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Electromagnetic Fields

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Preface

This is a textbook for use in the usual year-long course in electromagnetism at an intermediate level given for advanced undergraduates. I have done my best to make it as student oriented as possible by writing it in a systematic and straightforward way without any sleight of hand, and minimum use of "It can be shown that...." I have also tried to make clear the motivations for each step in a derivation or each new concept as it is introduced. At appropriate points I have pointed out the source of many of the simple mistakes commonly made by students and have suggested how they may be prevented. There is extensive cross referencing in the text so that there should be no doubt as to the detailed source of any specific result or its relation to the rest of the subject; this will also make the book much more useful as a reference source well after the course has been completed.

The emphasis is on the properties and sources of the field vectors, and I hope that I have succeeded in making clear the shifts in concepts and points of view that are involved in the change from action at a distance to fields. The overall treatment is generally that of a macroscopic and empirical description of phenomena, although the microscopic point of view is presented in the discussion of conductivity in Sections 12-5 and 24-8. However, Appendix B briefly surveys the microscopic origins of electromagnetic properties, and it is written and organized so that, if desired, it can be taken up section by section at an appropriate intermediate point. Thus Section B-1 could be discussed anytime after Section 10-7, and most of Section B-2 can follow after Section 20-5 while the last part on ferromagnetism can follow Section 20-7; finally, Section B-3 could be covered after Section 24-8 has been mastered and the student has worked Exercise 24-28. Similarly, even more flexibility is possible since separate sections of Appendix A that deals with the motion of charged particles can be studied anytime after the corresponding force term involving **E** and/or **B** has been obtained.

SI units are used throughout; in practice, this means we use MKSA units. It is virtually certain, however, that at some time a student will encounter material in Gaussian units and will need some guidance on what to do about it. This is the purpose of Chapter 23 in which other unit systems are discussed, but only after the general theory as given by Maxwell's equations has been systematically described. I have written this chapter primarily in terms of the purely practical aspects of how to recognize an equation written in other unit systems, how to put it in more familiar form if desired, and exactly what numbers should one put into an equation in Gaussian units in order to get a correct answer.

In Chapter 9, the boundary conditions satisfied by an *arbitrary* vector at a surface of discontinuity in properties are obtained in the general form involving its divergence and curl. Not only does this help the student by showing the importance of knowing these particular source equations but it simplifies later discussion since, as each new vector is defined, its boundary conditions can be found at once without having, in effect, to rederive them each time.

There are over 130 worked-out examples in the text. Virtually all of the standard ones are included, many of them done in more detail than is usually found, with particular emphasis on the crucial stage of setting up the problem in the first place, since this is so often what causes students so much difficulty. I have also included 555 exercises. Some are numerical to give an idea of typical orders of magnitude, some are similar to examples of the text, many refer to completely different situations, and some involve extensions of the theory. Many of these exercises will be found to be suitable for use as additional examples for classroom analysis. Answers to odd-numbered exercises are provided except, of course, for those in which the answer is included in the statement of the problem.

I have benefited over the years from discussions with and the questions of many students and my colleagues; I am grateful for their contributions to the final character of this book. Parts of Chapter 28 on Special Relativity have been taken verbatim or nearly so from my book *Introductory Topics in Theoretical Physics*, also published by John Wiley & Sons, and I acknowledge their permission to do so.

ROALD K. WANGSNESS

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Introduction

... Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

—*J. C. Maxwell,*
A Treatise on Electricity and Magnetism

It has been more than 100 years since Maxwell wrote the above in the preface to his now-famous book. His aim was to put the field concepts, which Faraday had been so instrumental in developing, into mathematical forms that would be convenient to use and would emphasize the fields as basic to a coherent description of electromagnetic effects. At that time, it had been only slightly more than 50 years since Oersted and Ampère had shown the relation between electricity and magnetism—subjects that had been studied and developed completely separately over a long period. The emphasis had been primarily on the forces exerted between electric charges and between electric currents and the idea of shifting to electric and magnetic fields as the primary features had little acceptance and was often, in fact, viewed with outright hostility.

As the title of this book indicates, times have changed, and our main interest here is the study of the nature, properties, and origins of electromagnetic fields, that is, of electric and magnetic vector quantities that are defined as functions of time and of position in space. Forces, and associated concepts such as energy, have not disappeared from the subject, of course, and it is desirable to begin with forces and to define the field vectors in terms of them. Nevertheless, our principal aim is to express our descriptions of phenomena in terms of fields in as complete a manner as we possibly can. This emphasis on fields has proved to be extremely rewarding and it is difficult to imagine how electromagnetic theory could have been developed to its present state without it.

This book contains more material than is normally covered in the usual one-year course; all of it, however, is of interest and value to a serious student of physics.

The points of view of all authors are generally not the same, and no book discusses every detail of a given subject. Here is a short list of relatively recent books on electromagnetism that are written at roughly the same level as this one.

W. B. Cheston, *Elementary Theory of Electric and Magnetic Fields*, Wiley, New York, 1964.

D. M. Cook, *The Theory of the Electromagnetic Field*, Prentice-Hall, Englewood Cliffs, N.J., 1975.

P. Lorrain and D. R. Corson, *Electromagnetic Fields and Waves*, Second Edition, Freeman, San Francisco, 1970.

J. R. Reitz and F. J. Milford, *Foundations of Electromagnetic Theory*, Second Edition, Addison-Wesley, Reading, Mass., 1967.

A. Shadowitz, *The Electromagnetic Field*, McGraw-Hill, New York, 1975.

The following books discuss electromagnetism at a more advanced level:

J. D. Jackson, *Classical Electrodynamics*, Second Edition, Wiley, New York, 1975.

W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, Second Edition, Addison-Wesley, Reading, Mass., 1962.

J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.

(Finally, a note on notation: in this book, the symbols $=, \simeq, \approx, \sim, \neq$ always mean, respectively, equal to, approximately equal to, of the order of magnitude of, proportional to, and different from.)

Chapter 1 Vectors

In the study of electricity and magnetism, we are constantly dealing with quantities that need to be described in terms of their directions as well as their magnitudes. Such quantities are called vectors and it is well to consider their properties in general before we meet specific examples. Using the notation and terminology that has been developed for this purpose enables us to state our results more compactly and to understand their basic physical significance more easily.

1-1 Definition of a Vector

The properties of the *displacement of a point* provide us the essentials required for our definition. If we start at some point P_1 and move in some arbitrary way to another point P_2, we see from Figure 1-1 that the *net* effect of the motion is the same as if the point were moved directly along the straight line D from P_1 to P_2 as indicated by the direction of the arrow. This line D is called the displacement and is characterized by both a magnitude (its length) and a direction (from P_1 to P_2). If we now further displace our point along E from P_2 to still another point P_3, we see from Figure 1-2 that the new net effect is the same as if the point had been given the single displacement F from P_1 to P_3. Accordingly, we can speak of F as the resultant, or sum, of the successive displacements D and E, so that Figure 1-2 shows the fundamental way in which displacements are combined or added to obtain their resultant.

A *vector* is a generalization of these considerations in that it is defined as any quantity which has the same mathematical properties as the displacement of a point. Thus we see that a vector has a magnitude; it has a direction; and the addition of two vectors of the same intrinsic nature follows the basic rule illustrated in Figure 1-2. Because of the first two properties, we can represent a vector by a directed line such as those already used for displacements. A vector is generally printed in boldface type, thus, **A**; its magnitude will be represented by $|\mathbf{A}|$ or by A.

A *scalar* is a quantity that has magnitude only. For example, the mass of a body is a scalar, whereas its weight, which is the gravitational force acting on the body, is a vector.

Because of the nature of a vector as a directed quantity, it follows that a parallel displacement of a vector does not alter it, or, in other words, two vectors are equal if they have the same magnitude and direction. This is illustrated in Figure 1-3 where we see that $\mathbf{A}=\mathbf{A}'$. Now we can investigate what mathematical operations we can perform with and on vectors.

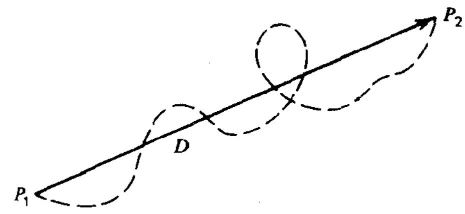

Figure 1-1. D is the displacement of the point from P_1 to P_2.

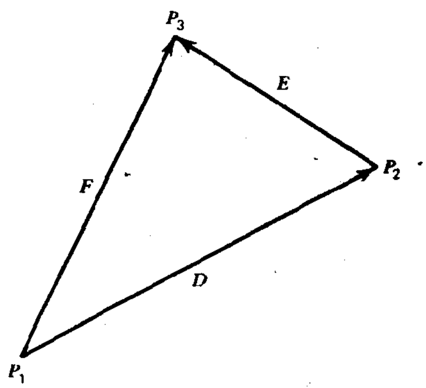

Figure 1-2. F is the resultant of the displacements D and E.

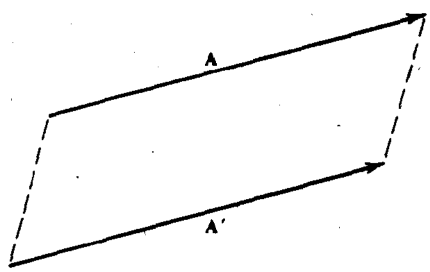

Figure 1-3. These two vectors are equal.

1-2 Addition

According to our basic rule we find that, if we take **A** and add **B**, we obtain the sum **C** shown as a solid line in Figure 1-4. We also see that, if we take **B** and then add **A**, we get the same vector **C**. Therefore addition of vectors has the property that

$$\mathbf{C}=\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A} \tag{1-1}$$

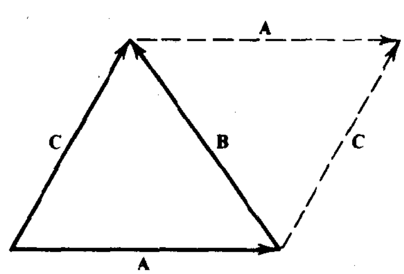

Figure 1-4. The sum of two vectors does not depend on the order in which they are added.

By proceeding in the same manner, one can establish the associative property of vector addition:

$$\mathbf{D}=(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{C})+\mathbf{B} \qquad (1\text{-}2)$$

and so on.

If we reverse a displacement such as D in Figure 1-1 by retracing it in the opposite direction, the net effect is then no displacement; hence it is appropriate to define the negative of a vector as a vector of the same magnitude but reversed in direction, for then we should obtain $\mathbf{A}+(-\mathbf{A})=0$, as we would want. Then we can easily subtract a vector by adding its negative:

$$\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B}) \qquad (1\text{-}3)$$

The product of a scalar s and a vector, which we write as either $s\mathbf{A}$ or $\mathbf{A}s$, is then merely the sum of s vectors $\mathbf{A}$, or is a vector with a magnitude equal to $|s|$ times the magnitude of $\mathbf{A}$, and is in the same direction as $\mathbf{A}$ if s is positive, and in the opposite direction to $\mathbf{A}$ if s is negative.

1-3 Unit Vectors

A *unit vector* is defined as a vector of unit magnitude and will be written with a circumflex above it, thus, $\hat{\mathbf{e}}$; since unit vectors are always taken to be dimensionless we will have $|\hat{\mathbf{e}}|=1$. If, for example, a unit vector $\hat{\mathbf{a}}$ is chosen to have the direction of $\mathbf{A}$, then we can write

$$\mathbf{A}=A\hat{\mathbf{a}} \quad \text{and} \quad \hat{\mathbf{a}}=\frac{\mathbf{A}}{A} \qquad (1\text{-}4)$$

This point is illustrated in Figure 1-5.

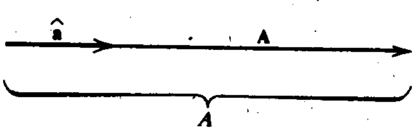

Figure 1-5. $\hat{\mathbf{a}}$ is a unit vector in the direction of $\mathbf{A}$.

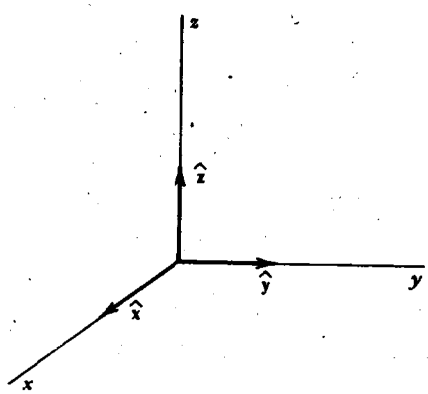

Figure 1-6. Unit vectors for rectangular coordinates.

A particularly convenient set of unit vectors can be associated with a rectangular coordinate system. They are written $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ and are defined to be in the directions of the x, y, and z axes respectively, as shown in Figure 1-6. In other words, each is in the direction of increasing value of the corresponding rectangular coordinate. We also see that any one of this set is perpendicular to each of the other two.

As we shall see, it is often convenient and advantageous to define other unit vectors.

1-4 Components

In order to proceed further, it is convenient to refer our vectors to particular coordinate systems. From Figure 1-7, we see that we can write a vector $\mathbf{A}$ as the sum of three properly chosen vectors, each of which is parallel to one of the axes of a rectangular coordinate system; that is, $\mathbf{A}=\mathbf{A}_x+\mathbf{A}_y+\mathbf{A}_z$. It is more useful, however, to write each of these terms as the product of a scalar and the unit vectors of Figure 1-6. Thus we write $\mathbf{A}_x=A_x\hat{\mathbf{x}}$, and so on, and the above expression becomes

$$\mathbf{A}=A_x\hat{\mathbf{x}}+A_y\hat{\mathbf{y}}+A_z\hat{\mathbf{z}} \tag{1-5}$$

The three scalars A_x, A_y, A_z are called the *components* of $\mathbf{A}$; hence we see that a vector can be specified by three numbers. The components can be positive or negative; for example, if A_x were negative, then the vector $\mathbf{A}_x$ of Figure 1-7 would have a direction in the sense of decreasing values of x.

From Figure 1-7, it is seen that the magnitude of a vector can be expressed in terms of its components as

$$A=|\mathbf{A}|=\left(A_x^2+A_y^2+A_z^2\right)^{1/2} \tag{1-6}$$

In Figure 1-8, we illustrate the fact that $\mathbf{A}$ makes specific angles with respect to each of the axes; these angles α, β, γ are called the *direction angles* of $\mathbf{A}$ and are measured from the positive directions of their respective axes. Figure 1-9 shows

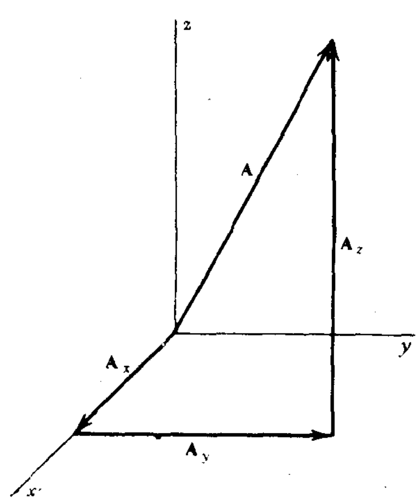

Figure 1-7. **A** is the sum of the rectangular vector components.

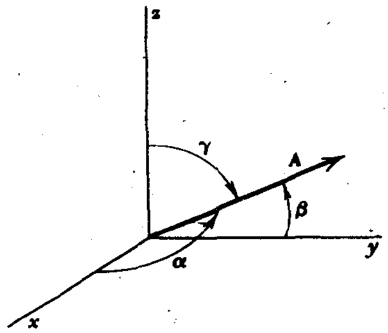

Figure 1-8. Definition of direction angles.

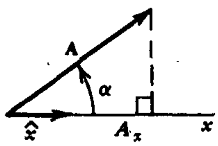

Figure 1-9. A_x is the x component of **A**.

the plane containing both **A** and $\hat{\mathbf{x}}$ and we see that A_x is given by $A_x = A\cos\alpha$. Combining this with (1-6), we get

$$l_x = \cos\alpha = \frac{A_x}{A} = \frac{A_x}{\left(A_x^2 + A_y^2 + A_z^2\right)^{1/2}} \tag{1-7}$$

where l_x is called a *direction cosine*. Similar expressions hold for the other two direction angles β and γ and their associated direction cosines l_y and l_z, so we see from (1-6) and (1-7) that, if we know the rectangular components of a vector, we can calculate its magnitude and direction.

If we now combine (1-4), (1-5), and (1-7), we find that the unit vector $\hat{\mathbf{a}}$ can also be written as

$$\hat{\mathbf{a}} = l_x\hat{\mathbf{x}} + l_y\hat{\mathbf{y}} + l_z\hat{\mathbf{z}} \tag{1-8}$$

so that the components of a unit vector in a given direction are simply the direction cosines associated with that direction. If we now apply the general result (1-6) to the specific vector $\hat{\mathbf{a}}$, we get the important relation involving direction

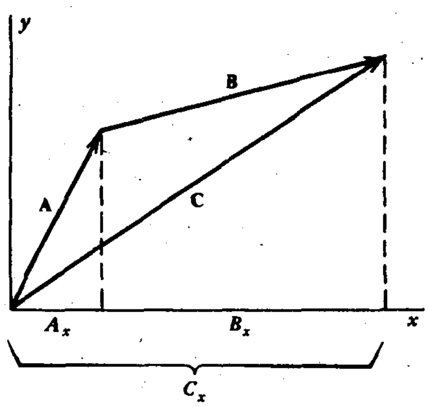

Figure 1-10. A component of a sum equals the sum of the corresponding components.