COLLEGE ALGEBRA AND TRIGONOMETRY

MICHAEL SULLIVAN

College Algebra and Trigonometry

Third Edition

Michael Sullivan

Chicago State University

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Maxwell Macmillan International New York Oxford Singapore Sydney On the cover: A painting from a series, "Stations of the Cross," by California artist Tony Ligamari. The work is acrylic on canvas and measures 3 ft \times 2 ft. The artist's use of vivid colors and hard-edge abstraction relates him to the abstract expressionist movement in California. He uses a masking technique to create negative space that gives a sense of depth and movement. His Italian heritage is evident in the subject matter he has chosen to abstract. Ligamari's work is in several private collections; he was a visiting artist at the American Academy in Rome in 1986 and has had shows in London, Florence, and Copenhagen as well as in the United States. He is represented by the John Natsoulas Gallery in Davis, California.

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Preface

Intent/Purpose

As a professor at an urban public university for over 25 years, I am aware of the varied needs of students—students who range from being less well-prepared and insufficiently motivated to those who are well-prepared and highly motivated. As the author of an engineering calculus text as well as texts in finite mathematics and business mathematics, I understand what students must know in order to be successful in such courses. At the same time, I am also aware of, and very concerned about, the student who has decided this course in algebra and trigonometry is to be a terminal mathematics course. Based on these experiences, I have written a book to serve students with varied backgrounds and goals.

For the student who requires it, a review of basic material is contained in Chapter 1. To help further, concepts are viewed from different perspectives whenever possible, and alternate methods of solutions are given. Every effort has been made to be clear, precise, and consistent. Whenever it seemed appropriate, special encouragement has been offered; and whenever necessary, warnings have been given. To provide motivation, I have included understandable, realistic applications that are consistent with the abilities of the student. The problem sets have been carefully graded to build the student's confidence.

At the same time, a conscious attempt has been made to preview material that will be seen by students who intend to take courses such as finite mathematics, business mathematics, or engineering calculus. These previews relate to important concepts, applications, examples, and exercises that will be encountered in later courses and can be viewed now as algebra or trigonometry problems.

To summarize, my purpose in writing this book is to serve the different needs of each type of student through suitable motivation and material.

New to This Edition

The elements of the previous editions that proved so successful remain in this edition. Nevertheless, many changes—some obvious, others subtle—have been made. Virtually every change is the result of thoughtful comments and suggestions from colleagues and students who used the previous editions. As a result of this input, for which I am sincerely grateful, this edition will be an improved teaching device for professors and a better learning tool for students.

Student Involvement In this edition, a determined effort has been made to motivate and involve students. Recent events have been used whenever possible for added interest. An increased use of visualization in the examples and exercises should help students see what is happening and thereby help them understand the underlying ideas that are being presented. To further help the students, each chapter begins with a list of topics to be reviewed in preparation for the chapter. Within each section, at selected places marked by __, the student is encouraged to pause, work a problem related to the material just presented, and determine whether the material has been mastered before going on to the next topic. At the end of each chapter, a new component has been added, called "Sharpening Your Reasoning: Discussion/Writing/Research." This component consists of a variety of questions—many open-ended—to encourage discussion, critical thinking, writing projects, and library work.

Calculators/Graphing Calculators This edition continues to encourage the use of a scientific calculator when one is required or helpful. For those who have access to a graphing calculator, this edition also contains comments, checks, and exploratory problems following selected examples to enhance and/or reinforce the material being presented. Exercises that are especially suited to the use of a graphing calculator also have been added.

Revised/Reorganized Material

The discussion of division of polynomials now appears in Chapter 1. The discussion involving the graph of a function now appears in the first section of Chapter 4. The first two sections of this chapter have been reorganized so that the graph of a function appears sooner and so that some of the more difficult ideas are postponed.

A more thorough approach to the graph of a polynomial function is given in Section 5.3

Section 7.4 now contains a discussion on solving right triangles, including using a calculator to find the angles. Several new applied problems are presented.

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Section 12.1 is now motivated by looking at the graphs of two linear equations containing two variables—the visual approach being preferred as a starting point.

Illustrations Many new illustrations have been inserted, especially in the exercises, to assist the student in visualizing concepts and to reinforce the graphical orientation of the text. This edition now contains over 1200 illustrations.

Examples New examples have been inserted where the level of content or the diversity of approach warrants. After selected examples, a reference to the graphing calculator is given that comments on the graphing calculator or asks the student to check or explore results. This edition now contains over 700 examples.

Exercises Many new exercises have been added. Many of these are visual in nature, asking the student to draw conclusions from a graph. Others are problems that require a graphing calculator. These have been selected to provide insights that are attainable using a graphing calculator. This edition now contains over 5500 exercises. As part of the chapter review section at the end of each chapter, "True/False Items" have been included. A more significant addition to the chapter review is the inclusion of a new section titled "Sharpening Your Reasoning: Discussion/Writing/Research." This section consists of a variety of relevant mathematical questions designed to stimulate classroom discussion, critical thinking, writing projects, and library research projects.

Contents and Organization

The topics contained in this book were selected and organized with the different types of students who take this course in mind. Thus, throughout the book, a deliberate attempt is made to preview topics and techniques that will be used in later courses.

The first part of the book, Chapters 1–3, provides a detailed presentation of material needed for later topics. Chapter 1 mostly consists of review material. Although complex numbers are introduced in Section 1.9, this section may be postponed at the discretion of the instructor. If the instructor chooses to cover complex numbers later, Section 2.4, "Quadratic Equations with a Negative Discriminant," also can be postponed and then covered along with Section 1.9. Whether complex numbers are covered early or late, all exercises dependent upon complex numbers are clearly identified in the directions for the exercise.

The coverage of geometry in Chapter 1 should also be noted. No review would be complete without a discussion of the Pythagorean Theorem and the formulas for perimeter and area of a rectangle, area and circumference of a circle, and volume of a sphere.

For some students, Chapters 2 and 3 will represent new material; for others, they will be review. The large majority, of course, fall somewhere in between. In Section 2.3, quadratic equations are treated solely from the real number point of view, while in Section 2.4, complex number solutions are discussed. This ensures flexibility of coverage for the instructor and clarity of understanding for the student.

The middle part of the book, Chapters 4–10, covers the essential topics of college algebra and trigonometry. In Chapter 4, special emphasis is placed on functions and the graphs of functions. Graphing is usually done in steps, all of which are illustrated. The graphing techniques introduced in Chapter 4 are utilized and reinforced in Chapters 5, 6, and 8 using the new functions introduced there.

Because of the need to evaluate polynomial functions, both synthetic division and the nested-form technique are utilized in Chapter 5 as alternatives to substitution. The nested-form technique will be especially appreciated by students of computer science. Additional methods for finding the zeros of a polynomial, such as the Rational Root Theorem and Descartes' Rule of Signs, are introduced to facilitate the graphing of polynomial and rational functions. Approximating results, such as upper and lower bounds on zeros and partition techniques, are discussed in a separate section to provide a clear sense of the difference between numerical and nonnumerical methods.

Chapter 6 treats the exponential and logarithmic functions in the detail necessary and with language consistent for subsequent use in calculus. Applications to compound interest, earthquake magnitude, and growth and decay are among those given.

With Chapter 7, we begin the study of trigonometry. The trigonometric functions are introduced using the right triangle approach, followed by the trigonometric functions of a general angle. Following a section on applications, the unit circle approach and the properties of the trigonometric functions are discussed. (For those who prefer to introduce the trigonometric functions using the unit circle approach, see the description of *Precalculus*, Third Edition, on page ii.)

Chapter 8 continues the study of trigonometric functions with a detailed presentation of their graphs. Here, we use the graphing concepts introduced in Chapter 4, as well as some additional techniques. A discussion of inverse trigonometric functions rounds out the chapter. Chapter 9 treats the analytic side of trigonometry: identities; sum and difference, double-angle, half-angle, sum-to-product, and product-to-sum formulas; and trigonometric equations.

In Chapter 10, additional applications of trigonometry to solving a general triangle are given, including a rather complete discussion of the area of a triangle. Two full sections have been devoted to polar coordinates and graphing polar equations to provide a thorough development of this subject. The final section treats DeMoivre's Theorem.

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Chapter 11 contains topics from analytic geometry. Beginning with a detailed presentation of the conics (Sections 11.1–11.4), the parabola, ellipse, and hyperbola are defined using geometric (distance-based) means. After a discussion of rotation of axes and the general form of a conic, a unified definition of the conics using eccentricity is given. Lastly, plane curves and parametric equations are studied.

Chapter 12 discusses a variety of methods (four in all) for solving systems of linear equations. Sections on systems of nonlinear equations and systems of linear inequalities are also included.

The remaining chapters of the book cover topics that may be selected according to the specific needs of the students. For example, Chapter 13 presents mathematical induction, the Binomial Theorem, and arithmetic and geometric sequences; Chapter 14 discusses sets, counting, and probability. Each of these chapters contains applications of interest to the student in computer science, business, or mathematics.

Finally, Chapter 15 includes an introduction to matrices and linear programming, topics of use to students intent on taking finite mathematics. Also included are sections on vectors and the dot product that would be of help to students who may go on to study calculus. Note that partial fraction decomposition is presented in Section 15.3, after systems of equations have been covered, rather than following rational functions in Chapter 5. In this way, it provides not only an application for solving systems of linear equations, but also allows for a full and detailed discussion. This should prove useful to students going on to calculus.

Applications

As we mentioned earlier, every opportunity has been taken to present understandable, realistic applications consistent with the abilities of the student, drawing from such sources as tax rate tables, the *Guinness Book of World Records*, and newspaper articles. For added interest, some of the applied exercises have been adapted from textbooks the students may be using in other courses (for example, economics, chemistry, physics, etc.). See, for example, Sections 2.2, 4.6, 7.4, and 12.2.

Historical Notes

William Schulz of Northern Arizona University has provided historical context and information in anecdotes that appear as introductory material and at the ends of many sections. In some cases, these comments also include exercises and discussion of comparative techniques. See, for example, the problems at the end of Exercise 5.5.

Examples, Exercises, and Illustrations

The text includes over 700 examples and 5500 exercises, of which over 900 are applied problems. The examples are worked out in appropriate detail, starting with simple, reasonable problems and working gradually up to more challenging ones. Selected examples that allow for checking or exploration using a graphing calculator are indicated by the symbol c

Exercises are numerous, well-balanced, and graduated. They usually begin with problems designed to build confidence, continue with drill-type problems that mimic worked out examples, and conclude with problems that are more challenging. Many of the beginning problems in an exercise set are visual in character—such as showing a graph and asking for conclusions. They also include a number of exercises where the student will need a scientific calculator for their solution; these are clearly marked with the symbol C. Others are marked with the symbol C to indicate that a graphing calculator is required. Answers are given in the back of the book for all the odd-numbered exercises.

Illustrations are abundant, numbering over 1200. Full use is made of a second color to help clarify and highlight.

A Word about Format and Design

Each chapter-opening page contains a list of topics the student should review before beginning the chapter, a table of contents for the chapter, and an overview—often historical—of the contents.

New terms appear in boldface type where they are defined. The most important definitions are shown in color.

Theorems are set with the word "Theorem" in the margin for easy identification; if it is a named theorem, the name also appears in the margin. When a theorem has a proof given, the word "Proof" appears in the margin to mark clearly the beginning of the proof, and the symbol is used to indicate the end of the proof.

All important formulas and procedures are enclosed by a box and shown in color.

Examples are numbered within each section and identified clearly with the word "Example" in the margin. The solution to each example appears immediately following the example with the word "Solution" in the margin to identify it. The symbol
indicates the end of each solution.

Within each section, several references, marked by a \square , are given that direct the student to "Now work Problem" These are provided so that the student can pause, work a problem based on the material

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just presented, and then ascertain whether the subject matter has been mastered.

Each section ends with an exercise set. Each chapter ends with a chapter review containing a list of "Things to Know" (a summary of the key elements of the chapter), a selection of fill-in-the-blank items and true/false items (to test vocabulary and formulas), a set of review exercises, and a collection of questions to encourage classroom discussion, critical thinking, written projects, and library research.

Supplementary Material

For the Instructor:

Instructor's Solutions Manual to Accompany College Algebra and Trigonometry, by Katy Murphy and Michael Sullivan, contains worked-out solutions to all the even-numbered problems. If used in conjunction with the Student Solutions Manual, the instructor will have worked-out solutions for every problem in the book.

Instructor's Resource Material contains approximately 150 transparency masters that duplicate important illustrations in the text.

Instructor's Computer Test Generator, DellenTest 4.0 (MS-DOS version) and DellenTest MAC 2.0, contain 25-30 model test questions for the objectives of each chapter in the text. These test generators are available in both free response and multiple choice formats. Carefully constructed algorithms use random number generators to produce different yet equivalent versions of these model questions each time they are printed. By using these test generators it is possible to create an unlimited number of test items comparable to the question models, each different from the other yet each testing the same concept at the same level. This system thus provides a virtually unlimited number of quizzes, chapter tests, or final examinations. The system also generates an answer key and student worksheet with an answer column that exactly matches the column on the answer key. Graphing grids are included for all problems requiring graphs. The multiple choice versions of DellenTest 4.0 and DellenTest MAC 2.0 utilize distractors that are representative of commonly made student errors. These versions are machine-gradable.

DellenTest 4.0 utilizes MS-DOS and will produce high-quality output on an IBM-compatible dot matrix printer, a Hewlett-Packard Laserjet II® compatible laser printer, or a DeskJet® printer. DellenTest MAC 2.0 is available for the Macintosh with a minimum of 1 MEG RAM running under system 6.0 or higher. Both MS-DOS and Macintosh platforms incorporate a unique editing function that allows the instructor to create additional problems or to alter any of

the existing problems in the test generator using a full set of mathematics notation.

Both DellenTest 4.0 and DellenTest MAC 2.0 are accompanied by user notes and annotated printouts of the test models on a chapter-by-chapter basis.

For the Student:

Student Solutions Manual to Accompany College Algebra and Trigonometry, by Katy Murphy and Michael Sullivan, contains complete step-by-step solutions to all the odd-numbered problems.

Acknowledgments

Textbooks are written by an author, but evolve from an idea into final form through the efforts of many people. Before initial writing began, a survey was conducted which drew nearly 250 instructor responses. The manuscript then underwent a thorough and lengthy review process, including class-testing at Chicago State University. I would like to thank my colleagues and students at Chicago State, who cooperated and contributed to this text while it was being class-tested.

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