Supplementary Appendices with Graphing Calculator Examples for
HOLDER, DEFRANZA, AND PASACHOFF'S

# CALCULUS, MULTIVARIABLE CALCULUS, & SINGLE VARIABLE CALCULUS

SECOND EDITIONS

HOLDER • DEFRANZA • PASACHOFF • ROYSTER

E D I T I O N

# SUPPLEMENTARY APPENDICES WITH GRAPHING CALCULATOR EXAMPLES

**for Holder, DeFranza, and Pasachoff's**

***Calculus, Multivariable Calculus, & Single Variable Calculus***


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# PREFACE

This volume is a supplement to *Calculus, 2nd edition; Single Variable Calculus, 2nd edition;* and *Multivariable Calculus, 2nd edition,* by Leonard Holder, James DeFranza, and Jay Pasachoff, published by Brooks/Cole. It contains appendices for all three texts as well as a chapter on graphing calculators. Answers to the exercises in the appendices are also included.

Appendices 1 and 2 provide a concise review of algebra and trigonometry. Appendix 3 contains a more detailed treatment of the conic sections than is contained in the texts. These appendices can be referred to as needed, or they can be assigned as an integral part of the course. Appendix 4 on mathematical induction and the Binomial Theorem is referred to in the texts in various proofs and exercises. Appendix 5 contains some of the more difficult proofs of theorems in the texts. Appendix 6, on vectors in two and three dimensions, is a chapter reproduced from *Calculus* and *Multivariable Calculus* for the convenience of instructors covering vector operations in the first two semesters of the course.

The chapter on graphing calculators parallels selected parts of the first ten chapters of the texts. It explains how best to use graphing calculators or graphing software to visualize the appropriate calculus topics. Thanks are due to David Royster for his contribution of this important material.

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# APPENDIX

# PRECALCULUS REVIEW: ALGEBRA

## A1.1 THE REAL LINE

Since calculus is based on the real number system, we devote this section to a review of the most important properties of real numbers. We denote the set of all real numbers by $\mathbb{R}$. An important subset of $\mathbb{R}$ is the set of all **rational** numbers. These are numbers that can be expressed as the ratio of two integers—that is, in the form $m/n$, where $m$ and $n$ are integers, with $n \neq 0$. This includes the integers themselves, since an integer $m$ can be written as $m/1$. The real numbers that are not rational are called **irrational**. Some examples are $\sqrt{2}$, $\pi$, and $\sqrt[3]{7}$.

The decimal representations of rational numbers are either *terminating*, such as $5/4 = 1.25$, or *repeating*, such as $5/3 = 1.666\ldots$ or $9/11 = 0.272727\ldots$. All other decimal quantities represent irrational numbers. For example, $1.010010001\ldots$ is neither terminating nor repeating and so is an irrational number. Similarly, the decimal expansion of $\pi$ is $3.14159\ldots$, which neither terminates nor repeats.

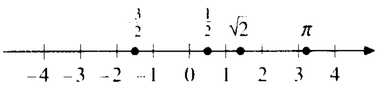


**FIGURE A1.1**
The real number line

One convenient way to visualize real numbers is by associating them with points on a line. By selecting points that correspond to 0 and 1, we establish both a scale and a direction on the line. The point corresponding to 0 is called the **origin**. Points that correspond to other integers can be determined as shown in Figure A1.1. All other real numbers can be made to correspond to points on this line. The remarkable thing about this correspondence is that not only does every real number correspond to a point on the line, but also every point on the line corresponds to one and only one real number. We say there is a *one-to-one correspondence* between the points and the real numbers. Because of this identification, we frequently do not distinguish between a real number and the point that corresponds to it. So we might say, for example, "the point 2" rather than "the point corresponding to the number 2." When a line has been *coordinatized* in the manner indicated, we call it the **real number line**, or simply the **real line**.

We assume that you are familiar with the usual arithmetic properties of real numbers having to do with addition and multiplication. We list these properties here.

**Addition and Multiplication Properties of $\mathbb{R}$**

The following properties hold true for all real numbers $a$, $b$, and $c$.

1. **Commutative properties** $a + b = b + a$; $ab = ba$
2. **Associative properties** $a + (b + c) = (a + b) + c$; $a(bc) = (ab)c$
3. **Distributive property** $a(b + c) = ab + ac$
4. **Identity elements** $a + 0 = a$; $a \cdot 1 = a$

   The number 0 is the **additive identity** for $\mathbb{R}$ and the number 1 is the **multiplicative identity** for $\mathbb{R}$.
5. **Inverses** For every $a$ in $\mathbb{R}$, there is a number $-a$ in $\mathbb{R}$, called the **additive inverse of *a***, such that $a + (-a) = 0$. For every $a \neq 0$ in $\mathbb{R}$, there is a number $a^{-1}$ in $\mathbb{R}$, called the **multiplicative inverse of *a***, such that $a \cdot a^{-1} = 1$.

Recall that subtraction and division are defined in terms of addition and multiplication by

$$a - b = a + (-b) \qquad \text{and} \qquad \frac{a}{b} = a \cdot b^{-1} \qquad \text{if } b \neq 0$$

Note carefully that the quotient $a/b$ is not defined if $b = 0$ since $b^{-1}$ is not defined. Thus, **division by 0 is excluded**.

## Inequalities

We concentrate now on the *order* properties of $\mathbb{R}$. If $a$ and $b$ are real numbers such that $b - a$ is positive, then we say $a$ **is less than** $b$ and write $a < b$. Thus,

$a < b$ means $b - a$ is positive.

When $a < b$, we may also say $b$ **is greater than** $a$ and write $b > a$. The symbol $a \leq b$ is shorthand for $a < b$ *or* $a = b$, and similarly for $b \geq a$. On the real line with positive direction to the right, if $a < b$, then $a$ is to the *left* of $b$ on the line. The following properties of inequality can be proved by using the definition of inequality and the addition and multiplication properties of $\mathbb{R}$.

**Properties of Inequality**

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$, then $\begin{cases} ac < bc & \text{if } c > 0 \\ ac > bc & \text{if } c < 0 \end{cases}$.
3. If $a < b$ and $b < c$, then $a < c$.
4. If $a < b$ and $ab > 0$, then $\frac{1}{a} > \frac{1}{b}$.

Pay particular attention to Property 2 when $c < 0$. In effect, this property says that multiplying both sides of an inequality by a negative number reverses the sense of the inequality.

In the two examples that follow, we illustrate ways to "solve" linear and quadratic inequalities. To *solve* an inequality means to find the set of all real numbers $x$ for which the inequality is satisfied.

**EXAMPLE A1.1** Solve the inequality

$$\frac{3 - x}{4} < \frac{x}{3} - 1$$

***Solution*** To eliminate fractions, we multiply both sides by the lowest common denominator, 12. Then we proceed by using the inequality properties shown:

$$9 - 3x < 4x - 12 \quad \text{Property 2}$$

$$-7x < -21 \quad \text{Property 1}$$

$$x > 3 \quad \text{Property 2}$$

Note that in the last step we multiplied both sides by the negative number $-\frac{1}{7}$ and therefore had to reverse the sense of the inequality. ■

A *set* is frequently designated by a symbol of the form

$$\{x : \underline{\qquad\qquad}\}$$

where a description of properties possessed by $x$ follows the colon. As an illustration, in Example A1.1 we could write the solution set as $\{x : x > 3\}$. This symbol is read as "the set of all $x$ such that $x$ is greater than 3." In such designations we will understand $x$ to be a real number unless otherwise specified.

**EXAMPLE A1.2** Solve the inequality $x(x - 1) \leq 2$.

***Solution*** We begin just as if this inequality were a quadratic equation, using Inequality Property 1 to bring all terms to one side, and then factoring:

$$x^2 - x - 2 \leq 0$$

$$(x - 2)(x + 1) \leq 0$$

**FIGURE A1.2**

The product on the left *equals* 0 when $x = 2$ or when $x = -1$. It is *less than* 0 when the factors $(x - 2)$ and $(x + 1)$ are of opposite signs. A convenient way to see where the product changes sign is to mark the points $x = 2$ and $x = -1$ on a real number line as in Figure A1.2. Then we test the product $(x - 2)(x + 1)$

for its sign in each of the three regions determined by these points. Because the product is of constant sign in each region, it is sufficient to test the sign at just one point in each region. For example, we could use $x = -2$, $x = 0$, and $x = 3$ as test values. The signs of the product are readily seen to be those shown above the regions in the figure. We can now write the answer as

$$\{x : -1 \leq x \leq 2\} \qquad \blacksquare$$

Figure A1.2 is an example of what we refer to as a **sign graph**. Here is another example illustrating the use of such a sign graph to solve a nonlinear inequality.

**EXAMPLE A1.3** Solve the inequality

$$\frac{x(x+5)}{x+1} > 3$$

***Solution*** First we add $-3$ to both sides and then combine in a single fraction, getting

$$\frac{x^2 + 2x - 3}{x+1} > 0 \quad \text{Verify.}$$

Now we factor the numerator and make a sign graph:

$$\frac{(x+3)(x-1)}{x+1} > 0$$

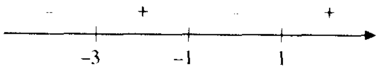


**FIGURE A1.3**

The points of division on the sign graph are points where either the numerator *or* the *denominator* is 0. The sign in each region is determined using a test value, with the results as shown in Figure A1.3. We can see that the solution consists of the points in either of the sets

$$\{x : -3 < x < -1\} \quad \text{or} \quad \{x : x > 1\} \qquad \blacksquare$$

Note carefully that in this example we did *not* clear the inequality of the fractions, since doing so would have involved multiplying by the factor $x + 1$, whose sign is sometimes positive and sometimes negative, so it would not be clear which part of Inequality Property 2 to use. Instead, we brought all the terms together as a single fraction and then tested the signs of the factors on numerator and denominator, just as we did for a product in Example A1.2.

The solution given in Example A1.3 can be indicated more briefly by using the following standard set notation. Let $A$ and $B$ denote two sets. Then the **union of $A$ and $B$**, written $A \cup B$, is defined as

$$A \cup B = \{\text{all elements in } \textit{either } A \textit{ or } B\}$$

So the solution in Example A1.3 can be written

$$\{x : -3 < x < -1\} \cup \{x : x > 1\}$$

It is also sometimes convenient to use the notation for the **intersection** of two sets $A$ and $B$, written $A \cap B$ and defined as

$$A \cap B = \{\text{all elements in } \textit{both } A \textit{ and } B\}$$

The symbol $\emptyset$ is used to denote the **empty set**. If $A \cap B = \emptyset$, we say $A$ and $B$ are **disjoint**.

## Interval Notation

Sets like those in the solutions to the preceding inequalities occur in many other contexts as well; we give them the following special names and symbols:

| Set | Name | Symbol |
|---|---|---|
| $\{x : a \le x \le b\}$ | Closed interval | $[a, b]$ |
| $\{x : a < x < b\}$ | Open interval | $(a, b)$ |

A set of the form $\{x : a < x \le b\}$ is designated $(a, b]$ and is said to be a **half-open** (or **half-closed**) interval. The same is true for $\{x : a \le x < b\} = [a, b)$. It is also convenient to introduce the symbol $\infty$, read *infinity* and interpreted roughly as meaning "beyond all bound." Although $\infty$ is *not* a number, we use it in interval notation as follows:

$$\{x : x \ge a\} = [a, \infty)$$

$$\{x : x > a\} = (a, \infty)$$

$$\{x : x \le a\} = (-\infty, a]$$

$$\{x : x < a\} = (-\infty, a)$$

$$\mathbb{R} = (-\infty, \infty)$$

## Absolute Value

The **absolute value** of a number means the "magnitude" of the number, without regard to its sign. More precisely, the absolute value of a real number $a$, denoted by $|a|$, is defined as

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

So $|a|$ is always nonnegative (that is, either positive or zero). For example, $|2| = 2$ and $|-2| = -(-2) = 2$. Geometrically, $|a|$ is the distance between $a$ and 0 on the real line. Similarly, for any two real numbers $a$ and $b$, $|a - b|$ is the distance between them on the real line.

$$\text{Distance between } a \text{ and } b = |a - b|$$

The basic properties of absolute value are given here.

### Properties of Absolute Value

1. $|a| \ge 0$ for all real numbers $a$. Furthermore, $|a| = 0$ if and only if $a = 0$.
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $|a + b| \le |a| + |b|$

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The first three properties can be seen to be true directly from the definition. Property 4 is called the **triangle inequality** and can be proved as follows. Since the absolute value of a number equals either the number or its negative, we have $-|a| \le a \le |a|$, and similarly, $-|b| \le b \le |b|$. Adding corresponding members gives

$$-(|a| + |b|) \le a + b \le |a| + |b|$$

Thus, $(a + b) \le |a| + |b|$ and $-(a + b) \le |a| + |b|$. It now follows that $|a + b| \le |a| + |b|$, since $|a + b|$ is either $(a + b)$ or its negative.

From the geometric interpretation of absolute value as the distance from $a$ to 0, we can readily see that for $a > 0$,

| | | | |
|---|---|---|---|
| | $|x| < a$ | if and only if | $-a < x < a$ |
| and | | | |
| | $|x| > a$ | if and only if | $x > a$ or $x < -a$ |

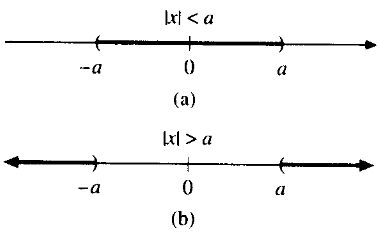


FIGURE A1.4

Figure A1.4 illustrates these relationships. They can also be verified by using the definition of absolute value. In interval notation, we see that the set of real numbers $x$ that satisfy $|x| < a$ is the open interval $(-a, a)$, and the $x$ values that satisfy $|x| > a$ are those in one or the other of the intervals $(-\infty, -a)$ or $(a, \infty)$—that is, in $(-\infty, -a) \cup (a, \infty)$.

**EXAMPLE A1.4** Solve the inequality $2|3 - 4x| - 1 > 7$.

***Solution*** Simplifying by using Inequality Properties 1 and 2, we get $|3 - 4x| > 4$, which is satisfied if and only if

$$3 - 4x > 4 \quad \text{or} \quad 3 - 4x < -4$$

Solving each of these linear inequalities, we get

$$-4x > 1 \qquad\qquad -4x < -7$$
$$x < -\frac{1}{4} \qquad\qquad x > \frac{7}{4}$$

The solution set can thus be written as

$$\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{7}{4}, \infty\right)$$

■

## The Completeness Axiom

We conclude this section with a brief discussion of one of the fundamental properties of the real number system that has to do with the order relation. A subset $S$ of $\mathbb{R}$ is said to be **bounded above** if there exists a real number $M$ such that $x \le M$ for all $x$ in $S$. When this inequality is true for all $x$ in $S$, $M$ is said to be an **upper bound** of $S$. Thus $M$ is an upper bound of $S$ if, on the real line, no element of $S$ is to the right of $M$. Clearly, if $M$ is an upper bound of $S$, any number to the right of $M$ is also an upper bound of $S$. The more critical question is whether there is an upper bound of $S$ smaller than all other upper bounds. If so, this upper bound is called the **least upper bound** of $S$. To illustrate, the set

$$\{1, 1\tfrac{1}{2}, 1\tfrac{3}{4}, 1\tfrac{7}{8}, 1\tfrac{15}{16}, \ldots\}$$

is bounded above by 3, or 4, or 10, for example, but the *least* upper bound is 2, since 2 is an upper bound and nothing less than 2 will do.

A fundamental property of $\mathbb{R}$ is that for every nonempty subset of $\mathbb{R}$ that is bounded above, such a least upper bound in $\mathbb{R}$ does always exist. This property of $\mathbb{R}$ is known as the *completeness axiom*.

**The Completeness Axiom**

Every nonempty set of real numbers that is bounded above has a least upper bound in $\mathbb{R}$.

**EXAMPLE A1.5** Determine the least upper bound of each of the following sets.

(a) $\{x : -1 \le x \le 3\}$
(b) $\{x : -1 < x < 3\}$
(c) $\{1 - 1/n : n \text{ a positive integer}\}$
(d) $\{x : x^2 < 2\}$

***Solution***

(a) An upper bound is 3, and since no number less than 3 is an upper bound, we conclude that 3 is the least upper bound.

(b) Again, 3 is an upper bound, and although 3 is not in the set, elements of the set come arbitrarily close to 3. So nothing less than 3 is an upper bound. For example, 2.99 is not an upper bound, since 2.999 is in the set and exceeds 2.99. Thus, 3 is the least upper bound.

(c) Some elements of the set are

$$1 - \tfrac{1}{1} = 0 \qquad 1 - \tfrac{1}{2} = \tfrac{1}{2} \qquad 1 - \tfrac{1}{3} = \tfrac{2}{3} \qquad 1 - \tfrac{1}{4} = \tfrac{3}{4}$$

found by setting $n = 1, 2, 3, 4$, respectively. Since every member of the set is less than 1, it follows that 1 is an upper bound. In fact, it is the least upper bound, since by taking $n$ large enough, we can make $1 - \frac{1}{n}$ as close to 1 as we wish. Nothing less than 1 is an upper bound to this set.

(d) Some upper bounds to the set are 3, 2, 1.5, 1.42, and 1.415, since the square of each of these numbers exceeds 2. (Check this.) In fact, the least upper bound is $\sqrt{2}$, whose decimal expansion is neither repeating nor terminating. This example shows, incidentally, that the rational numbers are not complete because the set of all rational numbers whose squares are less than 2, although bounded above, does not have a rational least upper bound. ■

## Exercise Set A1.1

**1.** Express each of the following rational numbers as either a terminating or a repeating decimal.

(a) $\frac{5}{8}$ (b) $\frac{8}{5}$

(c) $\frac{10}{27}$ (d) $-\frac{9}{37}$

(e) $\frac{10}{7}$

**2.** State which of the following are rational and which are not.

(a) $\frac{\sqrt{81}}{4}$ (b) $-2$

(c) $\sqrt{5}$ (d) $\sqrt[3]{-8}$

(e) $\frac{1}{2}+\frac{2}{3}$

**3.** Let $x = 1.242424\ldots$. By considering $100x - x$, express $x$ in the form $m/n$ in lowest terms.

**4.** Use the idea of Exercise 3 to write $x = 0.243243243\ldots$ in the form $m/n$ in lowest terms.

**5.** Replace the question mark with the correct inequality symbol.

(a) $-10 \mathbin{?} 2$ (b) $-7 \mathbin{?} 9$

(c) $\frac{2}{3} \mathbin{?} \frac{5}{9}$ (d) $3.6 \mathbin{?} \frac{15}{4}$

(e) $0 \mathbin{?} -100$

**6.** Write each of the following sets in interval notation and show it on the real line.

(a) $\{x : -1 < x < 2\}$ (b) $\{x : x \leq 2\}$

(c) $\{x : 2 < x \leq 5\}$ (d) $\{x : 0 \leq x \leq 1\}$

**7.** Write the meaning of each of the following intervals using set notation, and show it on the real line.

(a) $[2, 5]$ (b) $(-2, 3]$

(c) $(3, 5)$ (d) $(-\infty, 2)$

(e) $[0, \infty)$

**8.** Give the value of each of the following.

(a) $|-6|$ (b) $|0|$

(c) $|3 - \pi|$ (d) $|a - b|$ if $b > a$

(e) $|-x|$ if $x < 0$

*In Exercises 9–32, solve the inequality.*

**9.** $4 - 2x < 6 - 3x$

**10.** $\frac{x}{2} - \frac{3}{4} > \frac{5x}{6} - \frac{1}{3}$

**11.** $0 < \frac{3x-2}{4} \leq 2$

**12.** $-3 \leq \frac{5-3x}{2} < 6$

**13.** $|3 - 2x| \geq 2$

**14.** $|3x - 5| \leq 2$

**15.** $2|2x - 1| - 3 < 5$

**16.** $\frac{|1-x|}{3} < 2$

**17.** $x^2 - 3x - 4 \leq 0$

**18.** $2x^2 - 9x + 4 \geq 0$

**19.** $x(2x - 3) < 5$

**20.** $3x^2 \geq 4(1 - x)$

**21.** $\frac{3-x}{x^2+2x^2} < 0$

**22.** $\frac{x}{x+4} > 0$

**23.** $(2x - 1)(x + 4)(x - 3) \geq 0$

**24.** $(x - 1)(x + 2)(x - 4) < 0$

**25.** $\frac{3x+1}{x-2} > 2$

**26.** $\frac{2x-3}{x-1} \leq 4$

**27.** $\frac{x^2+4x}{x-3} > 0$

**28.** $\frac{x^2-3x-4}{x^2+x-6} \leq 0$

**29.** $|x - 3| < 0.01$

**30.** $|2x + 4| < 0.001$

**31.** $0 < |x - a| < \delta$

**32.** $|x^2 - L| < \varepsilon$

*In Exercises 33–35, show the set on the real line and give its least upper bound.*

**33.** (a) $\{x : x < 3\}$
(b) $\{x : |x - 2| < 1\}$

**34.** (a) $[-3, 7]$
(b) $(-5, 0)$

**35.** (a) $\left\{\frac{2n-1}{n} : n \text{ a positive integer}\right\}$
(b) $\left\{-1, -\frac{1}{2}, -\frac{1}{3}, \ldots\right\}$

**36.** Prove Inequality Property 1.

**37.** Prove Inequality Property 2. (*Hint:* Use the fact that the product of two positive numbers is positive. Also, if $c < 0$, then $-c > 0$.)

**38.** Prove Inequality Property 3. (*Hint:* Use the fact that the sum of two positive numbers is positive.)

**39.** Show that if $a > 0$, then $(1/a) > 0$. (*Hint:* Show $(1/a)$ cannot be negative or 0.)

**40.** Prove Inequality Property 4. (*Hint:* Use Exercise 39.)

**41.** Prove Absolute Value Property 2.

**42.** Prove Absolute Value Property 3.

**43.** Prove that $|a - b| \leq |a| + |b|$.

**44.** Prove that $||a| - |b|| \le |a - b|$. (*Hint:* Write the triangle inequality as $|x + y| \le |x| + |y|$. First substitute $x = a$ and $y = b - a$. Starting again with the triangle inequality, substitute $x = b$ and $y = a - b$.)

**45.** Prove that if $S$ is a set of real numbers that is bounded above, then $M$ is the least upper bound of $S$ provided the following two conditions are satisfied: (i) $x \le M$ for all $x \in S$ and (ii) if $L < M$, there is an $x$ in $S$ such that $x > L$.

**46.** Formulate a definition of a **lower bound** and the **greatest lower bound** of a set $S$ that is bounded below. Give conditions analogous to those in Exercise 45 that ensure that $m$ is the greatest lower bound of $S$.

# A1.2 THE CARTESIAN PLANE

Consider two real number lines perpendicular to each other so that their origins coincide, as in Figure A1.5. We name the horizontal line the **$x$-axis** and the vertical line the **$y$-axis**, and we call their point of intersection the **origin**. These axes divide the plane into four **quadrants**, which we number as in the figure. Through any point in the plane we pass vertical and horizontal lines. Their intersections with the axes determine the **$x$-coordinate**, or **abscissa**, and the **$y$-coordinate**, or **ordinate**, of the point. If the abscissa is $a$ and the ordinate is $b$, we represent the point by the ordered pair $(a, b)$. The context will make clear whether $(a, b)$ refers to a point in the plane or to an open interval on the line. The numbers $a$ and $b$ are called the **coordinates** of the point. For brevity, we often say the point has coordinates $(a, b)$. For example, in Figure A1.5 the point $P$ has coordinates $(3, 2)$, and the point $Q$ has coordinates $(-4, -3)$.

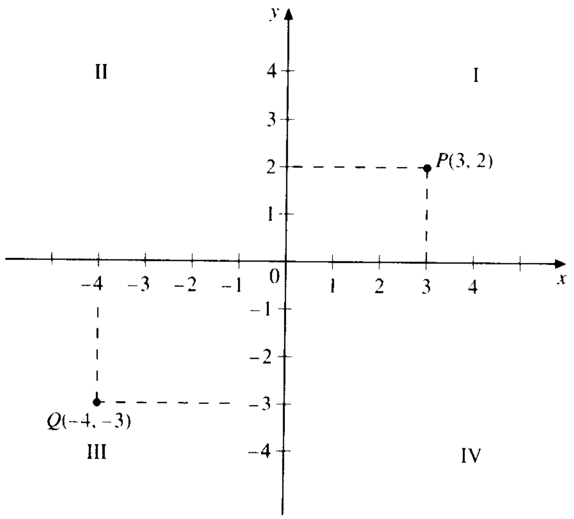


**FIGURE A1.5**

Conversely, if we are given an ordered pair of real numbers $(a, b)$, we can locate the unique point that has these coordinates from the intersection of a vertical line through $a$ on the $x$-axis and a horizontal line through $b$ on the $y$-axis. In this way we obtain a one-to-one correspondence between points in the plane and ordered pairs of numbers. Frequently we do not distinguish between a point and its coordinates, saying, for example, "the point (3, 2)" rather than "the point with coordinates (3, 2)."

What we have described is referred to as a **rectangular**, or **Cartesian**, coordinate system. The latter name is for the French mathematician and philosopher René Descartes (1596–1650), who is usually given credit for originating analytic geometry, which is based on representing ordered pairs as points in the plane. Actually, many of the ideas were first introduced by another French mathematician, Pierre de Fermat (1601–1665). The plane, when coordinatized as we have shown, is often referred to as the **Cartesian plane**.

## The Distance Formula

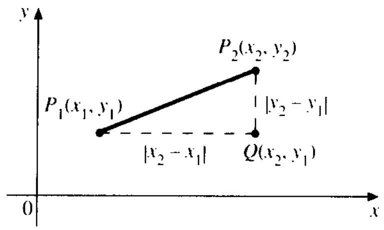


**FIGURE A1.6**

If we are given two points $P_1$ and $P_2$ in the plane with the coordinates $(x_1, y_1)$ and $(x_2, y_2)$, as shown in Figure A1.6, we can find the distance $d$ between them by using the **Pythagorean Theorem**, which says that in a right triangle the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs. We introduce the point $Q$ as shown, with coordinates $(x_2, y_1)$. Then $P_1QP_2$ is a right triangle, and since $P_1Q$ is horizontal, its length is $|x_2 - x_1|$. Similarly, $QP_2$ is vertical, and its length is $|y_2 - y_1|$. Thus, by the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Equivalently, since $|a|^2 = a^2$, we have the following formula.

**The Distance Formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For example, the distance between $(1, -2)$ and $(-4, 7)$ is

$$d = \sqrt{(1 + 4)^2 + (-2 - 7)^2} = \sqrt{25 + 81} = \sqrt{106}$$

Notice that it does not matter which point is labeled $(x_1, y_1)$ and which is labeled $(x_2, y_2)$.

## Graphs of Equations

The real power of the coordinate system devised by Descartes and Fermat is in representing equations, which are *algebraic* objects, by means of collections of points in the plane, which are *geometric* objects. If an equation involves only two variables $x$ and $y$, then the set of points in the Cartesian plane corresponding to all ordered pairs $(x, y)$ for which $x$ and $y$ satisfy the equation is called the **graph of the equation**. For example, if we wish to graph the equation $y = 2x - 1$, we can find several ordered pairs $(x, y)$ satisfying the equation, such as $(0, -1)$, $(1, 1)$, $(2, 3)$, and so on. Then we can locate these as points in the Cartesian plane and (going on faith) connect them with a smooth curve (which in this case is a straight line). This method of plotting points clearly has its drawbacks, even if we plot a very large number of points. How can we be sure, for example, that they should be connected with a single, unbroken curve? Calculus provides the necessary tools for analyzing curves so that questions such as this one can be answered. Even without calculus, analyzing certain curves helps us determine their equations; conversely, we can sometimes recognize the graph from the equation. For now, we do this analysis for only three particularly simple, but very useful curves: the straight line, the circle, and the parabola.

## The Straight Line

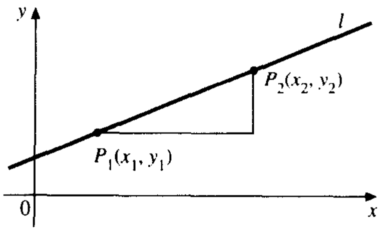

**FIGURE A1.7**

Consider a nonvertical line $l$, as shown in Figure A1.7, and let $P_1$ and $P_2$ be any two distinct points on $l$ with coordinates $(x_1, y_1)$ and $(x_2, y_2)$, respectively. Then we define the **slope** $m$ of $l$ by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

You should convince yourself, using similar triangles, that the value of the slope is independent of which two points on $l$ we use.

Lines that go upward to the right have positive slopes, and those that go downward to the right have negative slopes, as we can see from the definition. Also, the slope of a horizontal line is 0, and the slope of a vertical line is not defined. Figure A1.8 illustrates these facts.

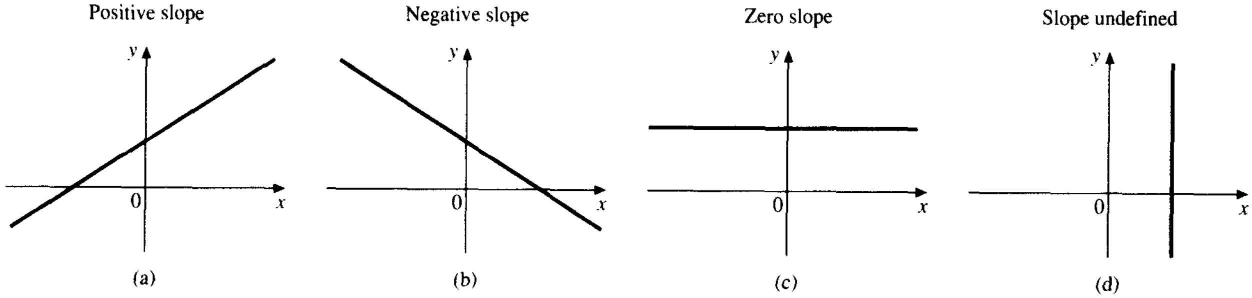

**FIGURE A1.8**

Suppose we know one point, say $P_1(x_1, y_1)$, on a nonvertical line $l$ and that its slope $m$ is given (or can be found). A point $P(x, y)$ distinct from $P_1$ will lie on $l$ if and only if the slope as calculated using $P_1$ and $P$ equals $m$:

$$\frac{y - y_1}{x - x_1} = m$$

If we clear this equation of fractions, we can write the equation of the line as

> **The Point-Slope Form**
>
> $$y - y_1 = m(x - x_1) \qquad \text{(A1.1)}$$

Note that in this form, the equation is satisfied even if $P = P_1$. Thus, the point $P(x, y)$ lies on $l$ if and only if its coordinates satisfy Equation A1.1. The graph

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