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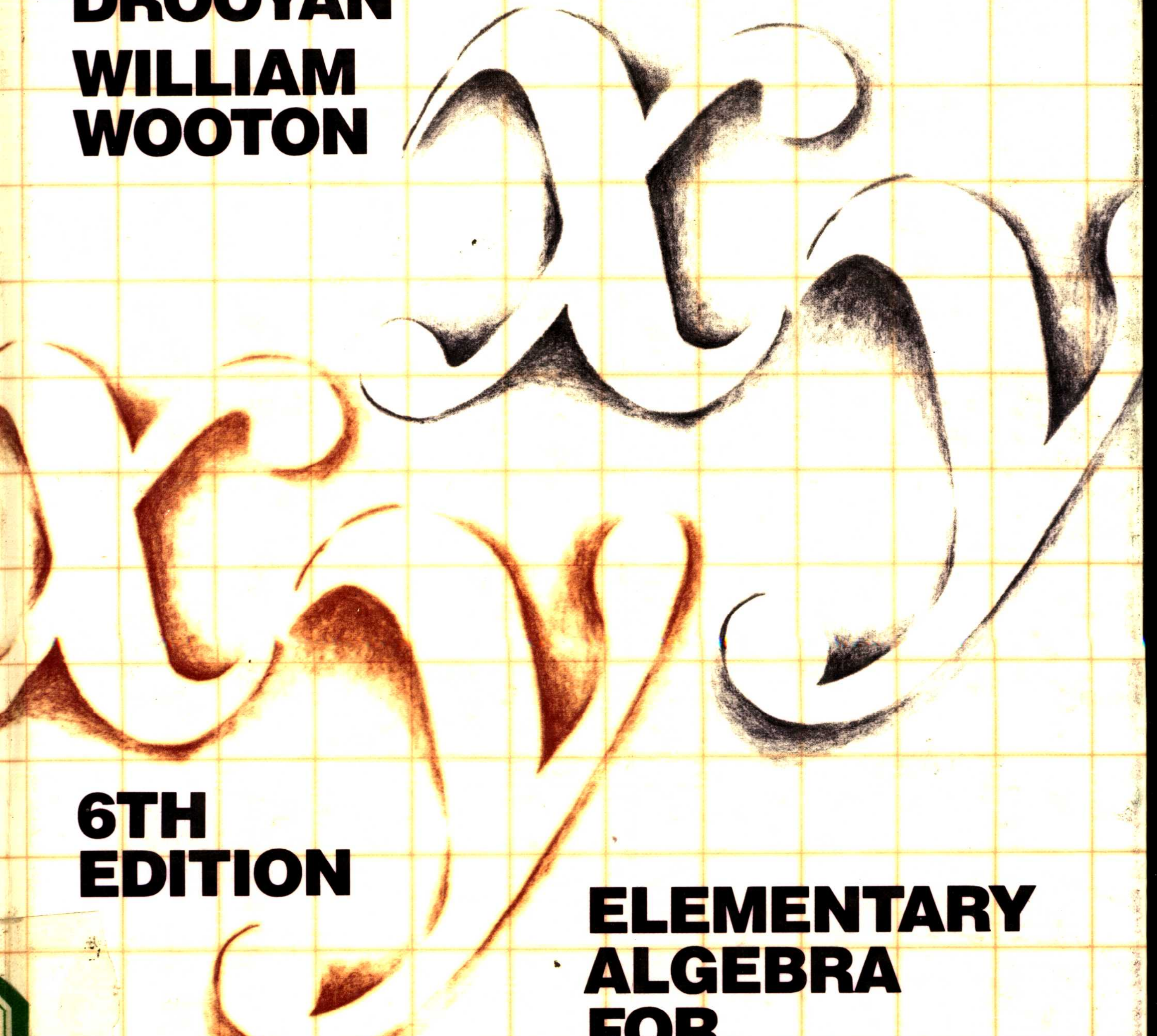

**6TH
EDITION**

ELEMENTARY ALGEBRA FOR COLLEGE STUDENTS

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6th Edition

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New York Chichester Brisbane Toronto Singapore

TO THE STUDENT: A student's solutions manual for the textbook is available through your college bookstore to accompany *Elementary Algebra for College Students* by Irving Drooyan and William Wooton. The solutions manual can help you with course material by acting as a tutorial, review, and study aid. If the solutions manual is not in stock, ask the bookstore manager to order a copy for you.

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PREFACE

This edition of *Elementary Algebra for College Students* retains the basic point of view of the preceding editions with respect to subject matter and pedagogy. As with each of our earlier revisions, we have rewritten parts of the text to improve clarity and revised many exercise sets to increase their effectiveness. Also, we have made several changes in the organization of the fifth edition and added new topics and features.

CHANGES IN ORGANIZATION

Chapter 5 of the previous edition has been separated into two chapters. In this edition, Chapter 5 treats properties of fractions and Chapter 6 covers operations with fractions. Products and quotients of fractions are presented before sums and differences.

Chapter 6 of the previous edition has been separated into two chapters. In this edition, Chapter 7 treats linear equations in two variables and their graphs, and Chapter 8 covers systems of linear equations.

The appendicies of the previous edition, Integer Exponents, Scientific Notation, and Solutions of Inequalities are included in Sections 3.9 and 5.5 of this edition.

NEW TOPICS

Equations of straight lines are introduced in Section 7.6.

Solutions of inequalities in two variables are introduced in Section 7.8.

Set concepts are introduced in Appendix A.

NEW FEATURES

Annotations are used to highlight parts of examples.

Subheadings are used to highlight different topics of each section.

Common student errors are emphasized in the text, exercise sets, and cumulative reviews.

Word problems are treated more extensively, with an emphasis on setting up a mathematical model.

EXCERPTS FROM THE PREFACE OF THE PREVIOUS EDITION THAT ARE APPLICABLE TO THIS EDITION

This textbook has been written for students who are beginning their study of algebra at the college level and who are scheduled to complete two semesters of high-school work in one semester.

The general organization of the material is traditional. Algebra is developed as a generalized arithmetic, and the assumptions underlying the operations of both arithmetic and algebra are stressed. The textual material is brief. However, a large number of sample problems is included.

REVIEW MATERIAL

Subject matter is continually reviewed through the use of chapter and cumulative reviews at the end of each chapter. Ten additional cumulative reviews are included at the end of the book.

REFERENCE MATERIAL

Chapter summaries appear at the end of each chapter. A glossary of new terms introduced in the text is placed at the end of the book. In addition, a list of symbols introduced in the text appears on the inside front cover and a summary of the properties and operations appears on the inside of the back cover.

ANSWERS

Answers are provided for the odd-numbered exercises and for all exercises in the chapter reviews and cumulative reviews. A solutions manual containing completely worked-out solutions to all even-numbered exercises is available as a student supplement.

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1 WHOLE NUMBERS AND THEIR REPRESENTATION

In this book, we focus our attention on numbers. We will use the same procedures and symbols we used in arithmetic, together with certain new symbols. The vocabulary used in arithmetic will apply in algebra. In short, we will be studying arithmetic, but from a more general point of view.

1.1 NUMBERS AND THEIR GRAPHS

KINDS OF NUMBERS

The numbers we use to count things are called **natural numbers.** The numbers

$$1, 2, 3, 4, 5, 6, 7, \underset{\text{└── Read "and so on."}}{\ldots}$$

are natural numbers, whereas $\frac{2}{3}$, 3.141, and $\sqrt{2}$ are not.

A **prime number** is a natural number greater than 1 that is exactly divisible only by itself and 1—that is, a multiple of no natural number other than itself and 1. For example,

$$2, 3, 5, 7, 11, \text{ and } 13$$

are prime numbers, whereas 4 and 21 are not since 4 is divisible by 2 and 21 is divisible by 7 and 3. We exclude 1 from the set of prime numbers for reasons we will explain on page 6.

When the number 0 is included with the natural numbers, the numbers in the enlarged collection

$$0, 1, 2, 3, 4, 5, \ldots$$

are called **whole numbers.** Thus, we can refer to numbers such as 2, 3, and 6 as natural numbers or whole numbers. Of course, we can also call 2 and 3 *prime* numbers.

The natural numbers 2, 4, 6, . . . are called **even numbers,** and the natural numbers 1, 3, 5, . . . are called **odd numbers.** The even numbers are always multiples of 2. (We can think of the whole number 0 as being even, since 0 is a multiple of 2.) A number is odd if it leaves a remainder of 1 when it is divided by 2.

Statements about numbers such as

$$4 = 2 \times 2, \quad 7 - 2 + 3 = 8, \quad \text{and} \quad 6 + 5 = 11$$

are called **equality statements.** In an equality statement, the symbols on the left-hand side of the **equals** sign (=) name the same number as the symbols on the right-hand side. Thus, 4 and 2×2 name the same natural number, and $7 - 2 + 3$ and 8 name the same number. We use the symbol ≠ when the left-hand side does *not* equal the right-hand side. For example,

$$5 \neq 2 \times 2 \quad \text{and} \quad 7 - 3 \neq 2.$$

NUMBER LINE

The whole numbers are ordered. That is, we can always say that a particular whole number is *greater than, equal to,* or *less than* another. We can use a **number line** to represent the relative order of a set of whole numbers.

To construct a number line:

1. **Draw a straight line.**
2. **Decide on a convenient unit of scale and mark off units of this length on the line, beginning on the left.**
3. **On the bottom side of the line, label enough of these units to establish the scale, usually two or three points. The point representing 0 is called the *origin.***
4. **Add a small arrow pointing to the right to indicate that numbers are larger to the right.**
5. **On the top side of the line, label the numbers to be graphed. Graph the numbers by placing dots at the appropriate places on the line.**

For example, the graph of the prime numbers less than 8 appears in Figure 1.1.

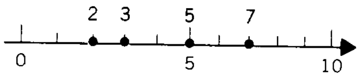


Figure 1.1

ORDER SYMBOLS

Given any two numbers, the number whose graph on a number line is to the left is *less than* the number whose graph is to the right. For example, in Figure 1.2, the graph of 3 is *to the left* of the graph of 7. Therefore, 3 *is less than* 7. We could also state this relationship as "7 *is greater than* 3."

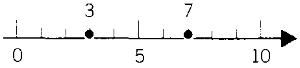


Figure 1.2

We use special symbols to indicate the order relationship between two numbers:

$<$ means "is less than";

$>$ means "is greater than."

For example,

$2 < 5$ is read "2 is less than 5,"

and

$5 > 2$ is read "5 is greater than 2."

Notice that the point of the symbols $<$ and $>$ always points to the smaller number.

Points to smaller number. $2 < 5$

$5 > 2$ Points to smaller number.

EXERCISES 1.1

■ *Which of the following are prime numbers?*

Sample Problems

a. 5 b. 15 c. 29

Ans.

a. 5 is prime (because it is exactly divisible only by 1 and itself).

b. 15 is not prime (because it is divisible by 3 and 5).

c. 29 is prime (because it is exactly divisible only by 1 and itself).

1. a. 4 b. 7 c. 9 d. 11

2. a. 8 b. 13 c. 14 d. 17

3. a. 21 b. 23 c. 25 d. 29

4. a. 31 b. 33 c. 36 d. 37

■ *List all prime numbers between (not including) the given numbers.*

5. 1 and 15 **6.** 16 and 25 **7.** 26 and 35

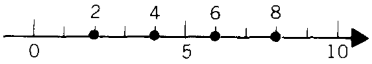
8. 36 and 45 **9.** 46 and 65 **10.** 66 and 100

■ *Graph the following numbers on a number line (use a separate number line for each problem).*

Sample Problem

The first four natural numbers divisible by 2.

Ans.



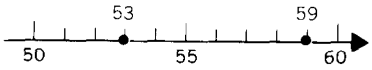
11. The natural numbers greater than 5 and less than 15.
12. The odd natural numbers greater than 7 and less than 12.
13. The natural numbers exactly divisible by 3 and less than 19.
14. The natural numbers exactly divisible by 4 and less than 19.
15. All prime numbers less than 10.
16. All prime numbers between 10 and 20.
17. The first four odd natural numbers.
18. The first four even natural numbers.
19. The first four natural numbers exactly divisible by 3.
20. The first six natural numbers not exactly divisible by 3.

Sample Problems

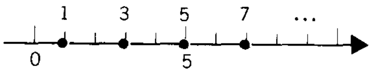
a. The prime numbers between 50 and 60.
b. The odd numbers.

Ans.

a.

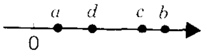


b.



21. The prime numbers between 60 and 80.
22. The prime numbers between 80 and 100.
23. All natural numbers exactly divisible by 3.
24. All natural numbers exactly divisible by 4.
25. All natural numbers.
26. All whole numbers.

■ *Let a, b, c, d represent whole numbers. Their graphs are shown on the number line below.*



Replace the comma in each pair with the proper symbol: $<$, $>$, *or* $=$.

Sample Problems

a. a, c

b. $a, 0$

Ans. a. $a < c$

b. $a > 0$

27. b, c

28. c, a

29. a, d

30. b, d

31. b, b

32. a, a

33. $0, b$

34. $c, 0$

35. a, b

36. c, d

37. d, a

38. d, b

39. $d, 0$

40. $0, c$

1.2 SUMS AND PRODUCTS

Mathematics is a language. As such, it shares a number of characteristics with any other language. For instance, it has verbs, nouns, pronouns, phrases, sentences, and many other concepts that are normally associated with a language. They have different names in mathematics, but the ideas are similar.

In language, we use pronouns such as he, she, or it to stand in the place of nouns. In mathematics, we use symbols such as x, y, z, a, b, c, and the like, to stand in the place of numbers. Letters used in this way are called **variables.** In this chapter, variables will always represent whole numbers.

In a language, the verbs are action words, expressing what happens to nouns. In mathematics, operations such as addition, multiplication, subtraction, or division express an action involving numbers. The symbols we use for these operations, and the properties these operations have, are the same in algebra as in arithmetic.

SUMS

When we add two numbers a and b the result is called the **sum** of a and b. We call the numbers a and b the **terms** of the sum.

$$\underbrace{\overset{\text{terms}}{2 + 3}}_{\text{sum}} \qquad \underbrace{\overset{\text{terms}}{x + 5}}_{\text{sum}}$$

PRODUCTS

When we multiply two numbers a and b, the result is called the **product** of a and b. We call numbers a and b the **factors** of the product. In arithmetic, we used the symbol $\times$ to represent multiplication. But in algebra, the symbol $\times$ may sometimes be confused with the variable x, which we use so frequently. So in algebra we usually indicate multiplication either by a dot between the numbers or by parentheses around one or both of the numbers.

$$\underbrace{\overset{\text{factors}}{2 \cdot 3}}_{\text{product}} \qquad \underbrace{\overset{\text{factors}}{2(3)}}_{\text{product}} \qquad \underbrace{\overset{\text{factors}}{(2)(3)}}_{\text{product}}$$

Multiplication of variables may be written the same way or may be written with the symbols side by side. For example,

$$\underbrace{\overset{\text{factors}\downarrow\downarrow}{ab}}_{\text{product}} \qquad \underbrace{\overset{\text{factors}\downarrow\downarrow}{3x}}_{\text{product}}$$

where

ab means "the number a times the number b," and
$3x$ means "the number 3 times the number x."

PRIME FACTORS

If we multiply 3 by 4, we obtain 12. We might also obtain 12 by multiplying the natural numbers 2 and 6, or 12 and 1, or 2, 2, and 3. In this book we are going to be interested primarily in the *prime factors* of a number. **Prime factors** are factors that are prime numbers. If we now ask for the prime factors of 12, we are restricted to the single set 2, 2, and 3. This is the reason we do not include 1 in the set of prime numbers. If 1 were included, another set of prime factors of 12 would be 1, 2, 2, and 3. For example,

the prime factors of 18 are 2, 3, and 3, since $18 = 2 \cdot 3 \cdot 3$, and 2 and 3 are prime numbers;

the prime factors of 21 are 3 and 7, since $21 = 3 \cdot 7$ and 3 and 7 are prime numbers.

PROPERTIES OF ADDITION AND MULTIPLICATION

A basic property of addition and multiplication, called the **commutative law**, states the following:

The order in which the terms of a sum (or factors of a product) are paired does not change the sum (or product).

Thus, it is always true that

$$\overbrace{a + b} = \overbrace{b + a}$$

The *order* of the terms has been changed.

and

$$\overbrace{a \cdot b} = \overbrace{b \cdot a}.$$

The *order* of the factors has been changed.

For example,

$$5 + 3 = 3 + 5$$

and

$$5 \cdot 3 = 3 \cdot 5.$$

Another useful property of addition and multiplication, called the **associative law,** states the following:

> **The way in which three terms in a sum (or three factors in a product) are grouped for addition (or multiplication) does not change the sum (or product).**

Thus, it is always true that

$$\overbrace{(a + b)} + c = a + \overbrace{(b + c)}$$

The terms are *grouped* differently.

and

$$\overbrace{(a \cdot b)} \cdot c = a \cdot \overbrace{(b \cdot c)}.$$

The factors are *grouped* differently.

For example,

$$(2 + 3) + 4 = 2 + (3 + 4)$$

and

$$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4.$$

In the above example, we used parentheses to indicate grouping. In $(2 + 3) + 4$, the parentheses indicated that the 2 and 3 are added first. In $2 + (3 + 4)$, the parentheses indicated that the 3 and 4 are added first. Brackets [ ] can be used in the same way that we use parentheses.

ALGEBRAIC EXPRESSIONS

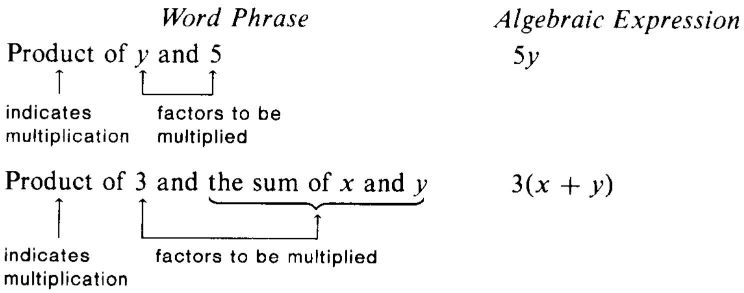
An **algebraic expression,** or simply, an **expression,** is any meaningful collection of numbers, variables, and signs of operation. For example,

$$4x, \quad 3x + y, \quad \text{and} \quad 2(x + 3)$$

are algebraic expressions. An important part of algebra involves translating word phrases into algebraic expressions. Here are some simple examples.

Word Phrase	*Algebraic Expression*
$\underbrace{\text{Sum}}_{\text{indicates addition}}$ of $\underbrace{3 \text{ and } x}_{\text{terms to be added}}$	$3 + x$ or $x + 3$

(continued)



By the commutative law, $5 \cdot y = y \cdot 5$ and $3 \cdot (x + y) = (x + y) \cdot 3$. However, it is customary to write products with the numeral first, as shown in the above examples.

EXERCISES 1.2

■ *Write each number as the product of prime factors.*

Sample Problems

a. 15

b. 16

Ans.

a. $15 = 3 \cdot 5$

b. $16 = 2 \cdot 2 \cdot 2 \cdot 2$

1. 4 **2.** 6 **3.** 8 **4.** 9

5. 14 **6.** 20 **7.** 24 **8.** 25

■ *Fill in the blank according to the indicated law.*

Sample Problems

a. Commutative law
$7 + 10 = 10 + \underline{?}$

b. Associative law
$(6 \cdot 4) \cdot 3 = 6 \cdot (4 \cdot \underline{?})$

Ans.

a. $7 + 10 = 10 + 7$

b. $(6 \cdot 4) \cdot 3 = 6 \cdot (4 \cdot 3)$

9. Associative law
$(3 + 6) + 9 = \underline{?} + (6 + 9)$

10. Associative law
$(x + 3) + y = x + (3 + \underline{?})$

11. Commutative law
$6 \cdot 8 = 8 \cdot \underline{?}$

12. Commutative law
$5 + 7 = 7 + \underline{?}$

13. Associative law
$(3 \cdot x) \cdot y = 3 \cdot (x \cdot \underline{?})$

14. Commutative law
$3 + 2x = 2x + \underline{?}$

15. Commutative law
$(8 \cdot 9) \cdot 3 = (9 \cdot \underline{?}) \cdot 3$

16. Commutative law
$(5 + y) + x = x + (5 + \underline{?})$