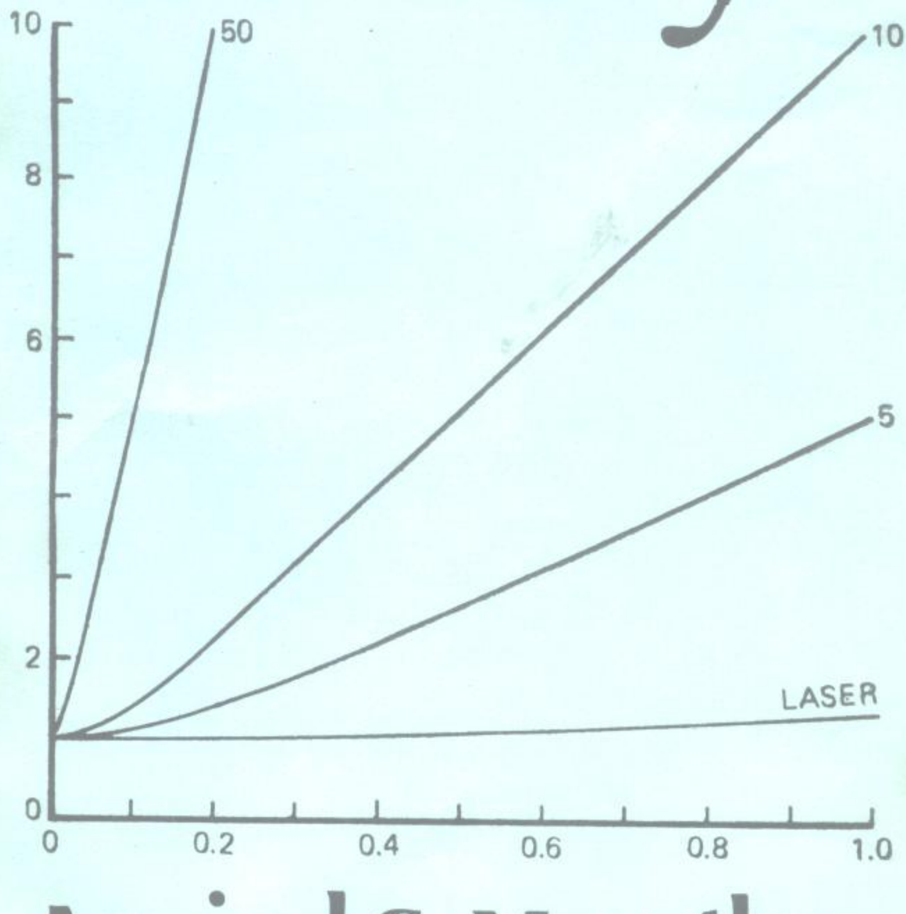
Elements Of Optical Coherence Theory



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Preface

The theory of partial coherence has been evolving over the past three decades or more. During this time, numerous papers and several research monographs have appeared, covering a wide range of topics at a relatively advanced level.

There is now a definite need for a textbook to introduce the student to the basic elements of optical coherence theory. The present book is intended to fill that need. The book presupposes a basic knowledge of interference and diffraction of light and a familiarity with Fourier theory. The required level of knowledge of interference and diffraction is that of a good general physics course at the undergraduate level. Some knowledge of statistics would be helpful, but it is not absolutely necessary since some basic discussion on this topic is included in the Appendixes to Chapter 3. The book may be used for a senior undergraduate or a first-year graduate course. It can also be used by scientists and engineers working in other fields of research. They can apply the elements and methods of coherence theory to their own fields of work.

The approach taken in writing this book is that of "back to basics." A good foundation will prepare the reader for applications and uses of coherence theory in the area of interest, without outside help. Homework problems are included wherever possible. For the most part, they are meant to further the understanding or to develop a new aspect of the topics covered in the text. Some references are supplied for the same purpose and also to give the reader sources for further research into the topic of interest. There are a few problems that call for short numerical calculations, in order to provide an understanding of the order of magnitude of the quantities involved in the theory.

Insofar as possible, without sacrificing clarity, the book is kept to a limited size so that a major portion of it may be comfortably covered during a one semester course. The selection of topics was a difficult task. Instead of covering a large number of topics inadequately, it was decided to limit the topics to what may be called "elements" of coherence theory, including basic properties of sources and the light they emit. As a result, some topics that might be expected to be covered in a book on coherence theory are not included here.

As the title suggests, the subject matter of the book is theoretical. Wherever appropriate, however, basic experiments are brought into the discussion so that the reader may appreciate the usefulness of theoretical concepts and quantities in relation to elementary optical experiments. Theory inevitably entails derivation of results. This can be lengthy but an attempt has been made to make it painless. In some cases the details of the derivation are given in an appendix; in others the "route" from one intermediate step to another is detailed in words. In the discussion of the diffraction of light in the language of the mutual coherence function (MCF), Appendixes 5.1 and 5.2 are used extensively to arrive at the result of the generalized van Cittert–Zernike theorem, which is the working equation for the rest of Chapter 5.

Chapter 1 discusses briefly the history of coherence theory. Chapter 2 details the complex analytic signal representation, and in Chapter 3 the MCF is introduced in this language, with a discussion of the field statistics. Chapter 4 develops mathematical familiarity with the MCF, along with the introduction of the concept of noncoherence. It is customary to discuss noncoherence in terms of sharply peaked narrow functions and an attempt has been made to introduce noncoherence with emphasis on the property of the constant or "uniform" spatial frequency spectrum and its interpretation in parallel with the "white" noise sources of electrical engineering. In this way, the properties of the noncoherent source follow more easily.

In the field of optics the term *intensity* is used loosely: Not properly defined, it has no place in the radiometric scheme. In order not to confuse it with the term *radiant intensity* defined in radiometry, intensity is always qualified with an appropriate adjective. Thus, when referring to the time average of the square of the field variable, we use the term *optical intensity* and denote it by I or by $I(x)$ to display the space coordinate(s). Frequently, it is necessary to distinguish optical intensity from its spectral version, which we call *optical spectral intensity* (OSI) and denote by $\hat{I}(x, \nu)$. We follow the Système International for units and nomenclature, and for the purpose of this book we display the radiometric symbols in sans serif type. Thus, irradiance is denoted by E to distinguish it from the electric field $\mathbf{E}$. By using a scaling factor C with appropriate units, we use $\mathsf{E} = \mathsf{C}I$ for irradiance, with units of W m^{-2}. Similarly, $\hat{\mathsf{E}} = \mathsf{C}\hat{I}$ is used for spectral irradiance [W m^{-2} Hz^{-1}]. Although this approach entails dealing with a wide range of symbols and terms, the author feels it is best to follow this scheme in order to clearly specify what is meant in each particular situation.

As much as possible, an attempt has been made to keep the notation uniform throughout the book. A list of symbols, notation, and abbreviations is also given.

Chapter 5 deals with the propagation of the mutual coherence function (MCF) and related topics. The generalized van Cittert–Zernike theorem and the van Cittert–Zernike theorem with noncoherent sources are discussed. By way of example, it is shown how the partially coherent-source result approaches the noncoherent-source result by going to the limit of the constant spatial frequency spectrum. The Thompson and Wolf experiment, the Michelson stellar interferometer, and the two-beam interferometer are studied to explain the spatial and temporal coherence of light. The book is concerned exclusively with the second-order statistics, namely, the mutual coherence function. The Hanbury-Brown and Twiss interferometer is mentioned only briefly, to make the reader aware of the relatively recent advances. The chapter continues with a discussion of how the beam energy is distributed (spread out) in the right half-space after it leaves the source. The measurement of the mutual coherence function is discussed at the end.

The propagation of the MCF is used to discuss image formation in Chapter 6. The discussion makes use of the object and image space spatial frequencies defined with the object and image distances measured from the entrance and exit pupil planes, respectively. In theoretical formulations, there is a tendency to omit the "constants" that occur outside of the integrals or to collect several constants under one common constant as the development proceeds. This "snowball" effect has been avoided. As a result, the relationships in the chapter may be verified for dimensional balance at every stage of the development. In Chapter 6, Hopkins' effective source is introduced in order to study the influence of partial coherence on optical imaging. The chapter ends with a summary and a brief discussion of resolution criteria. In particular, it is shown how the various resolution criteria are special cases of the one that may be formulated by use of the spatial coherence function and the van Cittert–Zernike theorem.

The last chapter, Chapter 7, is on radiometry. After a brief review of conventional radiometry, a study is made of the properties of the noncoherent source. The study is based on the well-established diffraction calculation. It is shown that the idealization of noncoherence in the theory of partial coherence is the same as the idealization of a Lambertian source of conventional radiometry. Two approaches to generalized radiometry applicable to sources of any state of coherence are presented. The results of the special cases derived in the first approach (largely unpublished) are tabulated for ease of reference. The second approach, pioneered by Walther, Marchand, and Wolf, is also discussed along with the results that follow for some of the special cases. Because the discussion is brief, the original paper of Marchand and Wolf is reproduced in Appendix 7.2. No attempt is made (at least not intentionally) to upgrade or downgrade one or the other approach; only the results as they follow for the special cases are displayed for ease of comparison by the reader. A word of caution to the reader at this point: the first approach is given in order to observe what the results would be if that path were followed; the approach that is widely used throughout the literature is the one due to Walther, Marchand, and Wolf.

I offer sincere thanks to my former professors, H. H. Hopkins and E. L. O'Neill, with whom I have had the pleasure of associating and studying optics. I should also like to express my appreciation to Professor E. Wolf, who has done so much for so long in the field of optics and from whose wide range of publications I learned coherence theory. I am extremely fortunate to associate with the distinguished faculty of the Optical Sciences Center, University of Arizona. In particular, my association with Professors H. H. Barrett, R. V. Shack, R. R. Shannon, P. N. Slater, and J. C. Wyant has proved to be a valuable learning experience through helpful discussions. It is a pleasure to acknowledge helpful discussions on radiometry with Professor W. L. Wolfe, Dr. F. O. Bartell, and Dr. J. M. Palmer. And, of course, in an institution of learning, the students form an important component: I am thankful for their discussions and, in particular, I should like to mention Dr. M. J. Lahart, Dr. V. N. Mahajan, and Dr. R. E. Wagner. Furthermore, I am especially grateful to Professor P. A. Franken, Director of the Optical Sciences Center, for providing an environment conducive to learning and for his constant encouragement.

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Symbols, Notation, and Abbreviations

a	radius of lens or entrance pupil
a'	radius of exit pupil
a_0	radius of circular source
a_0, b_0	rectangular source sides
$\boldsymbol{a}_c$	coherence area for spatial coherence
$A(\mathbf{x} - m\boldsymbol{\xi})$	Fourier transform of the exit pupil function
$\mathscr{A}$	symbol for area
$\text{Besinc}\,(x) \equiv 2J_1(x)/x$	J_1 is the Bessel function of order 1 and x stands for the unitless argument
BFP	back focal plane
c	speed of light in vacuum
c-radiometry	conventional radiometry
$\text{cyl}(r_s/a)$	$= \begin{cases} 1, & r_s < a,\ r_s = \left(x_s^2 + y_s^2\right)^{1/2} \\ 0, & r_s > a \end{cases}$
$C(\mathbf{f}'_1, \mathbf{f}'_2, \nu)$	Hopkins' frequency response (HFR) function
C	suitable constant for use with the scalar field ψ: $\mathsf{E} = \mathsf{C}\langle\psi^2\rangle = \mathsf{C}I$
$E\{\ \}$	ensemble average
EP	enclosed power
ESP	enclosed spectral power
$\mathbf{E}$	electric field vector
E	irradiance [W m^{-2}]
$\hat{\mathsf{E}}$	spectral irradiance [W m^{-2} Hz^{-1}]
f, g	pair of spatial frequencies: $f = (1/\lambda)p$, $g = (1/\lambda)q$
$f = \alpha/\lambda s,\ g = \beta/\lambda s$	object-space spatial frequencies

$\mathbf{f} = \hat{\mathbf{i}}f + \hat{\mathbf{j}}g$	two-dimensional vector for object-space frequencies
$f' = \alpha'/\lambda s'$ $g' = \beta'/\lambda s'$	image-space spatial frequencies
$\mathbf{f}' = \hat{\mathbf{i}}f' + \hat{\mathbf{j}}g'$	two-dimensional vector for image-space frequencies
FOV	field of view
FTS	Fourier transform spectrometry
g-radiometry	generalized radiometry
HFR	Hopkins' frequency response
$\mathfrak{H}[\cdots]$	Hilbert transform of $[\cdots]$; see Eq. (2.18)
$I = \langle\psi^2\rangle$	optical intensity, Eq. (2.3)
I	radiant intensity [W sr^{-1}]
$\hat{\mathsf{I}}$	spectral radiant intensity [W sr^{-1} Hz^{-1}]
$J_0, J_1, J_{3/2}$	Bessel functions of order 0, 1, 3/2 respectively
$k = 2\pi/\lambda$	propagation constant
$K(Q, P)$	approximate multiplicative free-space propagator for light from P to Q, Eq. (5-142)
LHS	left-hand side
L, $\hat{\mathsf{L}}$	radiance [W m^{-2} sr^{-1}] and spectral radiance [W m^{-2} sr^{-1} Hz^{-1}], respectively
$\mathfrak{L}(\boldsymbol{\alpha}')$	amplitude and phase transmittance of the system described in the exit pupil coordinates
$(\Delta l)_c$	coherence length
m	image magnification
m_p	pupil magnification
MCF	mutual coherence function
MOI	mutual optical intensity
MSDF	mutual spectral density function
MTF	modulation transfer function
M	radiant exitance [W m^{-2}]
$\hat{\mathsf{M}}$	spectral radiant exitance [W m^{-2} Hz^{-1}]
$(\mathrm{N.A.})_c$	numerical aperture of the condenser
$(\mathrm{N.A.})_o$	numerical aperture of the objective
OI	optical intensity
OPD	optical path difference

OSI	optical spectral intensity
OTF	optical transfer function
p, q, m	direction cosines: $m = \begin{cases} +\left(1-p^2-q^2\right)^{1/2}, & p^2+q^2 \le 1 \\ +i\left(p^2+q^2-1\right)^{1/2}, & p^2+q^2 > 1 \end{cases}$
PSF	point spread function
QH source	quasihomogeneous source
QM field	quasimonochromatic field
Q	energy in the field [joules] [J]
$r_{12} = (x_{12}^2 + y_{12}^2)^{1/2}$	radius vector in difference coordinates
$\mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z$ $= \hat{\mathbf{i}}r\sin\theta\cos\phi + \hat{\mathbf{j}}r\sin\theta\sin\phi + \hat{\mathbf{k}}r\cos\theta$	position vector with spherical polar coordinates
$\mathrm{Rect}(x/a_0)$	rectangular function, equals unity for $\lvert x \rvert \le a_0$ and zero for $\lvert x \rvert > a_0$
RHS	right-hand side
$s(t)$	temporal Fourier transform of $\hat{s}(\nu)$
$\hat{s}(\nu)$	step function, equals $+1$ for $\nu \ge 0$ and zero for $\nu < 0$
$\mathrm{sgn}(\nu)$	signum function, equals $+1$ for $\nu > 0$ and -1 for $\nu < 0$; see Eq. (2.23)
$\mathbf{s} = \hat{\mathbf{i}}x_s + \hat{\mathbf{j}}y_s + \hat{\mathbf{k}}z_s$	point on a surface $\mathbb{S}$
$S_{\mathrm{coh}}, T_{\mathrm{coh}}$	amplitude, impulse response, and transfer function of the lens system, respectively, for the coherent case
$S_{\mathrm{ncoh}}, T_{\mathrm{ncoh}}$	impulse response and transfer function of the lens system, respectively, for the noncoherent case
$S_{\mathrm{pcoh}}, T_{\mathrm{pcoh}}$	impulse response and transfer function of the lens system, respectively, for the partially coherent case
$S_{\mathrm{poly}}, T_{\mathrm{poly}}$	impulse response and transfer function of the lens system, respectively, for the polychromatic case in the noncoherent limit
SFS	spatial frequency spectrum

$\mathrm{Sinc}(x) = (\sin x)/x$	x is a unitless argument of the trigonometric sine function
t	time variable [s]
t_{ncoh}	normalized form of T_{ncoh}, OTF
$\hat{t}_{\mathrm{ob}}(\boldsymbol{\xi}, \boldsymbol{\nu})$	amplitude transmittance of the object
$(\Delta v)_c$	coherence volume
v	Michelson's visibility function for interference fringes
$V(t)$	$= (1/\sqrt{2})[V^{(r)}(t) + iV^{(i)}(t)]$, analytic signal
$V(\mathbf{r}, t)$	analytic signal for the field at point r
$V^{(i)}(t)$	imaginary part of the analytic signal
$V^{(r)}(t)$	real physical field
$\hat{V}(\nu)$	temporal Fourier transform of $V(t)$
$\hat{V}^{(r)}(\nu)$	complex temporal Fourier transform of $V^{(r)}(t)$
$\tilde{V}(\kappa p, \kappa q, z, t)$	two-dimensional spatial Fourier transform of $V(\mathbf{r}, t)$
$\mathring{V}(\kappa p, \kappa q, z, \nu)$	total (spatial and temporal) Fourier transform of $V(\mathbf{r}, t)$
v	visibility
w	coherence width
w_x	coherence width for spatial coherence along x
w_y	coherence width for spatial coherence along y
$W(\boldsymbol{\alpha}') \equiv W(\alpha', \beta')$	wavefront aberration in the exit pupil coordinates
WMW	Walther, Marchand, and Wolf approach
w	energy density (field) [J m^{-3}]
x, y, z	space coordinates
$x_{12} = x_1 - x_2$, $y_{12} = y_1 - y_2$	difference coordinates
$\mathbf{x} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y$	two-dimensional vector and coordinates for the image plane
$\boldsymbol{\alpha} = \hat{\mathbf{i}}\alpha + \hat{\mathbf{j}}\beta$	two-dimensional vector and coordinates for the entrance pupil
$\boldsymbol{\alpha}' = \hat{\mathbf{i}}\alpha' + \hat{\mathbf{j}}\beta'$	two-dimensional vector and coordinates for the exit pupil
$\boldsymbol{\alpha}_0 = \hat{\mathbf{i}}\alpha_0 + \hat{\mathbf{j}}\beta_0$	two-dimensional vector and coordinates for the effective source

$\gamma_{11}(\tau)$	complex degree of self-coherence
$\mid\gamma_{12}\mid$	degree of partial coherence of light from points 1 and 2
$\gamma_{12}(0)$	complex degree of spatial coherence
$\gamma_{12}(\tau) = \mid\gamma_{12}\mid \exp(+i\phi_{12})$	normalized MCF, complex degree of coherence
$\hat{\gamma}(\nu) = \hat{\gamma}_{11}(\nu)$	when points P_1 and P_2 coincide, normalized spectrum
$\hat{\gamma}_{12}(\nu)$	temporal Fourier transform of $\gamma_{12}(\tau)$
$\Gamma_{11}(\tau)$	self-coherence function; describes temporal coherence
$\hat{\Gamma}_{11}(\nu)$	optical spectral intensity (OSI), spectral density function, temporal Fourier transform of $\Gamma_{11}(\tau)$
$\Gamma_{12}(0)$	MOI for a pair of points P_1 and P_2, $\tau = 0$; describes spatial coherence
$\Gamma_{12}(t_1, t_2) = \boldsymbol{\Gamma}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$	mutual coherence function for the nonstationary case
$\Gamma_{12}(\tau) = \Gamma(P_1, P_2, \tau)$	mutual coherence function
$\hat{\Gamma}_{12}(\nu)$	MSDF for a pair of points P_1 and P_2; it is the temporal transform of $\Gamma_{12}(\tau)$
$\tilde{\Gamma}(f_1, g_1, f_2, g_2, z, \tau)$	spatial Fourier transform of $\Gamma_{12}(\tau)$; (f, g), pair of spatial frequencies
$\mathring{\Gamma}(f_1, g_1, f_2, g_2, z, \nu)$	spatial and temporal Fourier transform of $\Gamma_{12}(\tau)$
$\delta_{\mu\mu'}$	Kronecker delta, equals unity for $\mu = \mu'$, zero otherwise
$\delta(x - x')$	one-dimensional Dirac delta function
$\delta(\mathbf{x} - \mathbf{x}') = \delta(x - x')\delta(y - y')$	two-dimensional delta function using two-dimensional vectors $\mathbf{x}$ and $\mathbf{x}'$
$\delta^{(2)}(\mathbf{s}_1 - \mathbf{s}_2)$	two-dimensional delta function with three-dimensional vectors $\mathbf{s}$, with components x_s, y_s, z_s
$\Delta\nu$	spectral spread of QM fields
ε	enclosed power
$\boldsymbol{\xi} = \hat{\mathbf{i}}\xi + \hat{\mathbf{j}}\eta$	object-space two-dimensional vector
$\boldsymbol{\xi} \cdot \boldsymbol{\xi} = \xi^2 + \eta^2$	square of the vector length
$d\boldsymbol{\xi} \equiv d\xi\, d\eta$	area element
$f(\boldsymbol{\xi}) \equiv f(\xi, \eta)$	function of ξ and η
$\kappa = 1/\lambda$	kappa, reciprocal of the wavelength

λ	wavelength of light in vacuum
ν	temporal frequency [s^{-1}]
$\bar{\nu}$	mean frequency of a QM field
$\rho_j = \lvert \mathbf{r} - \mathbf{s}_j \rvert$	dijtance between P_j and Q, $j = 1, 2$
τ	time delay
τ_c	coherence time
Φ	average radiant power [W]
$\hat{\Phi}$	spectral radiant power [W Hz^{-1}]
ψ	scalar field function
$\omega = 2\pi\nu$	circular frequency
$\langle \ \rangle$	angular brackets for time average

Contents

1

Coherence

The rectilinear propagation of light has always been more or less self-evident to even the most casual inquirer. Less obvious has been the periodic nature of light. The first experiment to offer a glimpse of the periodicity of light was done by Grimaldi (1613–1663), who used sunlight to observe fringes in the shadow of a hair. The hair was arranged accurately parallel to a narrow vertical slit opening in an otherwise opaque window shade.

Newton (1642–1727) devised a variety of experiments in the study of light, and by the Newton's rings and related experiments he discovered clear evidence of light's periodicity. In explaining his results, Newton found it necessary to associate something periodic—he called it "fits"—with light rays, which until then had been regarded as uniform. He determined the interrelationships among length of period, color of light, and refractive index, and also found the law of the radii of the bright and dark rings in a single color of light. Newton was a thorough experimentalist, whose experiments were much more refined than those of his predecessors. However, he made it perfectly clear that he was not interested in idle hypotheses. At times he offered analogies, like the waves created in a pond by a falling stone, but he refrained from hypothesizing and emphasized the importance of accurately describing the findings. During Newton's lifetime, Huygens (in 1690) had proposed the wave hypothesis for light and had given his method for calculating the future shape of the wave front. But these considerations did not influence Newton's explanations of his own experiments.