The Uncertainty Principle and Foundations of Quantum Mechanics

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The Uncertainty Principle and Foundations of Quantum Mechanics

A Fifty Years' Survey

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Dedication

PROFESSOR SIR HERMANN BONDI, K.C.B.

A remarkable factor in the progress of science is the temporary concentration of interest on particular topics. Science is above all a social activity, the picture of the lonely scientist being largely a figment of an untutored imagination. Scientists hunt in packs and follow scents. Sometimes the scent is material: when the progress of technology has opened up a whole new method of experimental work, and the ways of using this newly available technique, the assimilation of the results obtained, the formulation of novel hypotheses and their means of being tested all attract a large pack of experimenters and theorists that in full cry produces astonishingly rapidly a large and novel output. On other occasions the scent is intellectual, when an awkward question has been asked and many try to find at least partial answers to it, answers that can often lead to fruitful insights and new and vital problems.

One characteristic of this pack hunting is that if a new theory leads in rapid succession to numerous and varied experimental tests, each passed with honours, each leading to yet newer applications, then hardly anybody will stop to examine critically and logically the philosophy and internal consistency of the theory. Nobody will want to do so, because if he finds no flaw, his work will be regarded as insignificant, while if he does find a flaw, his papers will be brushed aside with the comment: 'The theory works, so there must be some fault in his argument. Why waste time to sort it out when there are so many more fascinating things to be done?' Thus foundations cannot be coolly examined until well after the main part of the pack has passed the site of the excavations, many years later.

This volume brings together many illuminating phases of one of the most exciting and successful hunts in history, the formulation of quantum theory. Not only was this hunt outstanding in the range and wealth of experimental data it covered (including the extension of the applicability of a theory founded on the spectroscopy of atoms to nuclear physics), but also in its philosophical implications. Some of them appeared early, some were later grossly misunderstood and indeed exaggerated, but many are only now starting to be fully explored and begin to come into focus because only now is the site of the excavations sufficiently unencumbered to allow for deep investigations into problems of foundations.

Nothing could be more appropriate for such a hunt than to start with Heisenberg's own description of the origin of his celebrated uncertainty relations. Now that he is—alas—no longer with us, particular value attaches to this recent recollection of the most formative phase of modern physics by one of its foremost figures.

It is splendid to observe from the many contributions in the first two parts of the volume how lively and active the subjects of foundations and of measurement theory now are in so many parts of the world. The final two parts deal with novel aspects of formal theory and of applications, where again we live in a vigorous period of activity.

I trust that many will find this book thought provoking, enjoyable and indeed fascinating.

Foreword

HANS MATTHÖFER
Bundesminister für Forschung und Technologie, F.D.R.

Great achievements on the part of researchers are often the result of their having had the courage to leave familiar ground and to explore genuinely unknown fields. The discoverer of the quantum theory and the uncertainty principle was required to leave the solid ground of classical physics. One of the most significant changes in our comprehension of the universe—a change which is reflected in fields far removed from physics—was wrought by the departure from the determinacy of physical phenomena and by far deeper-reaching relativization of the law of causality. The quantum theory and the uncertainty principle are discoveries which have changed the basis of our way of thinking. We still cannot foresee their ultimate consequences.

Werner Heisenberg, whose passing we mourned during the preparation of this book, displayed not only the courage to leave the familiar terrain of classical physics. He also possessed the spirit to defend that which has been established as true in his field of science against nationalism and racism, even in the face of the most bitter political oppression. Both during and after the Second World War, he was therefore a guarantor of another Germany which desired peace and reconciliation among the peoples of the world.

Preface

The first thirty years of the twentieth century saw an explosive development in the physical sciences, the like of which it is improbable we shall see again. Many of the discoveries were made by comparatively young men and this has provided opportunities for the international scientific community to commemorate the fiftieth anniversary of some of the more fundamental discoveries during the lifetimes of their discoverers. This book, dedicated to Professor Werner Heisenberg, is one in a series of books, each designed as a tribute to one of the founders of modern physics. While the book was organized with the cooperation of Professor Heisenberg, it is with deep regret that we learned of his death on 15th February 1976, at the age of 74, just before going to press.

This book commemorates the formulation by Heisenberg in the Spring of 1925 of the system of mechanics known as quantum (or matrix) mechanics. The subsequent development of quantum mechanics by Heisenberg with Max Born and Pascual Jordan provided the basis for modern physics. One of Heisenberg's best known and far reaching contributions to the understanding of quantum mechanics was his Uncertainty Principle, which limits the precision of measurement of the dynamic variables of a system.

While Heisenberg's decisive contribution to physics, for which he received the Nobel Prize in 1932, was made at the age of 24, he continued to advance knowledge over a wide range of subjects: nuclear and sub-nuclear physics, S-matrix theory, solid state theory, plasma and thermonuclear physics, unified field theory, etc.

In compiling this volume, the editors have again been fortunate in securing the help and cooperation of scientists throughout the world. The aims were essentially similar to those of *Wave Mechanics, the First Fifty Years* (a tribute to Professor Louis de Broglie on the fiftieth anniversary of the discovery of the wave nature of the electron); to review aspects of the philosophical implications, past and current thinking and potential future developments in physics stemming from the fundamental discoveries associated with, in this case, Werner Heisenberg.

The Editors wish to record their thanks to the University of London King's College, for the facilities provided and to Professor David Bohm, Dr. R. J. Griffiths and Dr M. P. Melrose for reading various sections of the manuscript and for making helpful comments.

February 1976

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PART 1

Quantum Uncertainty Description

1

Remarks on the Origin of the Relations of Uncertainty

The late Professor WERNER HEISENBERG
Director Emeritus of the Max Planck Institut für Physik und Astrophysik, Munich, Germany

The situation of quantum theory in the summer of 1926 can be characterized by two statements. The mathematical equivalence of matrix mechanics and wave mechanics had been demonstrated by Schrödinger, the consistency of the mathematical scheme could scarcely be doubted; but the physical interpretation of this formalism was still quite controversial. Schrödinger, following the original ideas of de Broglie, tried to compare the 'matter waves' with electromagnetic waves, to consider them as real, measurable waves in three-dimensional space. Therefore he preferred to discuss those cases where the configuration space had only three dimensions (one-particle systems), and he hoped, that the 'irrational' features of quantum theory, especially quantum 'jumps', could be completely avoided in wave mechanics. The stationary states of a system were defined as standing waves, their energy was really the frequency of the waves. Born on the other hand had used the configuration space of Schrödinger's theory to describe collision processes and he took the square of the wave amplitude in configuration space as the probability of finding a particle. So he emphasized the statistical character of quantum theory without attempting to describe what 'really happens' in space and time.

Schrödinger's attempt appealed to many physicists who were not willing to accept the paradoxes of quantum theory; but the discussions with him in July 1926 in Munich and in September in Copenhagen demonstrated very soon, that such a 'continuous' interpretation of wave mechanics could not even explain Planck's law of heat radiation. Since Schrödinger was not quite convinced it seemed to me extremely important to decide beyond any doubt whether or not quantum 'jumps' were an unavoidable consequence, if one accepted that part of the interpretation of matrix mechanics, which already at that time was *not* controversial, namely the assumption that the diagonal element of a matrix represents the time average of the corresponding physical variable in the stationary state considered. Therefore I discussed a system consisting of two atoms in resonance. The energy difference between two specified consecutive stationary states was assumed to be equal in the two atoms so that for the same total energy the first atom could be in the upper and the second in the lower state or vice versa. If the interaction between the two atoms is very small one should expect that the energy goes slowly forth and back between the two atoms. In this case it can easily be decided whether the energy of one of the atoms goes continuously from the upper to the lower state and back again or discontinuously by means of sudden quantum jumps. If E is the energy of this one atom then the mean square of fluctuations $\overline{\Delta E^2}$ is quite different in the two cases [equation (1)]. The calculation does not require more than the *non*-controversial assumption of matrix mechanics mentioned above. The result decided clearly in favour of the quantum jumps and against the continuous change.

$$\overline{\Delta E^2} = \overline{(E-\bar{E})^2} = \overline{E^2} - \bar{E}^2 \tag{1}$$

The success of this calculation seemed to indicate, that the non-controversial part of the interpretation of quantum mechanics should already determine uniquely the complete interpretation of the mathematical scheme, and I was convinced that there was no room left for any new assumptions in the interpretation. In fact, in the example mentioned above the square of the elements of that matrix, which transformed from the state where the total energy of the system was diagonal to the state where the energy of the one atom was diagonal, had to be considered as the corresponding probability. In the autumn of 1926 Dirac and Jordan formulated the theory of those general linear transformations which corresponded to the canonical transformations of classical mechanics and which nowadays are called the unitary transformations in Hilbert space. These authors correctly interpreted the square of the elements of the transformation matrix as the corresponding probability; this was in line with Born's older assumptions concerning the square of Schrödinger's wave function in configuration space and with the example of the resonating atoms. It was in fact the only assumption which was compatible with the old non-controversial part of the interpretation of quantum mechanics; so it seemed that the correct interpretation of the mathematical theory had finally been given.

But was it really an interpretation, was the mathematical scheme a theory of the phenomena? In physics we observe phenomena in space and time; the theory should enable us, starting from the present observation, to predict the further development of the phenomenon concerned. But at this point the real difficulties started. We observe phenomena in space and time, not in configuration space or in Hilbert space. How can we translate the result of an observation into the mathematical scheme? E.g. we observe an electron in a cloud chamber moving in a certain direction with a certain velocity; how should this fact be expressed in the mathematical language of quantum mechanics? The answer to this question was not known at the end of 1926.

For some time Schrödinger had discussed the possibility, that a wave packet obeying his wave equation could represent an electron. But as a rule a wave packet spreads out so that after some time it may be extended over a volume much bigger than that of the electron. In nature, however, an electron remains an electron; so this interpretation would not do. Schrödinger pointed out, that in one special case, the harmonic oscillator, the wave packet did not spread; but this property had to do with the special fact, that for the harmonic oscillator the frequency does not depend on the amplitude.

On the other hand there could be no doubt that de Broglie's and Schrödinger's picture of the three-dimensional matter waves did contain some truth. In the many discussions we had in Copenhagen during the months after Schrödinger's visit it was primarily Bohr who emphasized this point again and again. But what does this term 'some truth' mean? We had already too many statements which contained 'some truth'. We could, for example, compare the statements: 'The electron moves in an orbit around the nucleus.' 'The electron moves on a visible path through the cloud chamber.' 'The electron source emits a matter wave which can produce interferences in crystals like a light wave.' Each of these statements seemed to be partly true and partly not true, and certainly they did not fit together. We got the definite impression that the language we used for the description of the phenomena was not quite adequate. At the same time we saw that at least in some experiments such concepts as position or velocity of the electron, wavelength, energy had a precise meaning, their counterpart in nature could be measured very accurately. It turned out that for a well defined experimental situation we finally always arrived at the same prediction, though Bohr preferred to play between the particle- and wave-picture while I tried to use the mathematical scheme and its probabilistic interpretation. Still we were not able to get complete clarity; but we understood that the 'well defined experimental situation' somehow played an important rôle in the prediction.

In the beginning of 1927 I was for some weeks alone in Copenhagen, Bohr had gone to Norway for a skiing holiday. In this time I concentrated all my efforts on the question: How can the path of an electron in a cloud chamber be represented in the mathematical scheme of quantum mechanics? In the despair about the futility of my attempts I remembered a discussion with Einstein and his remark: 'it is the theory which decides what can be observed'. Therefore I tried to turn around the question. Is it perhaps true that only such situations occur in nature or in the experiments which can be represented in the mathematical scheme of quantum mechanics? That meant: there was not a real path of the electron in the cloud chamber. There was a sequence of water droplets. Each droplet determined inaccurately the position of the electron, and the velocity could be deduced inaccurately from the sequence of droplets. Such a situation could actually be represented in the mathematical scheme; the calculation gave a lower limit for the product of the inaccuracies of position and momentum.

It remained to be demonstrated that the result of any well defined observation would obey this relation of uncertainty. Many experiments were discussed, and Bohr again used successfully the two pictures, wave- and particle-picture, in the analysis. The results confirmed the validity of the relations of uncertainty; but in some way this outcome could be considered as trivial. Because if the process of observation itself is subject to the laws of quantum theory, it must be possible to represent its result in the mathematical scheme of this theory. But these discussions demonstrated at least that the way in which quantum theory was used in the analysis of the observations, was completely compatible with the mathematical scheme.

The main point in this new interpretation of quantum theory was the limitation in the applicability of the classical concepts. This limitation is in fact general and well defined; it applies to concepts of the particle picture, like position, velocity, energy, as well as to concepts of the wave picture like amplitude, wave length, density. In this connection it was very satisfactory that somewhat later Jordan, Klein and Wigner were able to show that Schrödinger's three-dimensional wave picture could also be subject to the process of quantization and was then—and only then—mathematically equivalent to quantum mechanics. The flexibility of the mathematical scheme illustrated Bohr's concept of complementarity. By this term 'complementarity' Bohr intended to characterize the fact that the same phenomenon can sometimes be described by very different, possibly even contradictory pictures, which are complementary in the sense that both pictures are necessary if the 'quantum' character of the phenomenon shall be made visible. The contradictions disappear when the limitation in the concepts are taken properly into account. So we spoke about the complementarity between wave picture and particle picture, or between the concepts of position and velocity. In later literature, there have been attempts to give a very precise meaning to this concept of complementarity. But it is at least not in the spirit of our discussions in the Copenhagen of 1927 if the unavoidable lack of precision in our language shall be described with extreme precision.

There have been other attempts to replace the traditional language of physics with its classical concepts for the description of the phenomena, by a new language which should be better adapted to the mathematical formalism of quantum theory. But the development of language is a historical process, and artificial languages like Esperanto have never been very successful. Actually, during the past 50 years, physicists have preferred to use the traditional language in describing their experiments with the precaution that the limitations given by the relations of uncertainty should always be kept in mind. A more precise language has not been developed, and it is in fact not needed, since there seems to be general agreement about the conclusions and predictions drawn from any given experiment in this field.

2

In Praise of Uncertainty

GORDON REECE
Imperial College, London

1. THE PSYCHOLOGICAL BASIS OF OUR NEED FOR CERTAINTY

The first post-natal experiences of a human being are necessarily associated with learning about the world in which he or she lives. Ideally, his emotional needs will be satisfied in much the same way as his physical requirements. Indeed, these various aspects are inextricably intertwined, centring on the mother's breast, which supplies at once food, warmth, reassurance and companionship.

From the point of view of a very young baby, the idea of contentment cannot be separated from his confidence in the consistency and reliability of the world as he sees it. For him, happiness means the certainty that his food will arrive when he needs it, at the correct temperature and of a reliable composition.

Later he becomes aware of non-animate objects, some of which fail to interact with him (passive objects like floors and walls), while others (like mattresses, blankets and rattles) respond when pushed or shaken. Gradually, a baby builds up a library of objects in which he can have confidence. Floors can safely be crawled on; thin air cannot. Walls can be bumped without apparent damage (to the walls) while balls and bottles roll away when pushed. He learns to categorize the objects around him. Fine gradations are learned from the varying degrees of, for example, softness of floor coverings, and intensities of light, noise and warmth. None of these distinctions, however, rivals the fundamental importance of simple 'yes/no' questions such as 'Am I hungry?' or 'Am I wet?' It is not until a baby is much older—say a year, when his feelings about the world will already have begun to gel—that he begins to confuse the issue with questions like 'Am I very hungry?'

The real source of the baby's confidence in the external world is the certainty that if something is wrong it will be remedied. Uncertainty ('Where is Mummy?', 'Where am I?' or 'Why am I still hungry?') represents insecurity, a loss of confidence in the external world and consequent unhappiness. The baby's confidence relies also on a belief in causality: 'If I cry, then Mummy will come', 'If I get milk, then I shall no longer be hungry', and the action of crying represents this reliance.