Computer Aided Analysis and Optimization of Mechanical System Dynamics

Edited by Edward J. Haug

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OPTIMIZATION OF MECHANICAL SYSTEM DYNAMICS

IOWA CITY, IOWA, UNITED STATES, AUGUST 1-12, 1983

Director: Edward J. Haug Center for Computer Aided Design College of Engineering University of Iowa Iowa City, Iowa

Scientific Content of the Advanced Study Institute

The Advanced Study Institute was organized to bring together engineers, numerical analysts, and applied mathematicians working in the field of mechanical system dynamic analysis and optimization. The principal focus of the Institute was on dynamic analysis and optimization of mechanical systems that are comprised of multiple bodies connected by kinematic joints and compliant elements. Specialists working in this area from throughout North America and Western Europe presented alternative approaches to computer generation and solution of the equations of system dynamics. Numerical analysis considerations such as sparse matrix methods, loop closure topological analysis methods, symbolic computation methods, and computer graphics were explored and applied to system dynamic analysis and design. This forum provided ample opportunity for expression of divergent views and spirited discussion of alternatives and their pros and cons. Emerging developments in dynamics of systems with flexible bodies, feedback control, intermittent motion, and other interdisciplinary effects were presented and illustrated. Animated graphics was shown to be a valuable tool in visualization of system dynamics, as illustrated through applications in mechanism and vehicle dynamics. Recently developed methods of kinematic synthesis, kinematic and dynamic design sensitivity analysis, and iterative optimization of mechanisms and machines were presented and illustrated.

Scientific Program of the Advanced Study Institute

The scientific program began with a review (Haug) of alternative approaches that are possible and trade-offs that must be made in selecting an efficient, unified method of system dynamic analysis.

Fundamental analytical methods in machine dynamics were reviewed (Paul and Wittenburg) and computational applications discussed. methods for kinematic definition of system state were discussed (Wittenbury and Wehage). Lagrangian formulations of equations of mechanical system dynamics, using symbolic computation and a minimal set of generalized coordinates, were presented and applied to study vehicle dynamics (Schiehlen). An alternative formulation, using a maximal set of Cartesian generalized coordinates and the resulting simplified form of sparse equations, were presented and illustrated (Chace and Nikravesh). The potential for application of general purpose symbolic computation languages for support of dynamic analysis was considered and test problems illustrated (Noble). A comprehensive review of numerical methods that are available for solving differential equations of motion, regardless of how derived, was presented (Enright) and computer software that is available for applications was discussed. Special numerical analysis problems associated with mixed differential-algebraic equations and numerical methods for treating systems with both high frequency and low frequency content were discussed and the state of the art evaluated (Gear). Application of numerical integration methods to various formulations of equations of motion were discussed and use of high speed computer graphics to create an animation as output of dynamic analysis was illustrated (Nikravesh). Formulations for dynamic analysis of mechanisms and machines with flexible components were presented and their relationship with finite element structural analysis codes discussed (van der Werff). Systematic incorporation of feedback control and hydraulic effects in large scale mechanical system dynamics were discussed and illustrated (Vanderploeg). of kinematic synthesis were presented and their application using microcomputers illustrated (Rankers). Methods for design sensitivity analysis and optimization of large scale kinematically and dynamically driven systems were presented and illustrated (Haug). Iterative optimization methods that are suitable for application in kinematic and dynamic system synthesis were reviewed and their pros and cons discussed (Fleury and Gill).

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These proceedings contain lectures presented at the NATO-NSF-ARO sponsored Advanced Study Institute on "Computer Aided Analysis and Optimization of Mechanical System Dynamics" held in Iowa City, Iowa, 1-12 August, 1983. Lectures were presented by free world leaders in the field of machine dynamics and optimization. Participants in the Institute were specialists from throughout NATO, many of whom presented contributed papers during the Institute and all of whom participated actively in discussions on technical aspects of the subject.

The proceedings are organized into five parts, each addressing a technical aspect of the field of computational methods in dynamic analysis and design of mechanical systems. The introductory paper presented first in the text outlines some of the numerous technical considerations that must be given to organizing effective and efficient computational methods and computer codes to serve engineers in dynamic analysis and design of mechanical systems. Two substantially different approaches to the field are identified in this introduction and are given attention throughout the text. The first and most classical approach uses a minimal set of Lagrangian generalized coordinates to formulate equations of motion with a small number of constraints. The second method uses a maximal set of cartesian coordinates and leads to a large number of differential and algebraic constraint equations of rather simple form. These fundamentally different approaches and associated methods of symbolic computation, numerical integration, and use of computer graphics are addressed throughout the proceedings. At the conclusion of the Institute, participants agreed that a tabulation of available software should be prepared, to include a summary of capabilities and availability. A survey was carried out following the Institute to provide information on software that is available. Results of this survey are included in the introductory paper.

Basic analytical methods of formulating governing equations of mechanical system dynamics are presented in Part 1 of the proceedings. Implications of selection of alternative formulations of the equations of classical mechanics are identified and discussed,

with attention to their suitability for computer implementation. Algebraic and analytical properties of alternative generalized coordinate sets are discussed in some detail.

Part 2 of the proceedings focuses on methods of computer generation of the equations of dynamics for large scale, constrained dynamic systems. Both the loop closure Lagrangian generalized coordinate approach for formulating a minimal system of governing equations of motion and the cartesian coordinate approach that leads to a maximal set of loosely coupled equations are presented and illustrated. Use of symbolic computation techniques is presented as an integral part of the Lagrangian coordinate approach and as an independent method for analytical studies in system dynamics.

Numerical methods of solving systems of ordinary differential equations and mixed systems of differential-algebraic equations are treated extensively in Part 3. Theoretical properties of numerical integration algorithms are reviewed and their favorable and unfavorable attributes for application to system dynamics analyzed. A review of available computer codes for use in solution of equations of dynamics is presented. Applications of integration techniques and high speed computer graphics to aid in solution of dynamic equations and in interpretation of results are presented and illustrated.

Two important interdisciplinary aspects of machine dynamics are presented in Part 4. Methods of including the effects of flexible bodies in machine dynamics applications, based primarily on finite element structural analysis models, are presented and illustrated. A method for incorporating feedback control subsystems into modern mechanical system dynamic analysis formulations is presented and examples that illustrate first order coupling between control and phyical dynamic effects are illustrated.

Part 5 of the proceedings focuses on synthesis and optimization of kinematic and dynamic systems. An extensive treatment of methods of type and parameter synthesis of mechanisms and machines is presented and illustrated through applications on a microcomputer. Methods of design sensitivity analysis and optimization of both kinematically and dynamically driven systems, using large scale computer codes for formulation and solution of dynamic and design sensitivity equations, are presented. Finally, surveys are presented on leading iterative optimization methods that are available and applicable for design optimization of mechanical system dynamics.

The extent and variety of the lectures presented in these proceedings illustrate the contribution of numerous individuals in preparation and conduct of the Institute. The Institute Director wishes to thank all the contributors to these proceedings and participants in the Institute, who refused to be passive listeners and participated actively in discussions and contributed presentations. Special thanks go to C. Flack, S. Lustig, and R. Huff for their efforts in administrative planning and support of the Institute. Finally, without the financial support of the NATO Office of Scientific Affairs, the U.S. National Science Foundation, and the U.S. Army Research Office, the Institute and these proceedings would not have been possible. Their support is gratefully acknowledged by all concerned with the Institute.

February, 1984 E. J. Haug

^{*} The views, opinions, and/or findings contained in these proceedings are those of the authors and should not be construed as an official position, policy, or decision of the sponsors, unless so designated by other documentation.

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